

# **GAS TURBINE DESIGN**

## **A SIMPLE EXPLANATION OF THE BASIC PRINCIPLES**

BY

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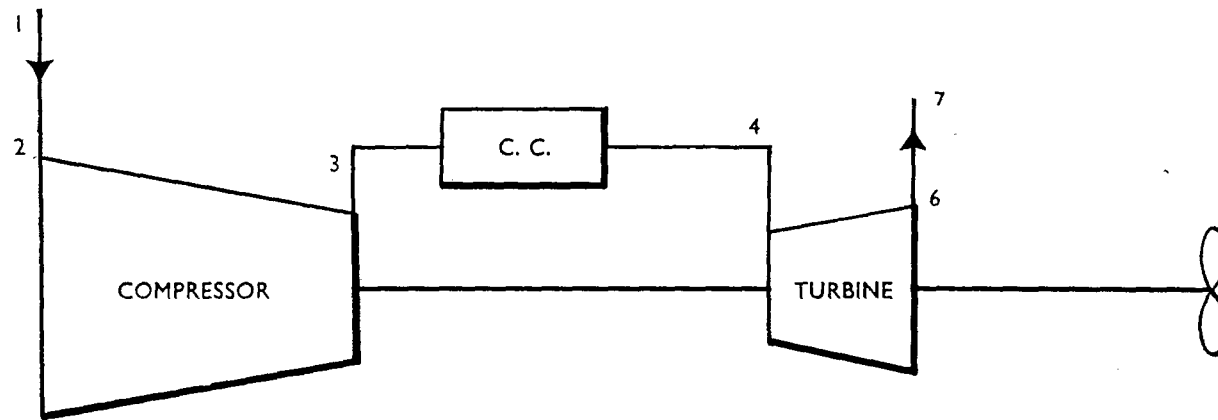
It is thought that those who have not been particularly concerned with gas turbines may be interested in a description of the considerations in the first stages of the design of a simple engine. It is hoped that this description builds up, in logical steps from basic theory, the cycle details of a boost engine. In explanation of the layout, the first part explains how the practical engine differs from the ideal and derives a simple expression for the efficiency and the second examines the elementary design of the engine. An appendix shows the derivation of the figures.

### **PART I**

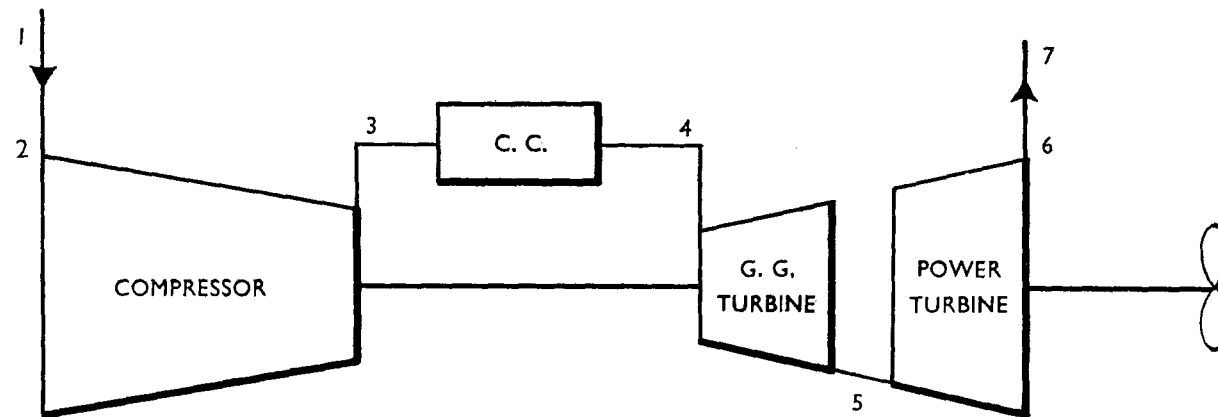
#### **WHAT IS A GAS TURBINE ?**

Like the majority of other prime-movers, a gas turbine is a device for transforming the chemical energy of fuel into useful mechanical power. The two usual methods of doing this are by external and internal combustion engines. The former includes the steam turbine installation and the closed-cycle gas turbine, which burn the fuel in a furnace and transfer the heat in a boiler to the working fluid which is circulated round a closed system. The latter includes the Diesel and petrol engines and the open-cycle gas turbine, which burn the fuel in the working fluid. The closed-cycle engine has only a limited application, and because of the large weight and size of the boiler and cooler, it is unlikely to be used in naval service, and it is not, therefore, proposed to discuss it further.

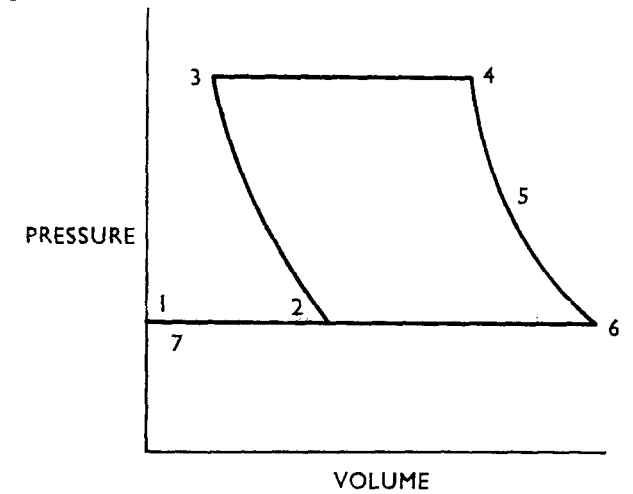
An open-cycle gas turbine performs the processes of induction, compression, heating, expansion and exhaust, in the same way as the Diesel engine, but



(a)



(b)



(c)

there the similarity ends. In the Diesel the processes are intermittent and all take place in the same part of the engine, the cylinder. It can be considered as a batch process in which a sample of air is taken in and has a series of operations performed upon it, during one of which it delivers power. Each batch of air delivers power for only a fraction of the time that it is in the engine and multiple cylinders are normally used to give some approximation to a steady output. An advantage of the intermittent flow is that the peak temperature occurs for only a very short time, and, since the metal does not have time to heat up, the maximum temperature can be much higher than could be accepted for a steady temperature.

The gas turbine is a constant flow machine in which each of the processes takes place in its own part of the engine. Each process is continuous and, while a given sample of air only produces power for part of its time in the engine, some of the air is always producing power. The output is therefore steady and a multiplicity of turbines or a flywheel is not necessary. Since the temperatures are steady, the maximum must be limited to what the metal can stand and it is necessary to have considerable quantities of excess air for this purpose.

The use of the word 'turbine' to describe the whole engine is most unfortunate. To be accurate, the turbine is only that part of the plant which actually produces power. One doesn't think of a boiler as being part of a steam turbine, yet a combustion chamber is decreed part of a gas turbine.

### What Happens to the Air

Stripped of all its trimmings an engine consists of a compressor, a combustion chamber, a turbine, and the necessary inter-connecting ducts. The turbine drives the compressor and has a surplus of power, usually about one third of the total, as useful output. This is shown in FIG. 1 (a), and usually referred to as a simple cycle. The first advance is to divide the turbine into two as shown in FIG. 1(b). The compressor, combustion chamber and H.P. turbine unit are known as the gas generator and may run at a different speed to the L.P. or power turbine. This is of particular value in a propulsion engine as the shaft may be stopped and the gas generator left idling, which it does at about 60 per cent speed.

FIGS. 1 (a) and (b) show flow diagrams for the simple cycle in the single and two-shaft forms, and (c) shows an ideal simple cycle on a pressure-volume diagram. The numbers shown in FIG. 1(c) will be used as suffixes to denote the different points in the cycle. By 'ideal' is meant a cycle without losses, but this does not mean that it is perfectly efficient, because it is impossible to transform heat completely into mechanical energy and there is an inherent inefficiency. The additional losses which occur in practice, compared with the ideal cycle, will be considered in some detail later.

The simple cycle can be made complex by the addition of various additional components. A heat exchanger can be fitted between the compressor and the combustion chamber and used to extract some of the exhaust heat ; the compression may be done in more than one cylinder, with inter-coolers between ; reheaters may be fitted between the turbines. These are all designed to increase the efficiency, or give other advantages, at the cost of increased weight and complication.

The five processes mentioned above are shown in FIG. 1 and are :—

Induction— 1.2—In the inlet duct and sometimes a silencer. In practice it involves a pressure loss, so giving a sub-atmospheric pressure at the compressor inlet.

Compression—2.3—In the ideal cycle is isentropic but in practice is usually 80–85 per cent efficient.

Heating— 3.4—By direct burning of the fuel in air. In the ideal cycle is at constant pressure, but in practice a loss occurs, a large proportion of which goes to producing the air distribution and turbulence necessary to give good combustion. The thermal efficiency of combustion is nearly 100 per cent.

Expansion— 4.5.6—In the ideal cycle is isentropic but in practice is usually 85–90 per cent efficient. Note that it is usual to quote the turbine efficiency related to Total conditions at inlet and outlet. (Total conditions include velocity head). This is different from steam turbine practice, where the leaving loss is counted as an inefficiency of the turbine. This difference often precludes simple comparison between quoted efficiencies of steam and gas turbines.

Exhaust— 6.7—In the exhaust ducting and in practice involving a pressure loss, thus giving a back-pressure to the turbine.

Examination of the above processes show that the engine operates on the constant pressure cycle, which is well known in the elementary theory of thermodynamics.

### Output and Efficiency

The maximum possible efficiency of the constant pressure cycle is given by the air standard efficiency :—

$$1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}$$

where :  $r$  = the pressure ratio

$\gamma$  = the isentropic coefficient for air

At this efficiency, the work output is equal to the difference in the isentropic work of the turbine and compressor. If the compressor absorbs  $W$  work units per pound of air flow, then the ideal output is  $KW - W$ , where  $K$  is the work ratio, turbine to compressor. If  $T$  is the absolute temperature and the specific heat is constant then :—

$$K = \frac{T_4 - T_7}{T_3 - T_2} = \frac{T_4}{T_3} \cdot \frac{1 - \frac{T_7}{T_4}}{1 - \frac{T_2}{T_3}} = \frac{T_4}{T_3}$$

$$\text{Since } \frac{T_7}{T_4} = \left(\frac{P_7}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_3}$$

That is,  $K$  is the ratio of the maximum cycle temperature to the compressor outlet temperature. Since it was derived for the ideal cycle it is the isentropic compression temperature in the ratio, denoted  $T'_3$ , and not the actual outlet temperature which in practice is somewhat higher due to the inefficiency of the compression process. For a given inlet temperature  $T'_3$  is dependent only on the compression ratio, so for a given pressure ratio  $K$  is proportional to the maximum cycle temperature.

No machine is perfect, and at present, the gas turbine is very far from it. The work absorbed by the compressor is increased, and that delivered by the turbine, reduced, compared with the ideal (isentropic). The actual work output is given by  $\eta_T KW = \frac{W}{\eta_C}$ , where  $\eta_C$  and  $\eta_T$  are the isentropic efficiencies of the compressor and turbine respectively. The fraction of the ideal work is therefore :—  $\frac{\eta_T \eta_C K - 1}{\eta_C (K - 1)}$

It follows that, for there to be any useful output at all,  $\eta_T \eta_C K$  must be greater than unity. Here is the reason that a successful gas turbine was not built earlier. To increase  $K$ , the maximum cycle temperature must be increased, and this was delayed until the metallurgists developed suitable heat-resisting alloys. Turbine, and particularly compressor, efficiencies were too low until advanced aerodynamic knowledge was obtained. An average value of  $K$  is between 2 and 2.5, which means that the component efficiencies must be at least 65 per cent. In practice they must be over 70 per cent to allow for losses in the rest of the engine, such as pressure loss in ducts ; friction in bearings ; windage, etc., which will be considered later.

It is interesting to note the different effects that variations in the turbine and compressor efficiencies have on the overall efficiency. If  $K$  is 2 and the component efficiencies are 80 per cent, the output is 35 per cent of the ideal. If the turbine efficiency is increased to 81 per cent the output is 37 per cent, but if the compressor is improved to 81 per cent instead, the work output only goes up to 36.5 per cent. This greater effect of the turbine efficiency is due to the fact that the turbine power is about 50 per cent greater than the compressor power, and any improvement in performance has a proportionately greater effect on output.

At first it seems paradoxical that the first successful gas turbine was made possible by advances in compressor design. However, this was because the compressor was relatively new ground while the turbine designers had steam turbine experience to draw on. Today turbines are usually more efficient than their associated compressors, but it does not follow that the thermodynamic design is necessarily better. If the stage efficiencies are the same, then the designs are about equally successful, but unfortunately in the compressor the reheat results in an overall efficiency lower than the stage efficiency, while in the turbine it works the other way and the overall efficiency is higher than the stage efficiency. A further important factor is that the compression process of diffusing velocity energy into pressure energy is inherently more difficult than the converse used in the turbine.

### Effect of Maximum Temperature and Pressure

It has been shown that the efficiency is given approximately by the expression :—

$$\frac{\eta_T \eta_C K - 1}{\eta_C (K - 1)} \left[ 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right] \quad \text{where } K = \frac{T_{\max}}{T_3}$$

If the maximum temperature in the cycle is increased, then  $K$  will increase and so will the efficiency. It follows that as high a temperature as possible should be used and, in practice, it is selected by considering the required life of the engine with the creep properties of the materials available.

An increase in pressure ratio will increase the air standard efficiency (i.e. the second term in the expression), but at the same time it increases the compression temperature and hence reduces  $K$ . Once the maximum temperature

has been selected, the efficiency can be plotted against pressure ratio and the optimum obtained. If efficiency is all important then this optimum ratio must be used, but for reasons which will be seen later a different ratio may be used in practice.

In a complex cycle the above is true in principle but the optimum pressure ratio will also depend on the degree of heat exchange and inter-cooling chosen.

### **Additional Losses in the Cycle**

So far, the only losses considered have been those in the compressor and turbine, which are conveniently expressed as isentropic efficiencies. There are quite a number of other losses in an engine, and the most important will now be considered.

### **Pressure Losses**

These occur in every part of the cycle, and the more complex it is the more places there are for pressure losses to occur. If space is limited the losses in the ducting have a very great effect on deciding the extent to which extra components, such as inter-coolers and multiple spool compression, can be introduced.

Pressure losses are easily allowed for in a detailed cycle calculation (see Part II), and in the simple expression for efficiency a good approximation is possible. No matter where in the cycle a pressure loss occurs, its effect on the whole cycle is dependent on its percentage of the total pressure at that point. If all losses are expressed as percentages and added together, then the pressure ratio in the expression can be reduced by the same amount. In a simple cycle the total pressure loss will be of the order of eleven or twelve per cent, and it increases with the complexity of the cycle.

For the first attempt at the cycle calculation an estimate of the pressure losses must be made. The results of this calculation enable first order estimates of the flow areas and velocities to be made and pressure losses can be calculated reasonably accurately. A more accurate evaluation of the cycle can then be made.

A loss in the inlet has the same effect on efficiency as a similar loss in the exhaust, but a loss in the inlet reduces the pressure level throughout the engine, thus increasing the specific volume of the gas and increasing the engine size and weight for a given output. It is particularly unfortunate that silencers and their associated pressure losses usually have to be placed in the inlet to mask the compressor noise and not, like the Diesel engine, in the exhaust.

There is usually an appreciable pressure loss in the combustion chamber, most of which goes to produce the turbulence so necessary to good combustion.

### **Friction Losses**

The friction losses in the main gas stream are allowed for in the component isentropic efficiencies and the pressure losses. In addition, there are two other important friction losses—bearing friction and disc windage. There can be no simple correction made to the approximate expression for efficiency, but the actual horse power loss can be estimated fairly accurately and during the cycle calculation a percentage can be added to the compressor power and deducted from the turbine output.

### **Auxiliary Losses**

It is usual to have a number of auxiliaries driven off the compressor shaft. These may include any, or all, of the following :—fuel pump, governor, tacho-

meter, lubricating pumps. The first three are relatively small, their total consumption being less than a quarter per cent of the compressor power, while the lubricating pump power can range from very small, if ball and roller bearings are used, to one or two per cent if gears are supplied and scavenge pumps fitted. To supply its own sleeve bearings and auxiliary gears only about a quarter per cent of the compressor power would be required. The power absorbed by the auxiliaries is a direct loss to the turbine output.

### Variation of Gas Properties

The specific heat,  $C_p$ , and the isentropic index,  $\gamma$ , are not constant throughout the engine, as assumed in the expression for air standard efficiency. There are two superimposed variations, one due to temperature changes, and one to the change in chemical composition of the gases after combustion. In general  $C_p$  is higher and  $\gamma$  lower in the turbine than in the compressor. The larger specific heat increases the heat drop for a given temperature drop, but the change in  $\gamma$  results in a smaller temperature drop for a given pressure ratio. The effects of these changes tend to cancel out and the total change in efficiency is quite small. No correction is possible in the approximate expression, but in a detailed cycle calculation it is fairly simple to use the appropriate figures at each point.

### Variation in the Mass Flow

The mass flow in the turbine is greater than that in the compressor by the amount of fuel burnt, and less by the amount of air tapped off for cooling. Air is frequently used for cooling the turbine discs and cylinder and in future will probably be used for cooling blades ; however, a turbine is only referred to as 'cooled' if there is some form of blade cooling. The fuel burnt is easily allowed for, but the cooling air can only be an approximation, except in the most detailed stage-by-stage calculation, because air is normally taken from several different stages of the compressor and returns to the main stream at several points in the turbine.

### Appendix to Part I

*Estimation of efficiency by simple expression derived in text*

This is only intended as a quick method of obtaining an approximate efficiency for a gas turbine of given maximum temperature and pressure ratio, based on intelligent guesses for the various losses compared with the ideal cycle. Consider a simple cycle of 800 degrees C. maximum temperature and 6.5 : 1 pressure ratio.

With 15°C. inlet, isentropic compression temperature = 218°C.

$$\therefore K = 1073/491 = 2.185$$

The total pressure loss will be about 11.5 per cent.

$$\therefore \text{effective pressure ratio} = r = 6.5 \times 0.885 = 5.75$$

$$\text{Efficiency} = \frac{\eta_r \eta_c (K-1)}{\eta_c (K-1)} \left[ 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right] = 0.576 \times 0.394 = 22.7 \text{ per cent.}$$

The friction loss will be about 2 per cent. Since the useful output is about one third the total turbine power, the loss to output is about 6 per cent.

There will also be a loss of  $\frac{1}{2}$  per cent to combustion inefficiency.

$$\therefore \text{Overall efficiency} = 22.7 \times 0.935 = 21.2 \text{ per cent.}$$

## PART II

### THE DESIGN OF A GAS TURBINE

In this section a gas turbine suitable for naval boost propulsion, and its type of cycle, will be considered, and its chief details estimated. The appendix shows a calculation for the cycle chosen, and the figures obtained are shown in FIG. 2.

The designer is usually asked to produce an engine to give a stated power and to be suitable for a particular application. For commercial applications the efficiency must reach a certain level in order to be competitive with other prime-movers, but for naval applications other considerations may be of overriding importance. It is for the designer to decide the cycle to be used, and the maximum temperature and pressure conditions to be employed.

#### First Thoughts on the Cycle

For most naval applications many conditions almost select themselves. Space and weight is invariably at a premium and ducting the air in, and the exhaust out, is a major problem. The result is a design based on optimum specific output rather than maximum efficiency, where specific output is defined as the work output per unit mass flow of air through the engine. The maintenance load is of great importance, and greater reliability with less maintenance is a goal for which it is often worth sacrificing some efficiency. It must not, of course, be thought that efficiency is of no importance at all, but its principal value is in reducing the total weight of machinery and fuel for a given duty. This is different from the case of a commercial engine where a base load plant demands maximum financial efficiency, and a stand-by set demands minimum first cost. By the same argument, the introduction of complex cycles into naval craft can only be justified if the increase in machinery weight is more than offset by the saving in fuel weight, and if the maintenance load is not excessively increased.

Details of a design are, as always, a matter for compromise, but it is possible to outline a simple approach to the problem. For a boost propulsion engine the above argument demands a simple cycle, and the need to drive a propeller demands a free power turbine. The installation will therefore have the minimum weight for the boost power, which is only used for a small percentage of the total machinery life, and the two shaft arrangement will give maximum flexibility of operation of the output shaft.

#### Selection of Maximum Temperature

Consideration of the creep data of the materials available allow relationships to be drawn up between the maximum cycle temperature, stress in the first stage of turbine blading, and blading life. The last is dependent on maintenance considerations, and for a boost engine 1–2,000 hours at full load is a reasonable figure. The highest stressed blade is the moving, which is at a lower temperature than the fixed, but, even making the heat drop in the nozzles as high as possible, the nozzle inlet temperature cannot at present exceed about 800 degrees C.

#### Selection of Compression Ratio

Since the pressure ratio determines the isentropic compression temperature the expression for work output given under 'Output and Efficiency' in Part I is completely determinate for a given pressure ratio and maximum temperature. The maximum temperature is already known so a curve of specific output against pressure ratio can be plotted. For 800 degrees C. maximum temperature the curve peaks at about 6 or 7 to 1, and although this is below the optimum



for maximum efficiency it will give the smallest engine. A further consideration is that 7 : 1 is about the highest ratio that can as yet be obtained in a single spool axial compressor without very extensive development running. Axial compressors are now almost invariably used because they can attain a higher efficiency than the centrifugal type.

### Arrangement of Turbines

In order to reduce the number of high temperature rows of blades it is usual to have high heat drops in the early stages. This policy can have the further advantage that if the power can be obtained in two stages the turbine may be overhung, with resultant saving of bearing loss and engine length. The power turbine does not require high heat drops to get the temperature down, but the advantage of an overhung rotor are considerable, since it may be positioned immediately after the gas generator turbine thus giving better gas flow.

High heat drops in the compressor driving turbine tend to produce high swirl in the leaving flow. In some applications this is acceptable as the swirl can be taken out in the power turbine, but if, as is often the case, it is desired to use a standard gas generator, with both left and right handed power turbines, the exit from the gas generator must be as near axial as possible. In all cases the exit from the power turbine must be near axial to reduce the leaving loss.

### Speed of Shafts

The speed of the gas generator shaft is usually fixed by the compressor blading, although the conditions in the turbine must also be checked. A method of estimating the speed based on the compressor inlet blading is as follows.

In order to keep down the annulus size and the number of stages the inlet velocity should be high, but not high enough to cause adverse Mach number effects. Past experience suggests that 450 ft/sec is a reasonable figure. The annulus area can then be calculated from the mass flow which has been obtained from the specific output. Experience indicates that the hub ratio (root diameter/tip diameter) should be 0.7 to 0.85 at compressor inlet. Selecting a value enables the blade height and rotor diameter to be calculated, and strength considerations of the rotor and blades allow a speed to be selected.

Velocity triangles may now be drawn ; the proportions for a successful stage are known from past experience and, if necessary, can be varied by altering the hub ratio and rotational speed to give a satisfactory compromise.

The power turbine speed is obtained in a similar way, based on the leaving velocity and stress in the last row of blades. Alternatively it is possible to select suitable blade angles and work back to the hub ratio. (See power turbine in the example in the appendix).

### Typical Cycle

The cycle conditions for a simple cycle based on the above arguments are shown in FIG. 2. The calculations are outlined in the appendix. The assumptions of efficiencies and losses are shown, and the areas and speeds are based on an output of 5,000 horse power. For simplicity the blades have been considered to have a simple taper in which the section area decreases uniformly until at the tip it is half the value at the root.

### CONCLUSION

The foregoing attempts to show how the gas turbine designer starts work. The figures obtained in the appendix are only a preliminary skirmish with the

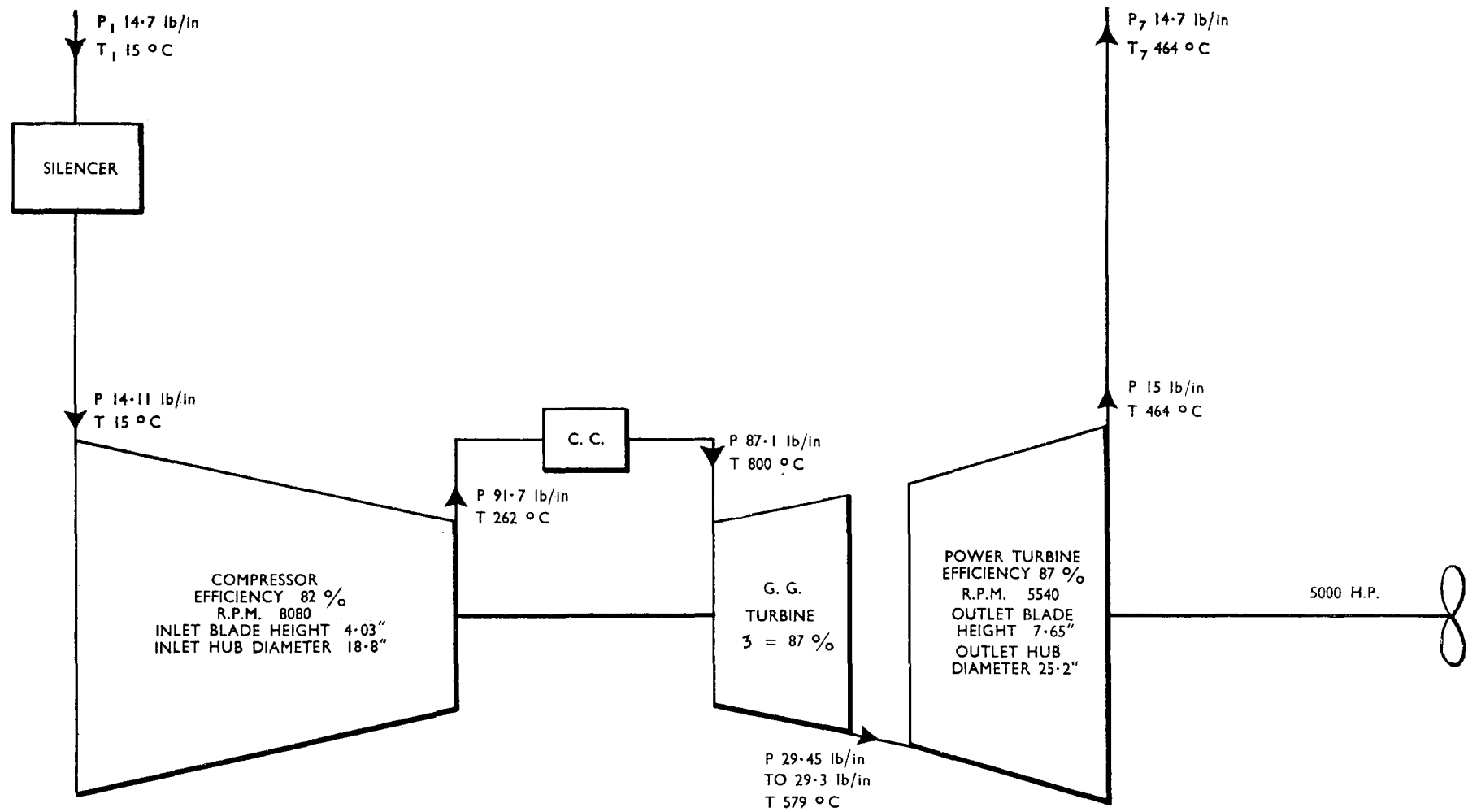


FIG. 2 (All lb/in to read as lb/in<sup>2</sup>)

problem. The thermodynamicist and mathematician go into very great detail and produce accurate dimensions and characteristics of the gas passages. However, this is only the beginning for the engineer, although from now on his problems are not special to the gas turbine. He spends his time pondering strengths, materials, machinability, and so on, just as does his colleague who designs a Diesel engine or a boiler. The design of a gas turbine is not a new technology, but the application of standard methods to the solution of variations on standard problems. New materials are employed to meet more strenuous conditions, new techniques may occasionally be used, but the fundamental engineering is still the same.

## APPENDIX

Data assumed in the calculations is based on the following :—

### *Material Available*

Maximum cycle temperature 800 degrees C. allowing centrifugal stress of 8 tons/sq in. in Nimonic 90 for turbine first stage moving blades with life of 1,000 hours at full power.

Compressor blade material, aluminium bronze allowing centrifugal stress of 8 tons/sq in.

Power turbine blade material, Nimonic 80A allowing centrifugal stress of 10 tons/sq in at temperature in last row.

Densities—	Nimonic 90	Nimonic 80A	Al-Bronze	
	0.299	0.295	0.290	lb/cu in.

### *Optimum Specific Output*

Pressure ratio 6.5 : 1. With 15°C. inlet this gives isentropic compression temperature of 218°C.

### *Past Experience*

Isentropic efficiencies— — Compressor— $\eta_c$ —82 per cent  
Turbine— $\eta_t$ —87 per cent

Combustion efficiency—99.5 per cent

Pressure losses—Inlet ducting —2 per cent  
Inlet silencer —2 per cent  
Exhaust ducting —2 per cent  
Combustion chamber —5 per cent  
Annulus between turbines — $\frac{1}{2}$  per cent

Friction losses—2 per cent on each shaft

Cooling air flow (disc and liner cooling only)—2 per cent

Gas properties—	Compressor	G.G. Turbine	P. Turbine	Comb. Chamber
$C_p$	0.24	0.275	0.265	0.260
$\gamma$	1.4	1.333	1.333	—

Axial flow velocity—Compressor inlet—450 ft/sec  
Power turbine outlet—400 ft/sec

**Cycle Calculation**

Standard atmospheric conditions are : 14.7 Lb/sq in and 15°C.

A loss of 2 per cent in ducting and 2 per cent in silencer gives : 14.11 Lb/sq in at compressor inlet.

Compression ratio of 6.5 and 82 per cent efficiency gives compressor outlet of : 91.71 Lb/sq in and 262.5°C. where actual temperature rise equals

$$\frac{\text{isentropic rise}}{\text{efficiency}} = \frac{T'_3 - T_2}{\eta_c} = 247.5^\circ\text{C}.$$

The work absorbed by the compressor is proportional to the actual temperature rise and equals

$$C_p \times 247.5 = 59.4 \text{ C.H.U./lb}$$

A loss of 5 per cent in combustion chamber gives outlet at 87.1 Lb/sq in and 800°C.

The calorific value of an average fuel is 10,000 C.H.U./lb

$$\begin{aligned} \text{fuel burnt} = w &= \frac{(800 - 262.5) \times (1 + w) \times C_p}{10,000} \\ &= 0.0142 \text{ lb/lb of air in combustion chamber.} \end{aligned}$$

With 2 per cent cooling air = 0.0139 lb/lb of air in compressor.

At 99.5 per cent combustion, fuel input  
= 0.014 lb/lb of air in compressor.

Mass flow in turbine equals compressor flow minus cooling flow plus fuel input = 0.98 + 0.014 = 0.994 lb/lb of air in compressor.

The gas generator turbine must produce sufficient power to drive the compressor and overcome the friction loss in the drive =  $1.02 \times 59.4$   
= 60.5 C.H.U./lb of air in the compressor.

$$\text{Temperature drop} = \frac{60.5}{0.994 C_p} = 221^\circ\text{C}.$$

$T_5$  = outlet temperature = 579°C.

The ideal temperature drop is given by  $\frac{\text{actual drop}}{\text{efficiency}} = \frac{221}{\eta_T} = 254^\circ\text{C}.$

Isentropic outlet temperature =  $T'_5 = 800 - 254 = 546^\circ\text{C}.$

$$\text{Pressure ratio} = \frac{P_4}{P_5} = \left( \frac{T_4}{T'_5} \right)^{\frac{\gamma}{\gamma-1}} = 2.96$$

Gas generator turbine outlet pressure =  $\frac{87.1}{2.96} = 29.45 \text{ Lb/sq in.}$

Loss of  $\frac{1}{2}$  per cent gives power turbine inlet 29.3 Lb/sq in and 579°C.

Ducting loss of 2 per cent gives back pressure of  $14.7 \times 1.02 = 15 \text{ Lb/sq in.}$

Pressure ratio = 1.95 and ideal outlet temperature  $T'_6 = 447^\circ\text{C}.$

Actual temperature drop =  $\eta_T (579 - 447) = 115^\circ\text{C}.$

Exhaust temperature = 464°C.

Work done is proportional to the actual temperature drop and mass flow  

$$= 0.994 \times 115 \times C_p = 30.3 \text{ C.H.U./lb air in compressor}$$

Friction loss of 2 per cent gives brake output of 29.7 C.H.U.

Efficiency related to calorific value of fuel

$$= \frac{29.7}{10,000 \times 0.014} = 21.2 \text{ per cent}$$

Specific output 
$$= \frac{29.7 \text{ C.H.U.} \times 1,400 \text{ ft Lb/h.p. sec}}{\text{lb C.H.U.} \times 550 \text{ ft Lb}}$$
  

$$= 75.6 \text{ h.p. per lb/sec air in the compressor}$$

Mass flow for 5,000 h.p. = 66.2 lb/sec

### Gas Generator Speed and Compressor Inlet Diameter

Based on the first stage of moving blades, which are the longest and hence the most heavily stressed.

Inlet axial velocity = 450 ft/sec ; air density = 0.0735 lb/cu ft

Annulus area for 66.2 lb/sec flow = 288 sq in

Experience suggests a hub ratio of 0.7

$$\text{Area} = 288 \text{ sq in} = \frac{\pi d^2}{4} \left[ \left( \frac{1}{.7} \right)^2 - 1 \right]$$

$$d = \text{hub diameter} = 18.8 \text{ in}$$

$$h = \text{blade height} = \frac{1}{2} \left( \frac{18.8}{.7} - 18.8 \right) = 4.03 \text{ in}$$

Suppose the blade is tapered uniformly so that the tip section is half the area of the root section, A.

Radius to c.g. of blade = 11.2 in

Mass of blade = 0.86A lb (Frustum formulae)

Centrifugal stress in the root section is given by

$$\frac{.86A \text{ lb} \times V^2 \times \text{Lb sec}^2 \times 12 \text{ in}}{A \text{ in}^2 \times 11.2 \text{ in} \times 32.2 \text{ ft lb ft}}$$

For a stress of 8 tons/sq in the blade velocity at c.g. =  $V = 790 \text{ ft/sec}$

Rim velocity = 663 ft/sec. r.p.m. = 8,080

### Power Turbine Speed and Outlet Diameter

Based on the outlet row of moving blades, which are the longest and hence the most heavily stressed by centrifugal loading.

Outlet axial velocity = 400 ft/sec ; air density = 0.03 lb/cu ft

Annulus area for flow of  $66.2 \times 0.994 \text{ lb/sec} = 789 \text{ sq in}$

The cycle calculation has shown that the heat drop in the power turbine is 30.3 C.H.U./lb of air in the compressor. It is reasonable to design the last stage for a drop of 16 C.H.U. with reaction varying from about 0.15 at the root to about 0.70 at the tip. To keep the leaving loss to a minimum, limit the outlet swirl to  $10^\circ$  from the axial. (Reaction is defined as the ratio of the heat drop in the moving blade to that in the whole stage).

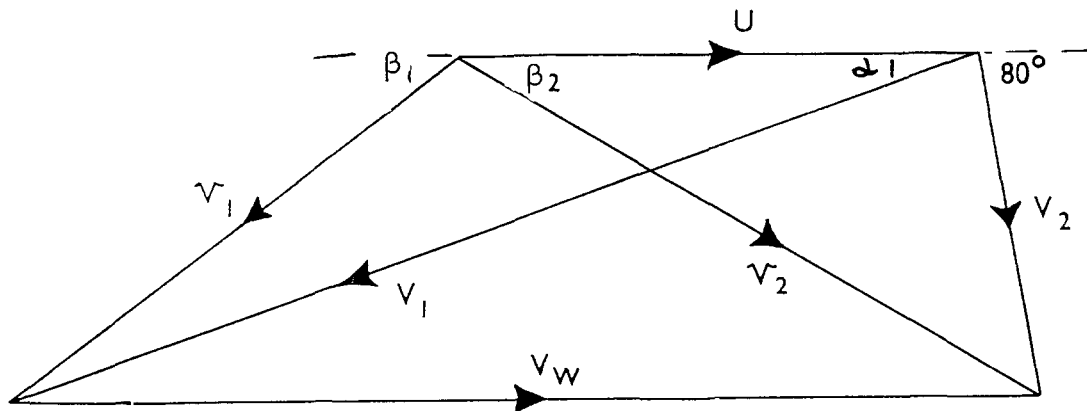


FIG. 3

FIG. 3 shows the velocity triangles at the root section of the last stage where

$U$	—	blade speed
$\alpha$ and $\beta$	—	blade angles
$v$	—	relative velocities
$V$	—	absolute velocities
$V_w$	—	velocity of whirl

Heat drop in moving blades is given by the difference of kinetic energy

$$= \frac{v_2^2 - v_1^2}{2g \times 1400} \text{ C.H.U.}$$

The work done in the stage is given by the product of the velocity of whirl and the blade speed, plus the change in kinetic energy between inlet and outlet. If axial velocity and swirl are constant through the turbine, then inlet and outlet velocities are the same and equal 406 ft/sec and work done is given by

$$16 \text{ C.H.U.} = \frac{U \cdot V_w}{1400 g}$$

The root section has been considered because it is the most difficult section from which to obtain the required work. Poor design can result in recompression in the root, instead of expansion

One possible solution gives the following :—

$$v_1 = 664 \text{ ft/sec} \quad v_2 = 781 \text{ ft/sec}$$

$$\text{Heat drop in moving blades} = 1.875 \text{ C.H.U.}$$

$$V_1 = 1198 \text{ ft/sec}$$

$$\text{Heat drop in fixed blades} = 14.125 \text{ C.H.U.}$$

$$\text{Reaction} = 11.7 \text{ per cent}$$

$$\text{Blade speed} = 600 \text{ ft/sec}$$

$$\text{Velocity of whirl} = 1200 \text{ ft/sec}$$

$$\text{Nozzle outlet angle, } \alpha_1 = 19.5^\circ$$

$$\begin{aligned} \text{Annulus area previously calculated} &= 789 \text{ sq in} \\ &= \pi [(R+h)^2 - R^2] \end{aligned}$$

$$\text{where } R = \text{hub radius}$$

$$h = \text{blade height}$$

$$\text{hence } 251 = 2hR + h^2$$

(A)

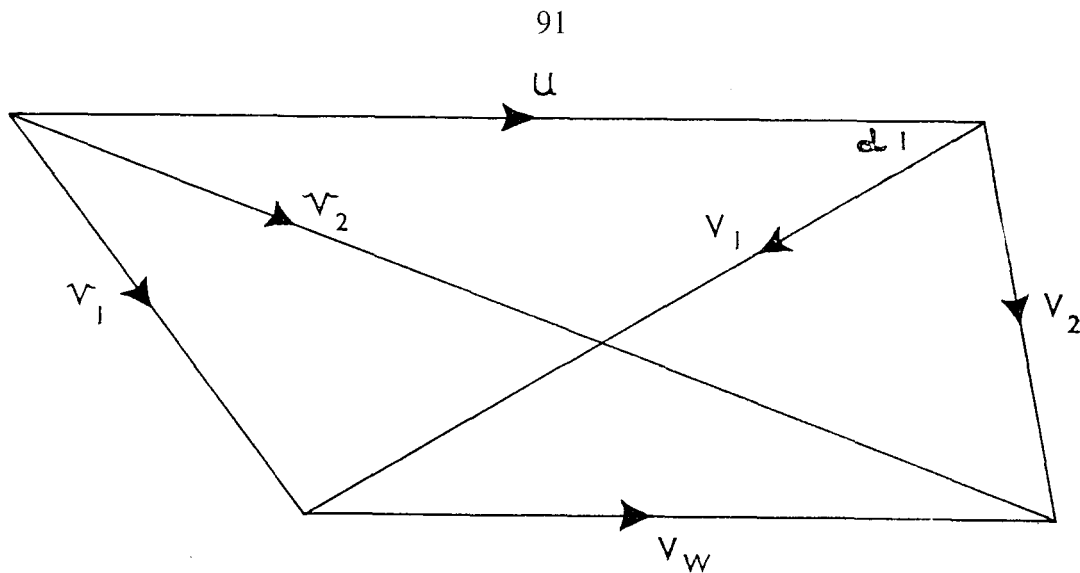


FIG. 4

If the blade is tapered uniformly so that the tip section is half the area of the root section, the use of simple frustum formulae show that

$$\text{Centrifugal stress at root} = \frac{0.0809 h (\text{velocity at c.g.})^2}{\text{radius of c.g.}} \quad (\text{B})$$

Solution of equations A and B for a centrifugal stress of 10 tons/sq in gives :—

Blade height	=	7.65 in
Disc diameter	=	25.2 in
Hub ratio	=	0.625
r.p.m.	=	5,540

For vortex flow  $U.V_w$  is constant at all radii  
Hence velocity triangles at tip— (Fig. 4)

$U$	=	965 ft/sec
$V_w$	=	745 ft/sec
$v_1$	=	494 ft/sec
$v_2$	=	1110 ft/sec
$V_1$	=	784 ft/sec

Heat drop	=	11 C.H.U. in moving blades
		5 C.H.U. in fixed blades

Reaction	=	69 per cent
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Nozzle angle, $\alpha_1$	=	$30.7^\circ$
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Note that the consideration of velocity triangles and hence blade sections at all radii is essential as a calculation at mean height can easily lead to impossible sections at root or tip. An assumption of hub ratio would lead to a simpler solution mathematically (as shown in the previous section on gas generator speed), but it is still necessary to check the velocity triangles to ensure that the work can be obtained with reasonable blade sections.

### Gas Bending Stress

Consideration of the blade bending stresses has been omitted in this simple outline. They must, however, be considered in detail due to the possibility of resonant vibrations.