# RADAR WAVEFORM AMBIGUITY FUNCTION DESIGN USING GENETIC ALGORITHMS

BY

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ABSTRACT

Modern military operations have been carried out in the littoral environment. The performance of the radar pulse in this arena can be degraded by the effects of clutter and interference. It is possible to design a radar waveform that minimizes the effect of this unwanted interference by careful specification of the waveforms' ambiguity function. Recent advances in waveform design have used genetic algorithms to find near optimum waveform parameters that produce a specified ambiguity function. This paper proposes a practical FSK PSK waveform that is capable of reducing the effects of interference and clutter in specified areas of range and Doppler in the ambiguity function. A new linear FM PSK waveform is proposed and its performance is compared with the FSK PSK waveform. The composite ambiguity function is used to compare a set of FSK PSK waveforms with a set of linear FM PSK waveforms. The results show that whilst the Linear FM PSK waveform gives better range and Doppler discrimination properties, it does not offer any appreciable performance when minimizing specified areas within the ambiguity function. During the initial research of this project, a genetic algorithm was designed to find new 64-bit bi-phase codes. Four codes were found that offer better performance than codes discovered in previous research.

## INTRODUCTION

### Background

Military operations over the last few years have been carried out predominantly in littoral areas. Here, the radar operating environment is extremely complex and difficult to characterize in terms of clutter and interference. Although it has been possible to model the environment, it is often not possible to modify in-service sensors to validate this model and subsequently improve radar performance.

Software based sensors are being developed that allow almost real time modifications of sensor parameters such as waveform generation. If the nature of clutter and interference are known, the radar waveform could be designed such that the effect of clutter and interference is minimized. The ambiguity function of a waveform can characterize its performance in clutter but there is no mathematical route from a desired ambiguity function to the waveforms parameters. Traditionally, radar waveforms have been designed by either trial and error or using past experience. As the waveforms become more complex, it is impractical to search exhaustively through all possible permutations to discover which waveform gives the best performance. A genetic algorithm is a stochastic search algorithm that can find optimized solutions in a large search space in a fraction of the time taken to search exhaustively the same space.

## Summary of recent research

Recent research has shown that radar waveforms can be designed, using genetic algorithms, such that they reduce the effects of clutter whilst still maintaining good Doppler and range discrimination properties. A waveform that gives the good range and Doppler discrimination is the FSK PSK waveform. A FSK PSK waveform had been designed such that sidelobes within a specified area of range and Doppler of the waveforms ambiguity function have been reduced. This reduction has the consequence of minimizing the effect interference or clutter within that specified area. The design of this waveform was completed using a genetic algorithm.

Previous research also showed that using multiple waveform sets, instead of single waveforms, the range and Doppler discrimination of radar could be enhanced by utilizing the composite ambiguity function. Here, the composite ambiguity function of a set of FSK PSK waveforms was calculated and it was shown that the sidelobes within a specified area of range and Doppler were reduced compared to the single waveform ambiguity function. It was also shown that the range and Doppler discrimination was improved. The FSK PSK pulses designed did not have realistic or practical values of phase and frequency such that these waveforms could be used in radar systems.

## Project aim

This project investigates the design of a practical FSK PSK waveform using genetic algorithms that generates an ambiguity function that minimizes the effect of clutter within a specified range and Doppler region. A new waveform, the Linear FM PSK waveform is introduced in this research. This waveform is a variation of the FSK PSK waveform. Its performance is compared with the FSK PSK waveform to determine which offers the best minimisation within specified areas or range and Doppler.

In addition, this project investigates the use of genetic algorithms to design practical FSK PSK and Linear FM PSK waveform sets. The performance of the composite ambiguity function of these waveform sets can be compared to the single waveforms' ambiguity function mentioned above.

During the initial stage of this projects research, the performance of a genetic algorithm searching for optimum 25-bit bi-phase codes was compared to the performance of an exhaustive search. A genetic algorithm was designed to search for an optimum 64-bit bi-phase code. A genetic algorithm was also designed to search for a 13-bit generalized BARKER sequence.

## Summary of results

The results show that there is no appreciable performance advantage of the new practical linear FM PSK waveform over the practical FSK PSK waveform when comparing the minimization of sidelobes in specified areas within the ambiguity function. The performance of the composite ambiguity function of both waveforms sets was similar.

The composite ambiguity function of the waveform sets allowed better sidelobe minimization when compared to the ambiguity function of the single waveform. The Linear FM waveform gave consistently better range and Doppler discrimination when compared with the FSK PSK waveform.

The bi-phase code investigation discovered two new 64-bit bi-phase codes that offered a better performance than codes found using a different search method in previous research. A new 13-bit generalised BARKER sequence was found.

# LITERATURE SEARCH

## Introduction

There are a number of pulse types available to the radar designer when designing a radar system. These pulses range from the simple bi-phase radar code to the more complex and novel FSK PSK pulse diverse waveforms which vary frequency and phase in each chip within the pulse. In order to explore all of these options with some detail, the literature search was divided into a number of sections.

- Bi-phase codes with minimum sidelobe levels.
- Polyphase codes with low autocorrelation sidelobes.
- Pulse diverse waveforms.
- Radar codes and the use of genetic algorithms to find codes with specified ambiguity functions.

## Bi-phase codes with minimum sidelobe levels

In 1953, BARKER[1] discovered a set of bi-phase codes that were deemed perfect such that the autocorrelation of these codes would yield sidelobes that fluctuated between 1 and 0. There are nine BARKER codes[2] in total; the longest is a 13 bit code. It has been calculated that BARKER codes do not exist up to at least 6,084 bits.[3] For lengths greater than 13 bits, bi-phase codes whose autocorrelation produces the minimum peak sidelobes for a given length are deemed the best.

COHEN[3] details bi-phase codes up to a length of 48 bits which, when autocorrelated, give the minimum peak sidelobe levels. For example, Cohen discovered that there exists two 25-bit bi-phase codes that when autocorrelated, have a minimum peak sidelobe level of 2. COHEN did not discover a pattern to his results and thus the search process was extremely lengthy. The search up to 48 bits was carried out using a tree structured search algorithm. This greatly reduced the time to search compared to an exhaustive search method of taking each possible code and computing the autocorrelation. No reference to exhaustive searches beyond 48 bits has been found.

In 1993, a Simulated Annealing Search Algorithm was developed to search for long bi-phase codes.[4] This reference outlines two resulting 64-bit codes which when autocorrelated give low sidelobes and have good cross correlation properties. Work carried out during this project will show that better alternative 64-bit codes have been found using a genetic algorithm. Although this previous research was completed in 1993 and computing power has been increased since, no reference to follow up work has been found.

## Polyphase codes producing Generalized BARKER Sequences

In the early stages of this project, it was intended to design a genetic algorithm to search for long polyphase radar codes with specified ambiguity functions. The literature search revealed that this subject had been researched comprehensively using different search methods. During this project a genetic algorithm was designed to search for a 13-bit generalized BARKER sequence that proved the principle that genetic algorithms could be used in this area of radar code design. The literature search of polyphase codes is outlined below for completeness.

In the previous section, the term bi phase coding referred to the phase being selected from either 0° or 180°. The term polyphase coding means that the phase of each bit in the code can be chosen from some alphabet (M). They are time-discrete complex sequences with constant magnitude and variable phase.[5] The alphabet can be any length from 2 (bi-phase) to infinity (continuous phase).

As discussed above, BARKER codes were defined as perfect. It is possible, with polyphase codes with a magnitude of unity and a suitable phase alphabet, to design polyphase codes such that the auto correlation has sidelobe levels of unity or less. These codes are called Generalized BARKER Sequences and they were discovered in 1965.[6]

In 1989, sixty-phase Generalized BARKER Sequences were discovered up to a length of 19 bits.[7] At the same time, an iterative search method was used to find generalised BARKER sequences up to length 25.[5]

As the length of code increases, so to does the search space and possible permutation of auto correlation outputs. It was not until 1994, when computing power had improved, that polyphase Generalized BARKER Sequences up to length 31 were discovered and documented.[8] Here, the search method utilized was the heuristic Great Deluge Algorithm. Instead of a limited alphabet of phases, the alphabet tends to infinity such that the phase becomes continuous. Whilst in theory the approach of continuous phase gives the required Generalized BARKER Sequence, in practice, continuous phase systems are not, at present, technically feasible.[9]

Generalized BARKER Sequences to length 36 were reported in 1996.[10] The search algorithm used was the Great Deluge Algorithm, but this time, the search was initialized using starting vectors derived from HUFFMAN sequences. Instead of being continuous, the phase alphabet was selected from values that were viable.

The final paper relating to Generalized Barker Sequences outlines sequences up to length 45 using small alphabets.[9] The search strategy is a modified Great Deluge Algorithm with random starting vectors. The maximum alphabet of length (M) used was 120 i.e. in 3° steps (360/120) for sequence lengths of 40 to 44. This maximum length of M=45 is claimed to be possible to implement.[9]

## Pulse diverse waveforms

In the previous sections, only phase has been altered within the relevant pulse. In FSK PSK pulse diverse waveforms, both the frequency and the phase of chips within a pulse can be varied from chip to chip to give a desired output from a matched filter. A generic FSK PSK pulse diverse waveform is shown in (FIG.1)

| CHIP 1 | CHIP 2 | CHIP 3 | CHIP 4 | CHIP 5 | CHIP 6 |
|---|---|---|---|---|---|
| F1<br>PH1 | F2<br>PH2 | F3<br>PH3 | F4<br>PH4 | F5<br>PH5 | F6<br>PH6 |

**SINGLE PULSE**

FIG.1 - FSK PSK WAVEFORM COMPOSITION

It should be noted that the frequency and the phase values within each chip remain constant throughout the chip duration. The issue of frequency sweep (linear FM) within each chip is the basis for investigation later.

The technique of using FSK PSK pulse diverse waveforms is relatively new and there are few references found to this work.

GUEY and BELL[11] developed the theory for designing multiple pulse diverse waveform sets that offer superior Doppler and range discrimination. They introduced the composite ambiguity function that allows superior discrimination to be achieved.

Two papers have been published that use a genetic algorithm and the theory developed by GUEY and BELL to design multiple pulse diverse waveform sets that produce tailored ambiguity functions. WONG[12] designed a four-signal waveform set where the sidelobes of the ambiguity function were minimized. This was achieved using a simple genetic algorithm and using non-realistic radar frequencies and phases in steps of 1/1024 radians.

This project investigates the effect of using realistic frequencies in pulse diverse waveforms and addresses the issue of using practical phase increments.

### Use of genetic algorithms in radar waveform design

In addition to the work completed above by WONG, two other references have been discovered that use genetic algorithms for radar code design. The first paper[14] details the use of genetic algorithms to develop an eight-phase polyphase code that has good Doppler tolerance and low range sidelobes around the main peak of the autocorrelation function. In practice, this clearance around the main peak allows good range discrimination of two very close targets. It details two codes of length 256 that, in addition to the autocorrelation criteria, have good cross correlation properties. The paper only considers eight-phase polyphase codes and does not investigate different phase alphabets.

It should be noted that although this reference is relatively recent, it was not discovered in the initial literature search. Furthermore, this paper was not discovered using the library search but during an Internet search on the subject in week eight of the project. This paper has not been referenced in work carried out in this area since 1997. PARISI has confirmed personally that to his knowledge, this work has not been pursued further. The discovery of this reference was a setback because the original thrust of the project was along similar lines.

The second paper in this field[15] examines the design of a generic spread spectrum radar polyphase code using a genetic algorithm. It then compares the results with other search methods such as Implicit Enumeration Techniques, Multi-level Tabu Search, Monte Carlo method and non-linear programming. The results show that the Multi Level Tabu Search and genetic algorithm performed comparably and both found the generic solution. However, the genetic algorithm was deemed superior because it was able to exploit its parallelism and perform searches for codes of longer length.

### Other forms of radar waveforms

Other forms of waveforms used in radar include:

a. Pseudo-random Binary Phase codes (M Sequence).[2]
b. Random binary phase.[16]
c. Step frequency modulation.[16]
d. Polyphase Frank codes (P1, P2, P3 and P4).[16]
e. HUFFMAN codes.[16]
f. Complementary codes.[16]

These codes do not offer simultaneous good range and good Doppler discrimination properties and are not part of this research.

## BACKGROUND KNOWLEDGE

### An overview of a simple genetic algorithm

Genetic algorithms are part a wider field of evolutionary algorithm which is based largely on Charles DARWIN's theory of evolution and survival of the fittest. The parallel drawn here is that in a genetic algorithm, a solution to a problem is evolved over a number of generations to meet a desired objective and hopefully find a good solution.

The genetic algorithm is a stochastic global search method. A number of possible solutions, known as a population, are evolved using the principle of survival of the fittest. The values within the initial population are random and therefore, no knowledge of the search space is required. At each generation, members of the population are selected for breeding based on their level of fitness and how well they solve a problem. New offspring are formed and evaluated for fitness. Fitter members supersede weak or less fit members of the population. This method of evolution leads to a population that is better suited to their environment.

The computer coding for the genetic algorithm was conducted in MATLAB. The genetic algorithm utilized the Genetic Algorithm Toolbox developed by Sheffield University.[17] The objective functions, that describe the search problem, were designed for each particular search

*The basic genetic algorithm*

The basic genetic algorithm is shown in (FIG.2).

1. START - INITIAL RANDOM POPULATION
2. EVALUATE POPULATION AGAINST AN OBJECTIVE FUNCTION
3. GENERATION LOOP
   - ASSIGN FITNESS to each member of population
   - SELECTION of parents for breeding
   - CROSSOVER of parents to create offspring
   - MUTATE some of the offspring
   - EVALUATE OFFSPRING against an objective function
   - REINSERT offspring into population
4. END ON MAXIMUM NUMBER OF GENERATION

FIG.2 – BASIC GENETIC ALGORITHM

*Chromosomes and population*

The chromosome is the basic building block for the genetic algorithm. Each chromosome contains a number of encoded parameters called genes. Genes are selected from some finite alphabet that can be binary, ternary or integer value that together can be decoded to form the parameters needed to solve the problem of interest. A population is made up from n individual chromosomes. The initial chromosome contains genes with random values

*Objective function, evaluation and fitness of the population*

Each chromosome in the population is evaluated using an objective function. This function takes in the chromosome, and produces an output. The individual chromosome output is compared with the outputs produced by other members of the population. A fitness value is then determined for each chromosome that will describe how it compares with the other members of the population. Consider the simple example shown in (FIG. 3).

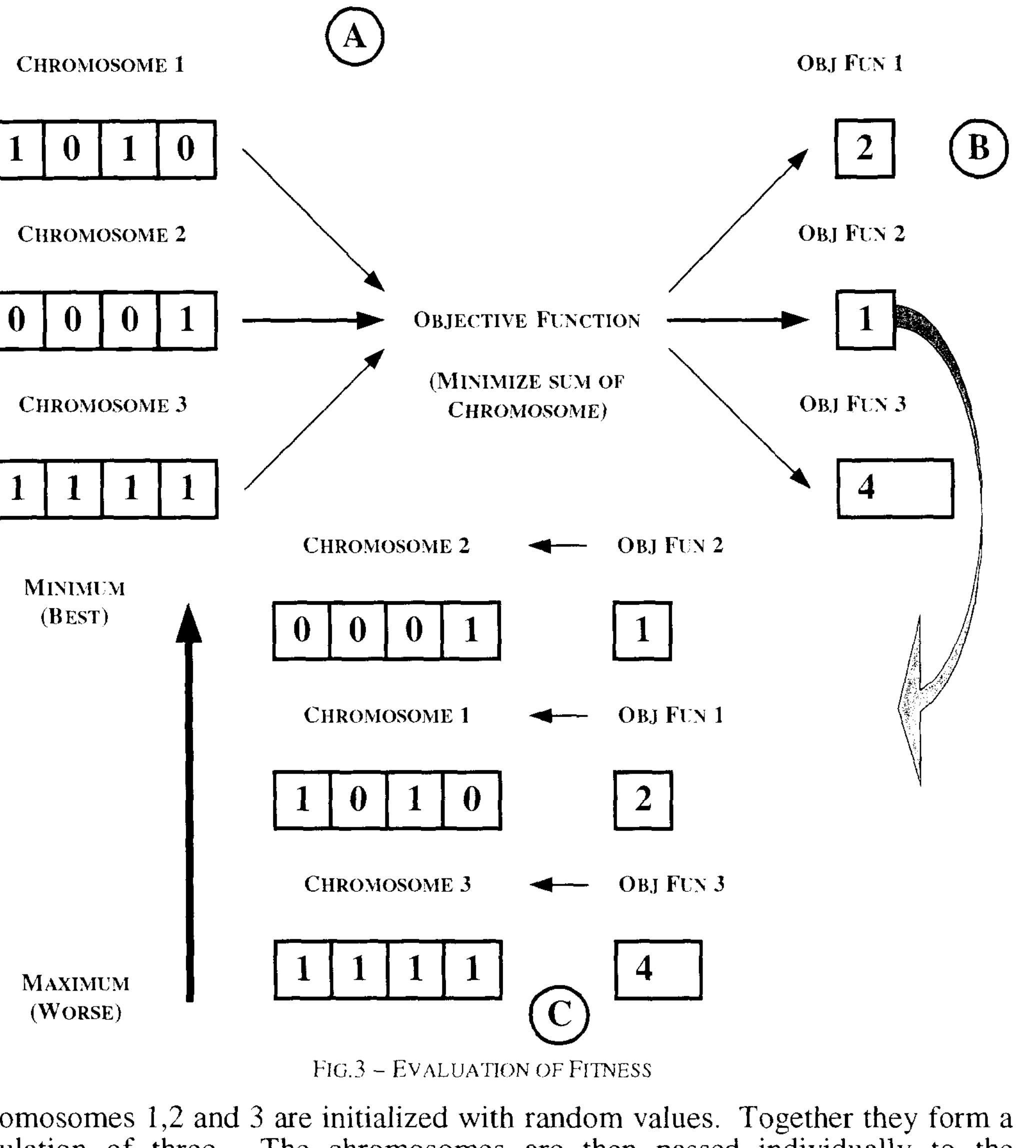



FIG.3 – EVALUATION OF FITNESS

Chromosomes 1,2 and 3 are initialized with random values. Together they form a population of three. The chromosomes are then passed individually to the objective function (Shown at point **A** in FIG.3). Here, the objective function is simply adding all the genes in each chromosome together. The objective function is defined by the user and is search space dependent. It is common practice to design the objective function such that its output will be minimized. The objective functions used in this project are discussed later.

The output of the objective function (point **B**) is now applied to the fitness function. The fitness function used here is a simple ranking function whereby each objective function output is ordered with the minimum (best) at the top and the maximum (worse) at the bottom (Point **C**).

*Selection of chromosomes for breeding*

Selection for breeding is based on the fitness of the individual chromosome. The level of fitness determines how many times an individual is chosen for reproduction. The better chromosomes (high fitness) are more likely to become parents for breeding. Poor chromosomes (low fitness) are not as likely to be selected for breeding. It is normal practice to allow worse fit individuals to breed, albeit less likely, as they may contain some useful genetic material.

*Crossover and mating operators*

Any two chromosomes can breed. During breeding, the offspring are formed by selecting genes from the parents. This selection of genes and formation of offspring is carried out by a recombination operator. The simplest operator is the single point crossover. This operator selects a single point within the parent chromosome at which the genetic information is swapped. This is shown graphically in (FIG.4).

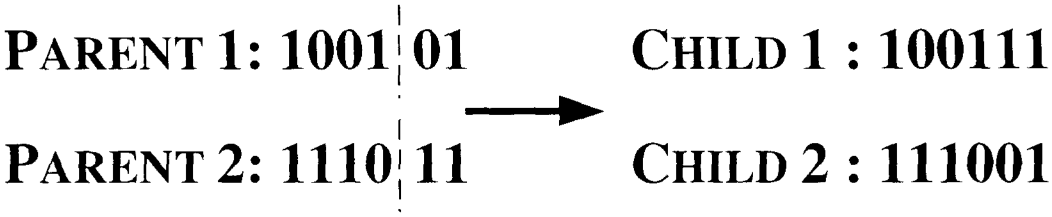



FIG.4 – SINGLE POINT CROSSOVER

The multi-point crossover operator was used during this research and this works by dividing the chromosome up into gene segments. Each segment has a 50:50 chance of swapping with its counterpart segment in the other parent. Two new offspring are produced. This operator is shown in (FIG.5). The grey values indicate those genes that are to be swapped.

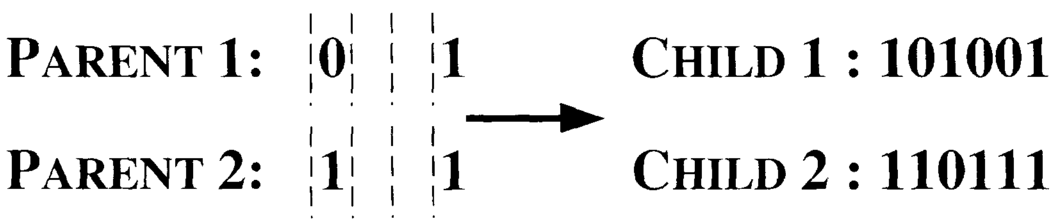



FIG.5 – MUTI-POINT CROSSOVER

*Mutation*

Another genetic operator used in conjunction with crossover is called mutation. Mutation is applied to each gene in the new offspring with a set probability. Mutation of a binary sequence chromosome is shown in (FIG.6).


FIG.6 – MUTATION OPERATOR

Mutation rate has the effect of allowing the genetic algorithm to increase its search space. This should prevent the genetic algorithm settling on a local minimum as opposed to a global minimum. If the mutation rate is too high, the genetic algorithm behaves as if it were a random search. Conversely, if too low, the genetic algorithm may converge on a local minimum.

After mutation has been applied, the new chromosomes are passed to the objective function and the evolution cycle is repeated. This full cycle is known as one generation. The whole genetic algorithm process can be summarized in (FIG.7).

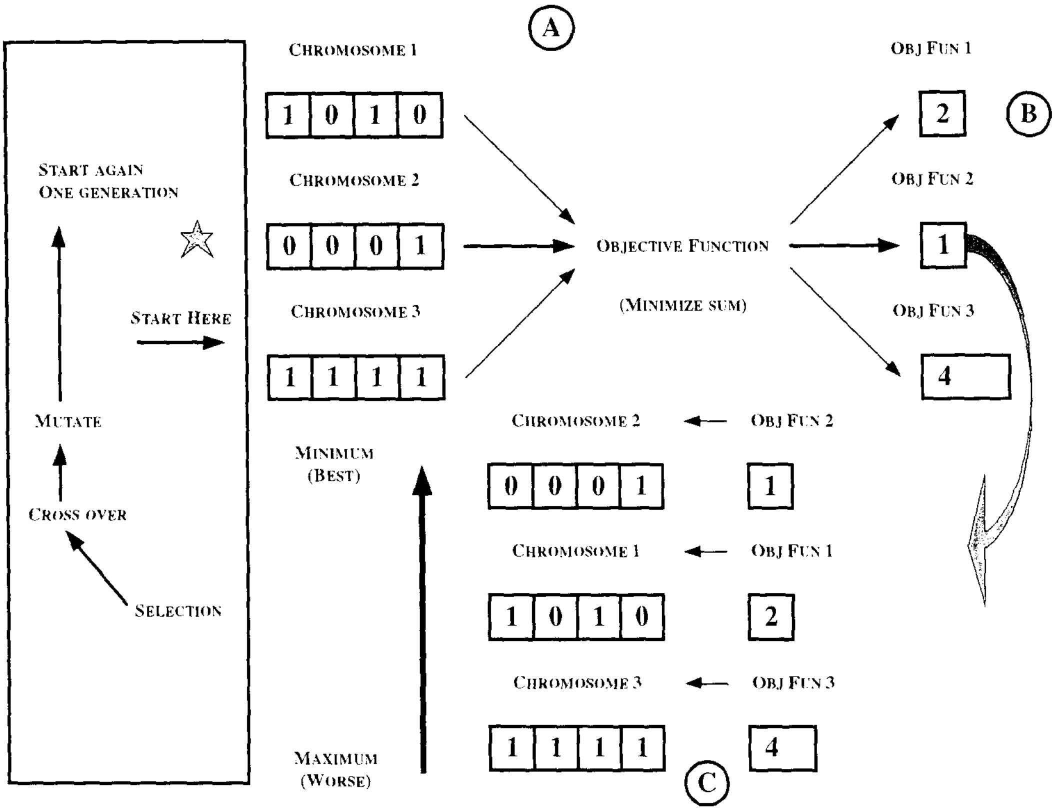



FIG.7 – GENETIC ALGORITHM PROCESS, ONE GENERATION

The number of generations in the Algorithm is set at initialization. Once the genetic algorithm has completed the required number of generations, the process is halted.

## Ambiguity Function

### *Introduction*

The ambiguity function is utilized during this project. This section aims to highlight the salient points of this function. There are many references to the computation of the ambiguity function and often there is different use of terminology and symbols to calculate essentially the same result.

### *Definition of the ambiguity function*

The ambiguity function is a time-frequency transform of a waveform used by radar or sonar to characterize the response of the matched filter as a function of delay and Doppler errors.[18] The ambiguity function characterizes the waveform in such a way that it can be used to determine the effects of Doppler at the output from the matched filter. From the function, one can also determine the resolutions of Doppler and range, effect of clutter and the effects of range-Doppler coupling.

The mathematical definition of the output of the matched filter is given below in equation 1.[2] $T_r$ indicates a target return at a Doppler frequency $F_d$. A positive $F_d$ denotes an incoming target.

$$\chi(T_r, F_d) = \int_{-\infty}^{\infty} s(t)s^*(t + T_r)e^{j2\pi F_d t}\,dt \quad - - \quad (1)$$

The squared magnitude of equation 1 is called the ambiguity function and when it is plotted, it is called the ambiguity diagram. The ambiguity function has a number of important properties that are described in detail at references 2 and 19. The most important ones are summarized below:

(*a*) The maximum value of the ambiguity Function is found at the origin and its value is $(2E)^2$ where E is the energy in the Pulse.

(*b*) The total volume under the Ambiguity Function is $(2E)^2$.

*The ideal Ambiguity Function*

The ideal ambiguity function would be a single spike with an infinitesimal small width as shown in (FIG.8).[2]

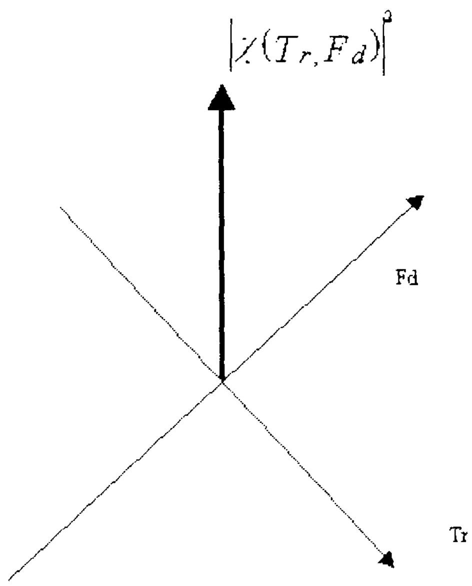



FIG.8. – IDEAL AMBIGUITY FUNCTION

The ideal ambiguity function would allow both extremely high simultaneous range and Doppler resolution. This would mean that there would be no output from the matched filter unless the Doppler of the echo matched that for the matched filter and the range resolution would be accurate and targets close together could be discriminated. However, in practice, this is an impossible ambiguity function because it does not fulfil the peak and volume properties of $(2E)$.[2] An approximation of the ideal ambiguity function, which fulfils the peak and volume properties, is represented in (FIG.9).[19]

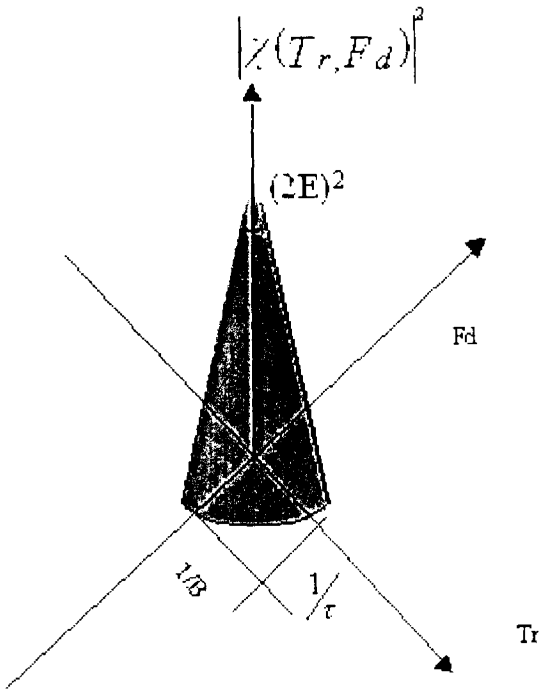



FIG.9 – APPROXIMATED IDEAL AMBIGUITY FUNCTION

Here, B is the signal bandwidth and $\tau$ is the signal pulse width. Although this may be ideal in term of satisfying the properties of the ambiguity function, the practical limitations of the range and Doppler resolutions ( $1/\tau$ and 1/B), may mean that this is ambiguity function not ideal to the radar designer.

Working within the properties of peak and volume, it is possible to view the ambiguity function as a mass of clay with a volume of $(2E)^2$. The peak at the origin must always have amplitude of $(2E)^2$. The clay can be distributed wherever one likes but no clay may be added or taken away. As discussed before, the ideal ambiguity function is one with a peak at the origin of small width, this would give good range and Doppler resolutions. This would mean that the remaining clay has to be distributed elsewhere. It may be concentrated at a peak elsewhere, or maybe distributed evenly across the range/Doppler floor. (FIG.10) shows the ambiguity function having a small peak with the consequence of a higher floor around the main peak. This ambiguity function is often referred to as the Thumbtack ambiguity function. The floor around the main peak should be as smooth as possible, any spikes or peaks in this are undesirable as they may be confused with target returns. Similarly, areas of range and Doppler in the floor may be subject to the effects of external radar clutter and therefore the height of the floor at these points may need to be minimised. The minimization of the floor in certain areas is investigated during this article.

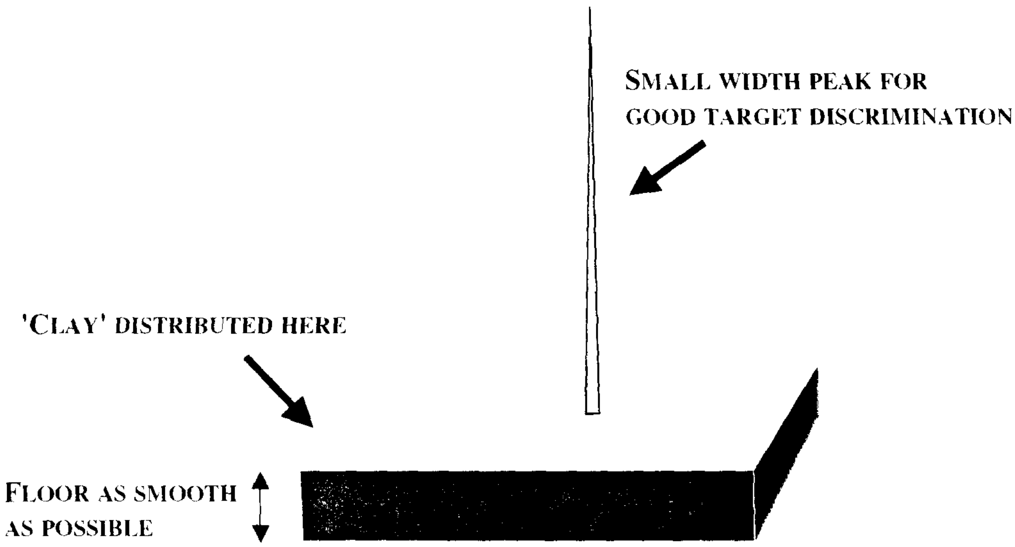



FIG.10 – THUMBTACK AMBIGUITY FUNCTION

*Example of an ambiguity function*

The ambiguity function is normally drawn as a three-dimensional plot with power (response), time and Doppler frequency as the axes. An example of a three-dimensional ambiguity function for a 13 bit Barker sequence is shown at (FIG.11). All the ambiguity functions in this article have been produced using a modified MATLAB routine.[18]

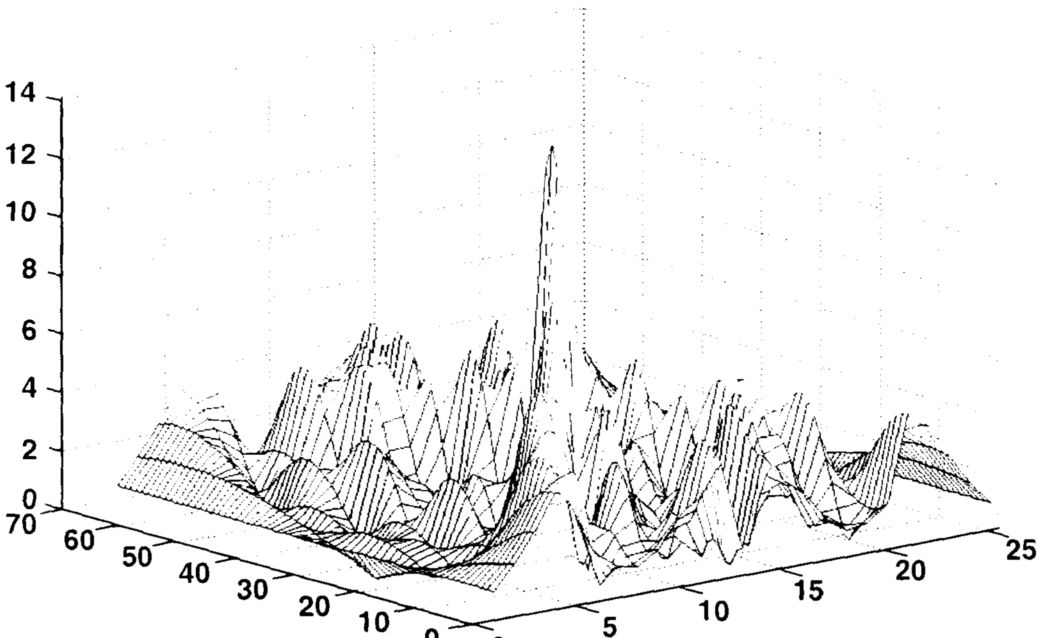



FIG.11 – AMBIGUITY FUNCTION FOR A 13 BIT BARKER CODE

Ambiguity diagrams can be plotted in two dimensions and are commonly plotted at zero Doppler and at zero range. The zero Doppler plot is the result of autocorrelation of the waveform. From this plot, the range discrimination and pulse compression of the pulse can be determined. The zero range plot can be used to determine the compressed pulses Doppler discrimination properties. These two plots for a 13-bit Barker code are shown in (FIGS.12A & B).

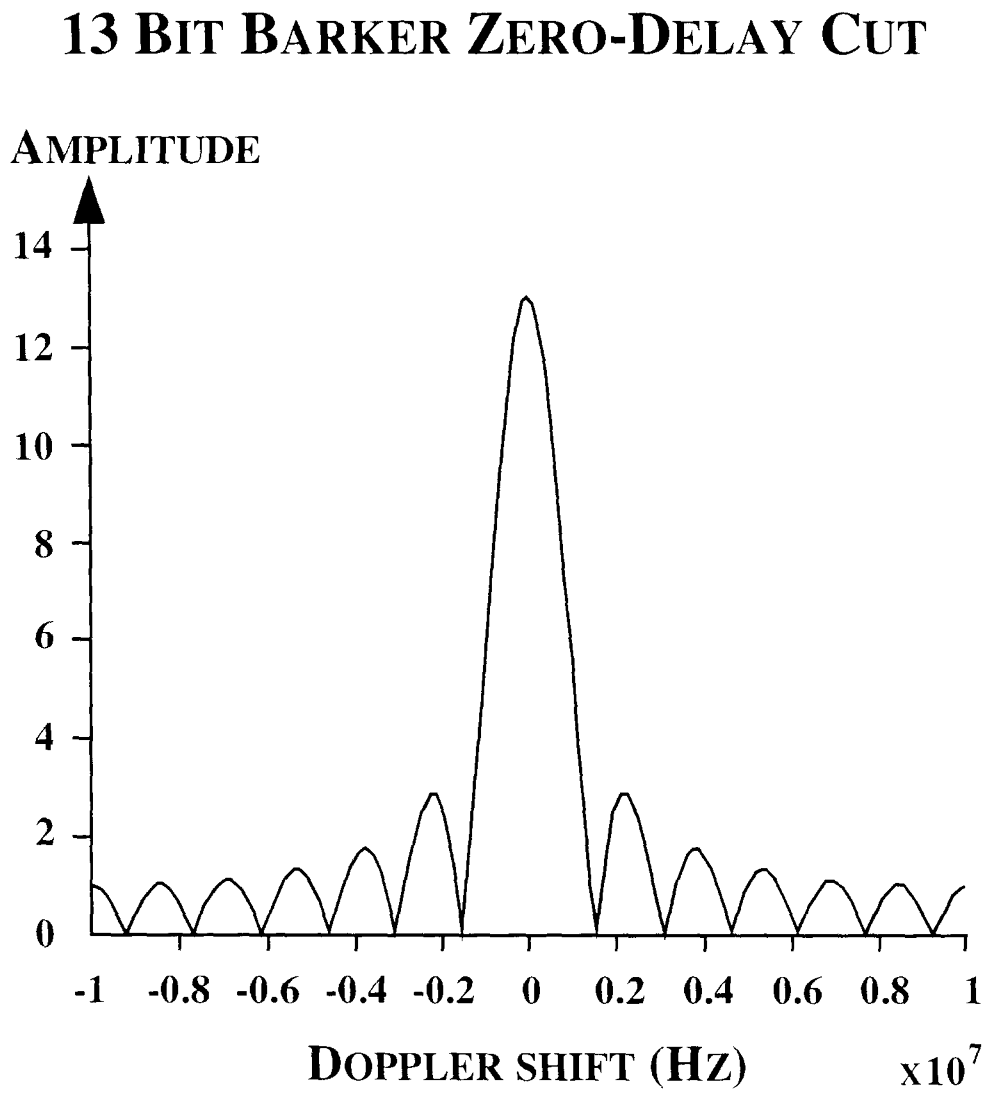



FIG.12A - ZERO RANGE PLOT OF A 13-BIT BARKER CODE

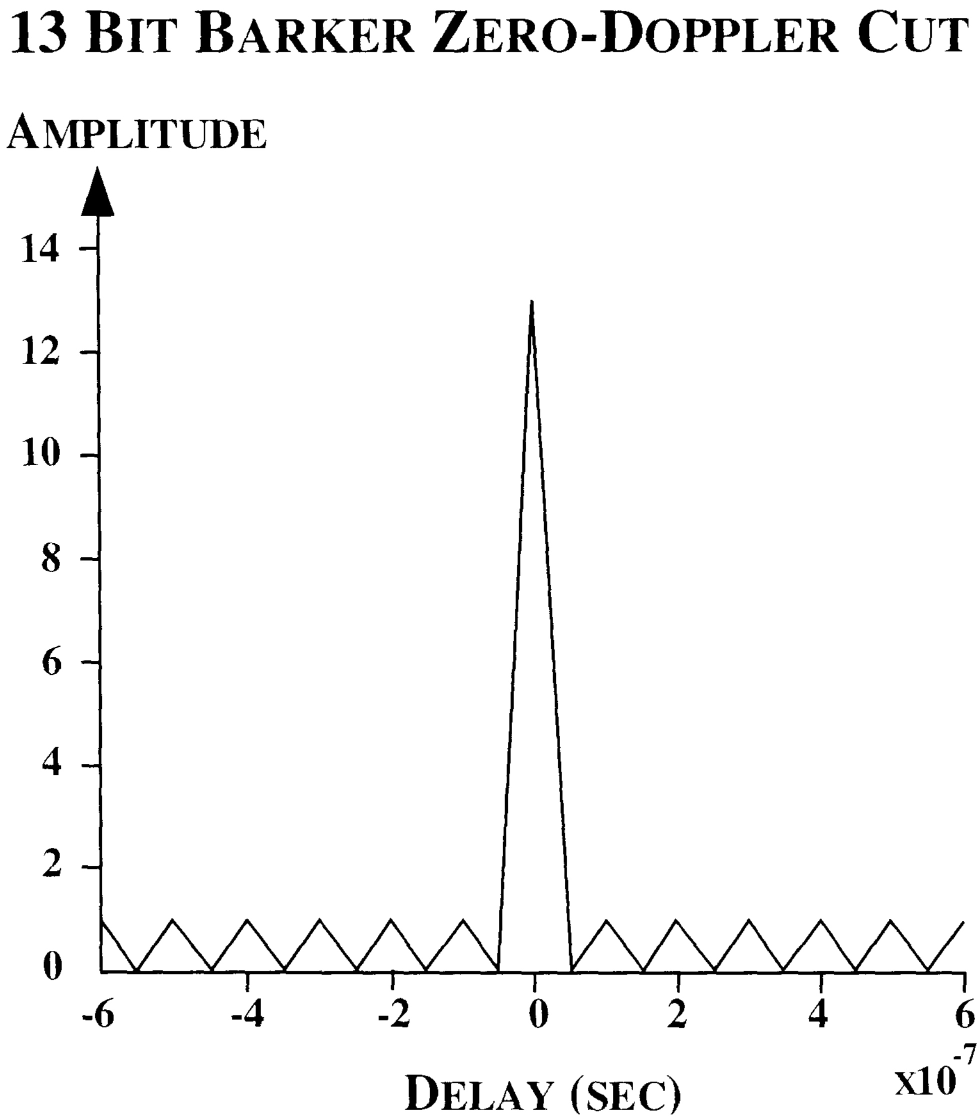



FIG.12B – ZERO DOPPLER PLOTS OF A 13-BIT BARKER CODE.

Finally, another useful plot method is the contour plot that can be tailored to show different levels from the peak of the main lobe. (FIG.13) shows the contour plot of a 13-bit Barker code.

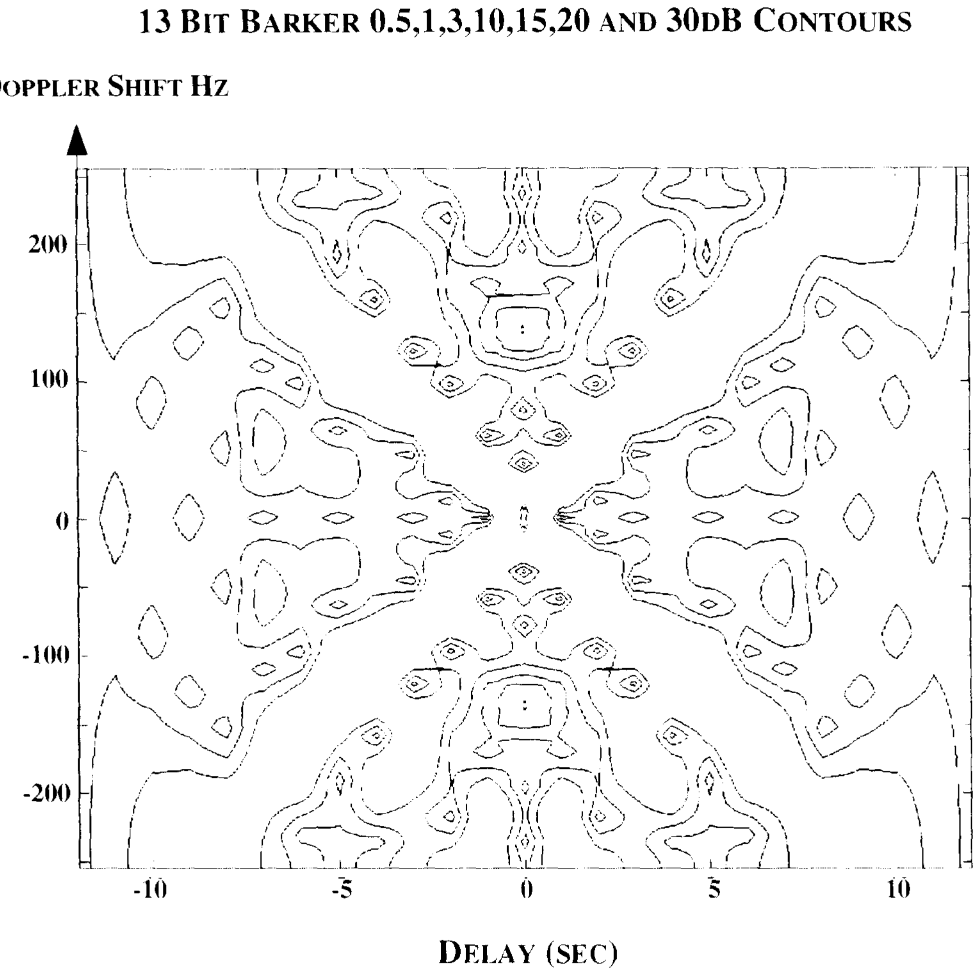



FIG.13 – CONTOUR PLOT OF A 13-BIT BARKER CODE

*Generation of a specified ambiguity function*

Producing a signal waveform to meet a specified ambiguity function is very difficult because there is no mathematical route from the desired ambiguity function to a waveform. Methods of selecting waveforms that meet a particular ambiguity function were discussed earlier. In practice, radar system designers use ambiguity functions to determine the performance of a selected waveform. Selection of a suitable waveform is either by trial and error or from previous system experience. This article examines using genetic algorithms to select practical pulse diverse waveforms to meet specified ambiguity functions.

## Composite Ambiguity Function

*Introduction*

The composite ambiguity function was introduced by GUEY and BELL[11] as a tool to measure the delay and Doppler discrimination characteristics of pulse diverse PSK FSK waveform sets. They proved that the composite ambiguity function produced a better Doppler and range resolution than single ambiguity function. The composite ambiguity function is plotted on the same axes as the ambiguity function. WONG and CHUNG[20] showed that by using a genetic algorithm as a design tool, an area of the composite ambiguity function could be minimized as to reduce the effect of interference such as clutter.

*Definition of the composite ambiguity function*

In mathematical terms, the composite ambiguity function is defined as the magnitude of the coherent sum of the individual signals' output from their matched filter. The mathematical expression is shown in equation 2.

$$C(\tau, \nu) = \left| \sum_{i=0}^{k-1} \chi s_i(\tau, \nu) \right| \quad \text{------------} \quad (2)$$

The individual signals' sidelobe level output from the matched filter is such that when they are summed coherently, the overall sidelobe level is reduced. Similarly, the main peak of the output of each individual matched filter is coherently summed. The overall effect is to enhance the main peak whilst suppressing the unwanted sidelobes.

The project examines the use of the composite ambiguity function to design practical waveform sets that offer enhanced Doppler and range discrimination. Practical FSK PSK pulse diverse waveform composite ambiguity functions will be compared to practical linear FM PSK composite ambiguity functions.

## PRELIMINARY INVESTIGATION OF RADAR CODE DESIGN AND GENETIC ALGORITHMS

### Introduction

This section outlines the results of the preliminary investigation into using genetic algorithms to search for basic radar codes that have low sidelobes when autocorrelated. This investigation was carried out in the early stages of this project and yielded some interesting results. Two types of radar codes were investigated, bi-phase and continuous polyphase codes and in each case a genetic algorithm was designed and compared to an exhaustive search of the same codes. A genetic algorithm was designed and found new 64-bit bi-phase codes with low sidelobes. In addition, a genetic algorithm was designed to search for a 13-bit generalised Barker sequence continuous polyphase code.

### Bi-phase code investigation

It was decided that the definition of a best code was one whose autocorrelation had the minimum peak sidelobe level and minimum RMS level within the sidelobes. This definition is based on the property of Barker codes, which have a minimum peak sidelobe level of unity. A MATLAB programme was designed to exhaustively search the autocorrelation of bi-phase codes of length 3-31bits and record the minimum peak sidelobe levels for each n bit code. Complementary codes have the same autocorrelation function. For example, the autocorrelation function of (1, -1, 1, -1) is the same as (-1, 1, -1, 1). This property was used in the exhaustive search to halve the search space for a given n bit code.

*Results of exhaustive search*

The results of the exhaustive search are detailed in Table 1.

TABLE.1 – *Results of exhaustive search*

| Bits | Number of Codes to Check | Min Peak SSL | SSL dB | Rms dB | Number of 'best' codes | % total codes that are 'best' |
|---|---|---|---|---|---|---|
| 3 | 4 | 1 | -9.54 | -12.55 | 2 | 500E+01 |
| 4 | 8 | 1 | -12.04 | -13.8 | 4 | 500E+01 |
| 5 | 16 | 1 | -13.98 | -16.99 | 2 | 1.25E+01 |
| 6 | 32 | 2 | -9.54 | -14.1 | 14 | 4.38E+0.1 |
| 7 | 64 | 1 | -16.9 | -19.91 | 2 | 3.13E+00 |
| 8 | 128 | 2 | -12.04 | -17.48 | 8 | 6.25E+00 |
| 9 | 256 | 2 | -13.06 | -17.32 | 8 | 3.13E+00 |
| 10 | 512 | 2 | -13.98 | -18.4 | 20 | 3.91E+00 |
| 11 | 1024 | 1 | -20.83 | -23.84 | 2 | 2.00E-01 |
| 12 | 2048 | 2 | -15.56 | -22 | 8 | 3.90E-01 |
| 13 | 4096 | 1 | -22.28 | -25.29 | 2 | 5.00E-02 |
| 14 | 8192 | 2 | -16.9 | -21.27 | 36 | 4.40E-01 |
| 15 | 16384 | 2 | -17.5 | -21.37 | 32 | 2.00E-01 |
| 16 | 32768 | 2 | -18.06 | -21.37 | 12 | 4.00E-02 |
| 17 | 65536 | 2 | -18.59 | -21.6 | 16 | 2.00E-02 |
| 18 | 131072 | 2 | -19.08 | -23.43 | 8 | 6.10E-03 |
| 19 | 262144 | 2 | -19.55 | -22.45 | 4 | 1.53E-03 |
| 20 | 524288 | 2 | -20 | -23.01 | 4 | 7.63E-04 |
| 21 | 1.05E+06 | 2 | -20.42 | -24.14 | 8 | 7.63E-04 |
| 22 | 2.10E+06 | 3 | -17.31 | -24.16 | 12 | 5.72E-04 |
| 23 | 4.19E+06 | 3 | -17.69 | -23.94 | 12 | 2.86E-04 |
| 24 | 8.39E+06 | 3 | -18.06 | -25.66 | 4 | 4.77E-05 |
| 25 | 1.65E+07 | 2 | -21.94 | -25.33 | 4 | 2.38E-05 |
| 26 | 3.36E+07 | 3 | -18.76 | -25.75 | 12 | 3.58E-05 |
| 27 | 6.71E+07 | 3 | -19.08 | -27.09 | 2 | 2.98E-06 |
| 28 | 1.34E+08 | 2 | -22.92 | -26.27 | 4 | 2.98E-06 |
| 29 | 2.68E+08 | 3 | -19.71 | -25.8 | 4 | 1.49E-06 |
| 30 | 5.37E+08 | 3 | -20 | -26.48 | 4 | 7.50E-07 |
| 31 | 1.07E+09 | 3 | -20.28 | -26.34 | 4 | 3.70E-07 |

Column 2

Details the number of codes to check which is derived from the number of bits n.

Column 3

Details the minimum peak sidelobe level of the n bit code.

Column 4

Expresses column 3 as a ratio in decibels with respect to the main peak of height n

Column 5

Details the RMS value of the sidelobes.

Column 6

Shows the number of best codes.

It should be noted that the number of best codes is the complementary value. The actual number of best codes is half the value indicated in column 6. The best codes are codes that when autocorrelated satisfy the minimum peak sidelobe criteria and the minimum RMS criteria. The final column shows the percentage of the best codes out of the total number of codes. As n increases, this percentage decreases. This implies that when searching for the best codes, the genetic algorithm will have to be very good to find these codes. This result is useful when determining the performance of the genetic algorithm search.

It was decided that the reference for the genetic algorithm would be the 25-bit code and therefore, the statistics of the 25-bit code will be discussed. A histogram of the minimum peak sidelobe levels found by the exhaustive search of a 25-bit code is shown at (FIG.14).

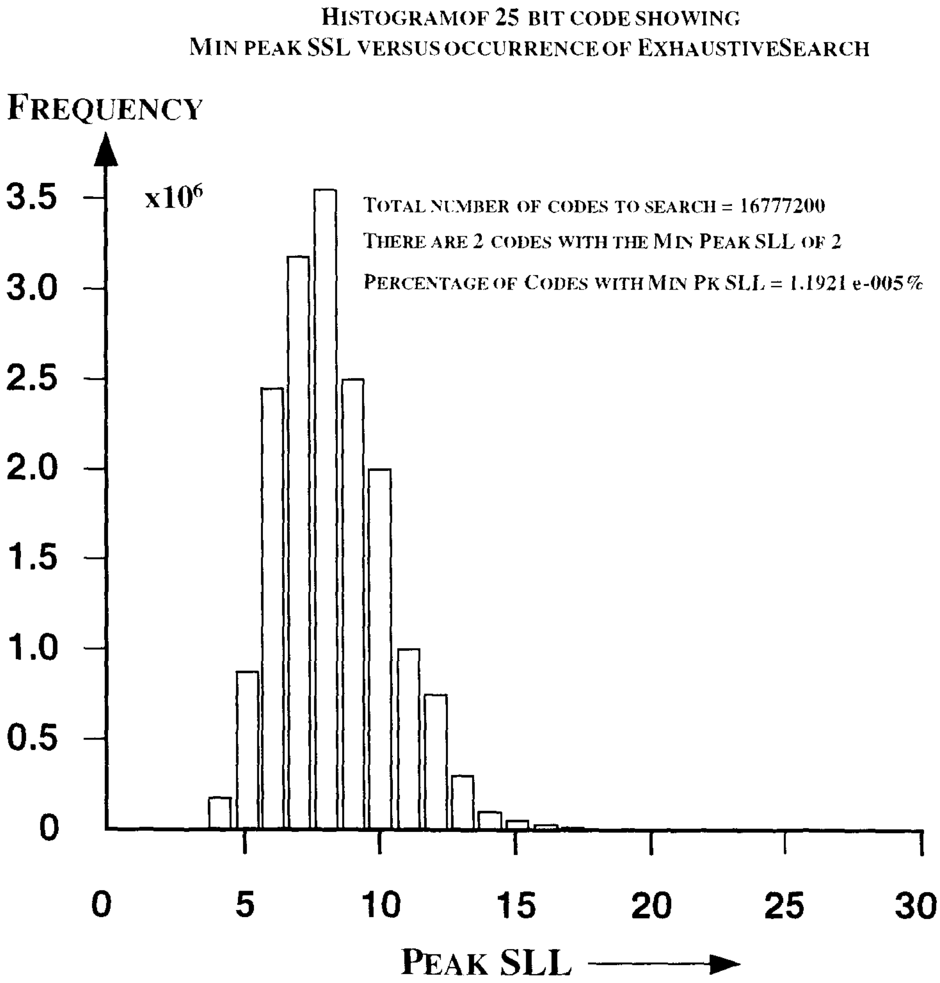



FIG.14 – HISTOGRAM OF MINIMUM PEAK SIDELOBE LEVELS FOR A 25 BIT CODE

There are only two codes from all the possible combinations of 25 bit codes that have a peak sidelobe level of 2. This highlights the fact that the genetic algorithm will have a difficult task in finding the best code due to the very small percentage of best codes ($1.92 \times 10^{-5}$ %) of the total search space. The search space for a 25-bit code is now known. A genetic algorithm can be designed and its performance tested against this reference.

*Genetic algorithm search for 25-bit bi-phase codes*

The operation of a genetic algorithm has been discussed earlier. A genetic algorithm was designed, using the Genetic Algorithm Toolbox in MATLAB[17] to search for the best 25-bit code found in the exhaustive search.

*Genetic Algorithm Design*

The number of variables in the chromosome was set to the length of the code (n). The value of the individual gene within the chromosome could be either +1 or -1. The chromosome is passed to the objective function for evaluation. The objective function simply autocorrelated the code (chromosome) and recorded the value of the peak sidelobe level. A desired minimum peak sidelobe level was introduced as a target for the genetic algorithm. The sidelobe levels of each code produced by the chromosome were compared to the desired level. A difference matrix was generated and the sum of this matrix was used as the output from the objective function. This output had to be minimized and once the difference is zero, the genetic algorithm had found the code that had the desired sidelobe level. This process is shown in (FIG.15).

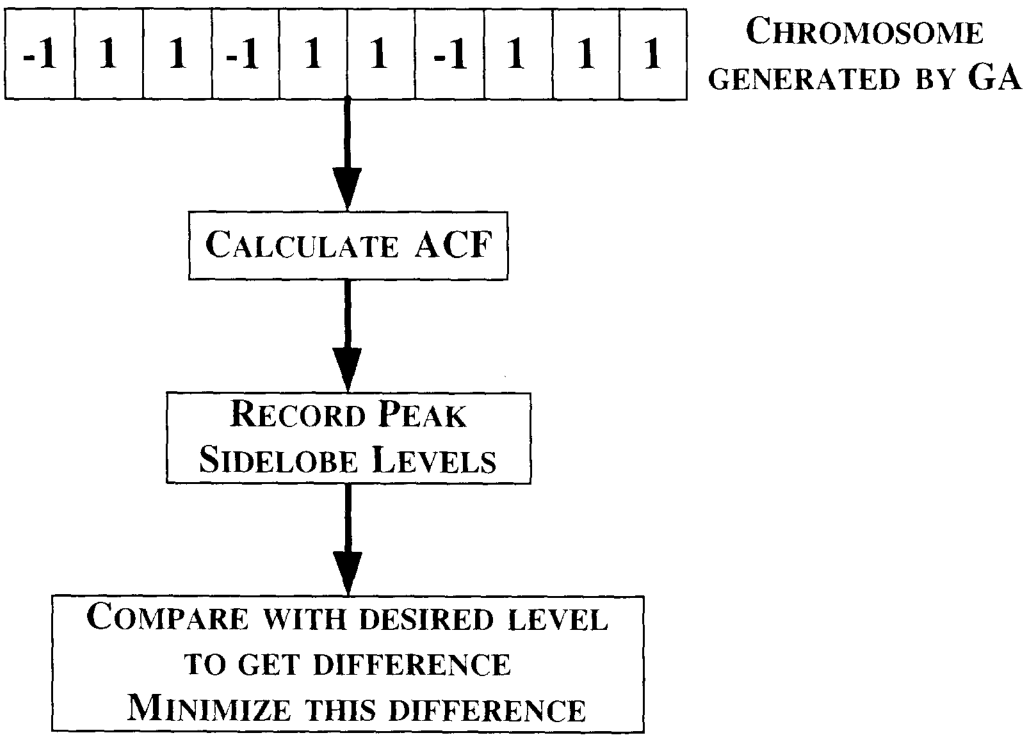



FIG.15 – BI-PHASE CODE GENETIC ALGORITHM REPRESENTATION

*Results of Genetic Algorithm search of a 25 bit Bi-phase code*

The desired sidelobe level was set at 2. The genetic algorithm was repeated 110 times and the results are shown in the histogram at (FIG.16).

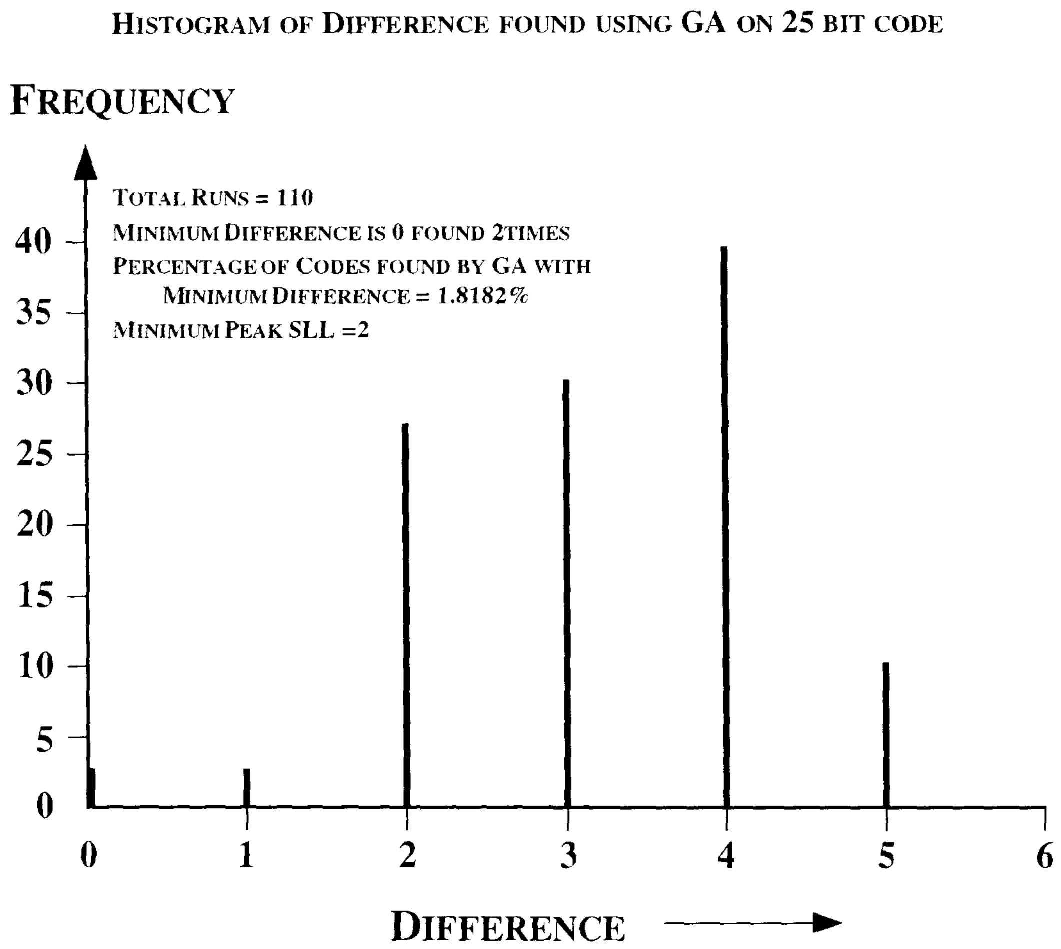



FIG.16 – RESULTS OF THE GENETIC ALGORITHM SEARCHING FOR THE IDEAL 25 BIT CODE

The results show that the genetic algorithm managed to find the desired sidelobe level of 2 twice in 110 runs (1.81%). The autocorrelation of the best code found by the genetic algorithm is shown in (FIG.17). The desired sidelobe level of 2 is plotted for reference. The parameters used in the genetic algorithm are also detailed on FIG.17.

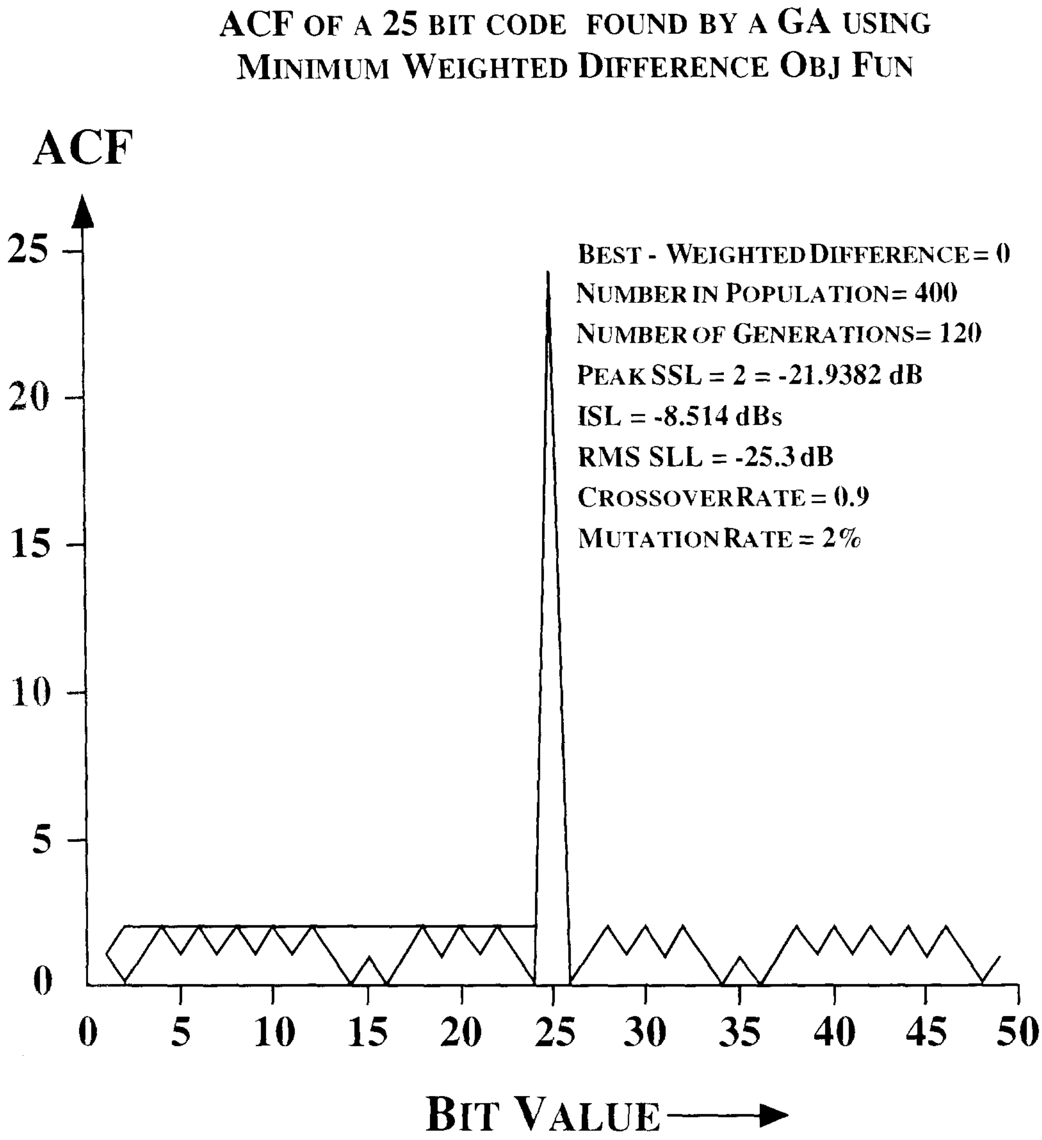



FIG.17 – AUTO CORRELATION OF 25-BIT BI PHASE CODE FOUND BY GENETIC ALGORITHM

This result proved that the genetic algorithm was capable of searching and finding a small number of optimal solutions in a very large search space. This result gave the necessary confidence to develop the genetic algorithm to search for longer codes (n>31) and that any result gained could be close to, if not at, the global minimum.

*Genetic algorithm search of a 64-bit bi-phase code*

It was decided to extend the search to n=64bits. There are no exhaustive search results for codes of length 64 bits. In 1990, an exhaustive search of 48 bits took 16 days.[3] If one assumes that this 48-bit exhaustive search could be carried out in today in 24hrs, an exhaustive search of 64 bits would take approximately 180 years.

*Results of genetic algorithm search of a 64-bit bi-phase code*

The desired sidelobe level was set at an arbitrary level of 5. The genetic algorithm was run 140 times. Various crossover rates and initial population sizes were used until a combination was found that would achieve the desired sidelobe level. The histogram of the results is shown in (FIG.18).

HISTOGRAM OF DIFFERENCE FOUND USING GA ON 64 BIT CODE

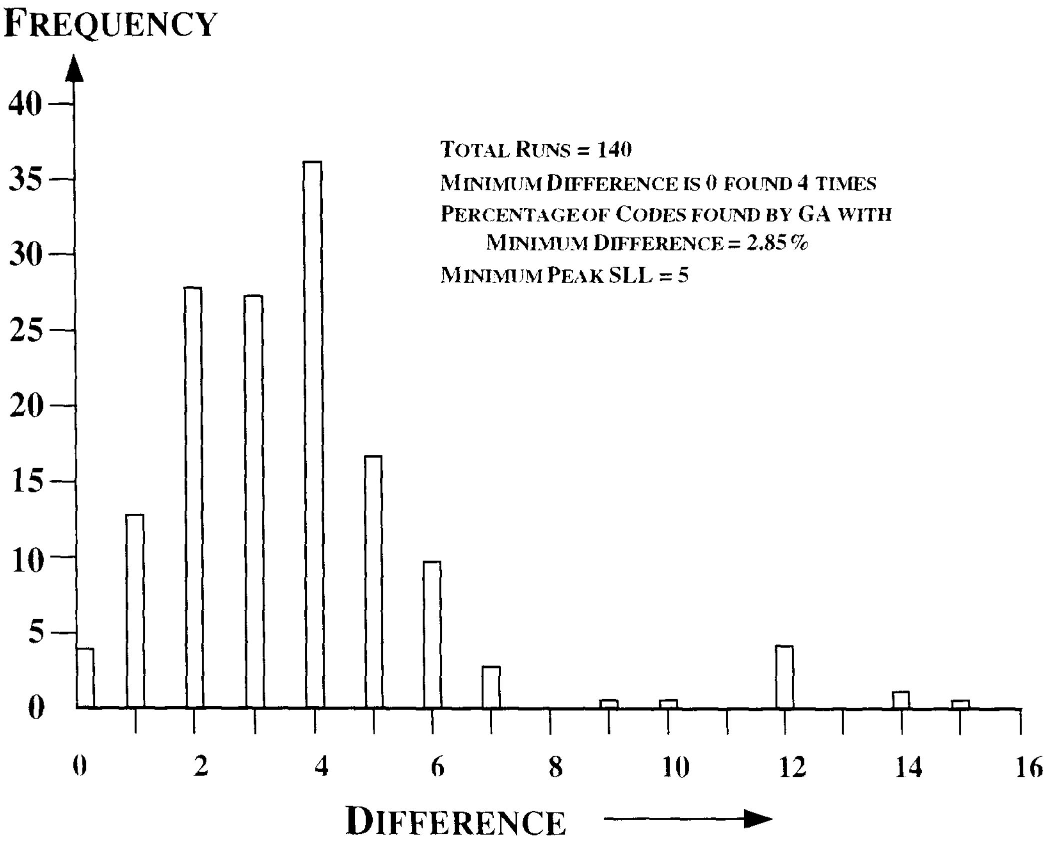



FIG.18 – HISTOGRAM OF RESULTS OF 64 BIT GENETIC ALGORITHM SEARCH

FIG.18 shows that four codes of minimum peak sidelobe level of 5 were found. The genetic algorithm is finding the codes that produce minimum peak sidelobe close to the desired level of 5. There were no codes found with a minimum peak sidelobe level of less than this. This is indicated by the grouping of frequency of occurrence towards the left in FIG.18.

Four best codes were found. The autocorrelation of two of them, along with the genetic algorithm parameters used is shown in (FIG.19).

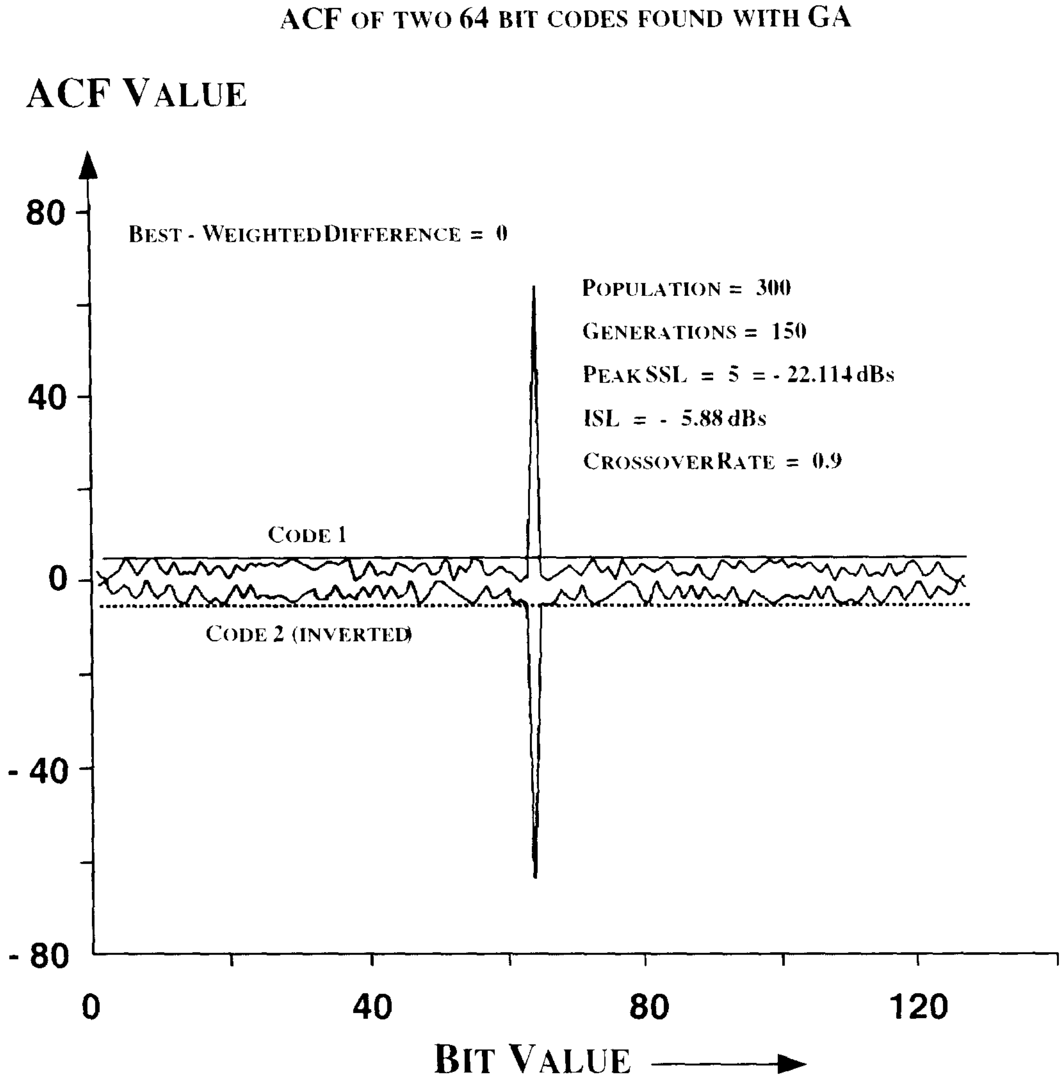



FIG.19 – AUTO CORRELATION OF TWO 64-BIT CODES FOUND BY GENETIC ALGORITHM

The 64-bit codes are too long to display in bi-phase form and therefore they are displayed in hexadecimal. The two codes are:

(*a*) 7A2242EE92714800 (hex) Code 1

(*b*) C491C95E6FFA0000 (hex) Code 2

These two codes are shown because the cross-correlation of these two particular codes gives the smallest peak cross correlation product when compared to the cross correlation products of the other codes in turn. Code 2 has been inverted such that both codes can be shown on the same plot.

*Comparison of 64 bit code result to other search methods*

Two 64 bit codes were discovered using Simulated Annealing in 1993.[4] These two codes had a minimum peak sidelobe level of 11 and a minimum peak cross correlation product of 11. The cross correlation product of two codes is an important issue when two codes are used for radar and communications applications. Here, the cross correlation product needs to be reduced as much as possible so that two separate codes of the same length can be used on the same channel.

Clearly, the codes found in this project have a better minimum peak sidelobe level of 5 (cf. 11) however, the best cross correlation product was found to be 15, which is poorer than 11 found by Simulated Annealing.[4] The cross correlation product of codes 1 and 2 are shown in (FIG.20).

**CROSS CORRELATION OF TWO 64 BIT CODES FOUND WITH GA**

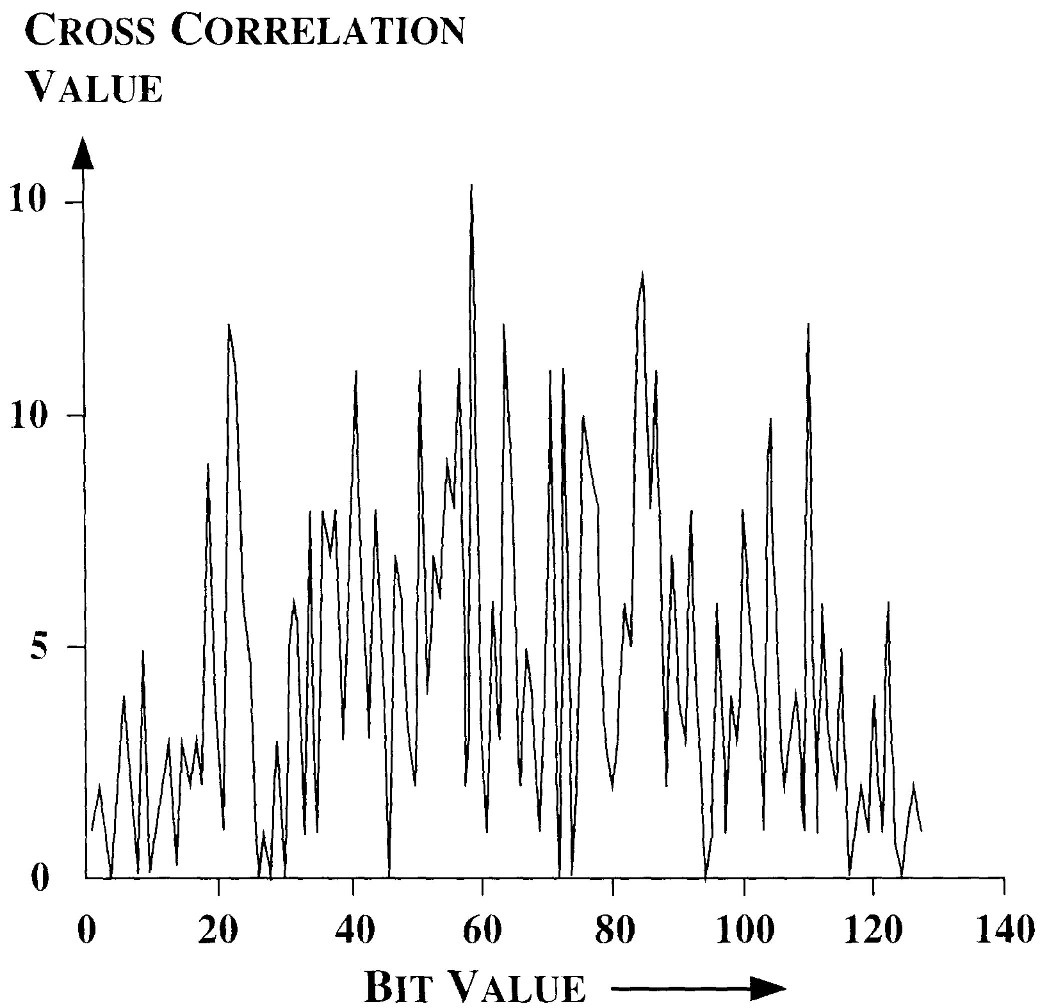



FIG.20 – CROSS CORRELATION OF TWO 64 BIT CODES FOUND BY GENETIC ALGORITHM

The objective function of the genetic algorithm was not designed, in this case, to search for two codes with minimum peak sidelobe levels and minimum cross correlation product. In the opinion of the author a genetic algorithm could be designed to search for this objective, however, due to limiting time, it was decided that this would not be pursued. Furthermore, best codes of length greater than 64-bits (128, 256, 512 etc.) could have been sought but this was not the main aim of this project. It is left as an area of further research.

### Generalized BARKER Sequences investigation

Having completed the investigation into bi-phase codes, the next logical step was to investigate polyphase coding. This investigation was not as detailed as the bi-phase research for a number of reasons outlined at the end of this section. To demonstrate the applicability of the use of genetic algorithms as a search tool in this area, a genetic algorithm was developed to search for a 13-bit continuous phase Generalized BARKER Sequence.

*Definition of Generalised BARKER Sequences*

The term polyphase coding means that the phase of each bit can be chosen from an alphabet of size (M>2). They are time-discrete complex sequences with constant magnitude and variable phase.[5] The alphabet (M) can be any length from 2 (bi-phase) to infinity (continuous phase). It is possible, using polyphase codes with a suitable phase alphabet (M), to design polyphase codes such that the autocorrelation in the zero Doppler plane has sidelobe levels less than unity. These codes are called Generalized Barker Sequences.[6]

*Design of genetic algorithm to search for Generalised BARKER Sequences*

Previous research[5] using an iterative algorithm, defined the phases used to generate a 13 bit continuous phase Generalized BARKER Sequence. A genetic algorithm was developed to prove its applicability in this area.

The genetic algorithm produced a population of chromosomes with individual genes that each had a phase value (in radians). Each chromosome was passed to the objective function where, using the phases defined in the chromosome, a waveform was produced. The peak sidelobe level of the autocorrelation of this waveform was computed. The genetic algorithm attempted to minimize the peak sidelobe level to below unity to find a Generalized BARKER Sequence. The objective function is summarised in (FIG.21).

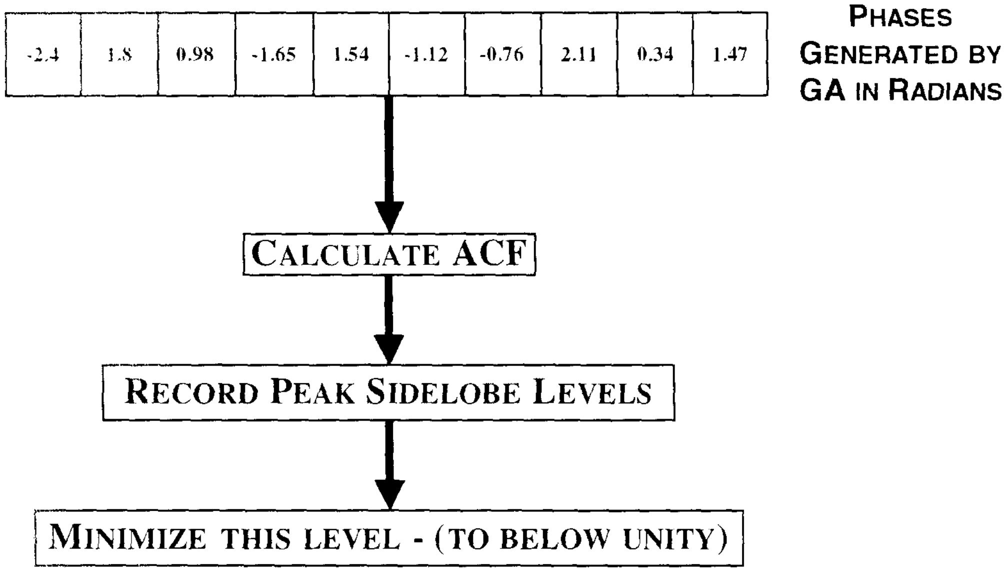



FIG.21 – GENETIC ALGORITHM REPRESENTATION FOR GENERALIZED BARKER SEQUENCE

*Results of genetic algorithm search*

The parameters used by the genetic algorithm are shown in Table 2

TABLE 2 – *Genetic algorithm parameters for 13 bit continuous phase code*

| Parameter | Value |
|---|---|
| Population | 200 |
| Generations | 300 |
| Crossover rate | 0.85 |
| Mutation Rate | 1% |
| Generation Gap | 0.9 |

The phases (in radians) of the best code found by the genetic algorithm are:

| Phase | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Radians | 5.56 | 5.42 | -1.18 | 5.62 | 4.07 | 1.87 | 7.81 |
| Phase | 8 | 9 | 10 | 11 | 12 | 13 | |
| Radians | -0.93 | 2.78 | 5.27 | 1.05 | 1.51 | 3.73 | |

These are different from previous recorded results.[5] This code gives a 13-bit Generalized BARKER Sequence autocorrelation function shown in (FIG.22). It can be seen that the peak sidelobe level is unity with the remaining sidelobe levels below that value. The peak sidelobe is –22.29dB from the main lobe peak, the same as a 13-bit BARKER code.

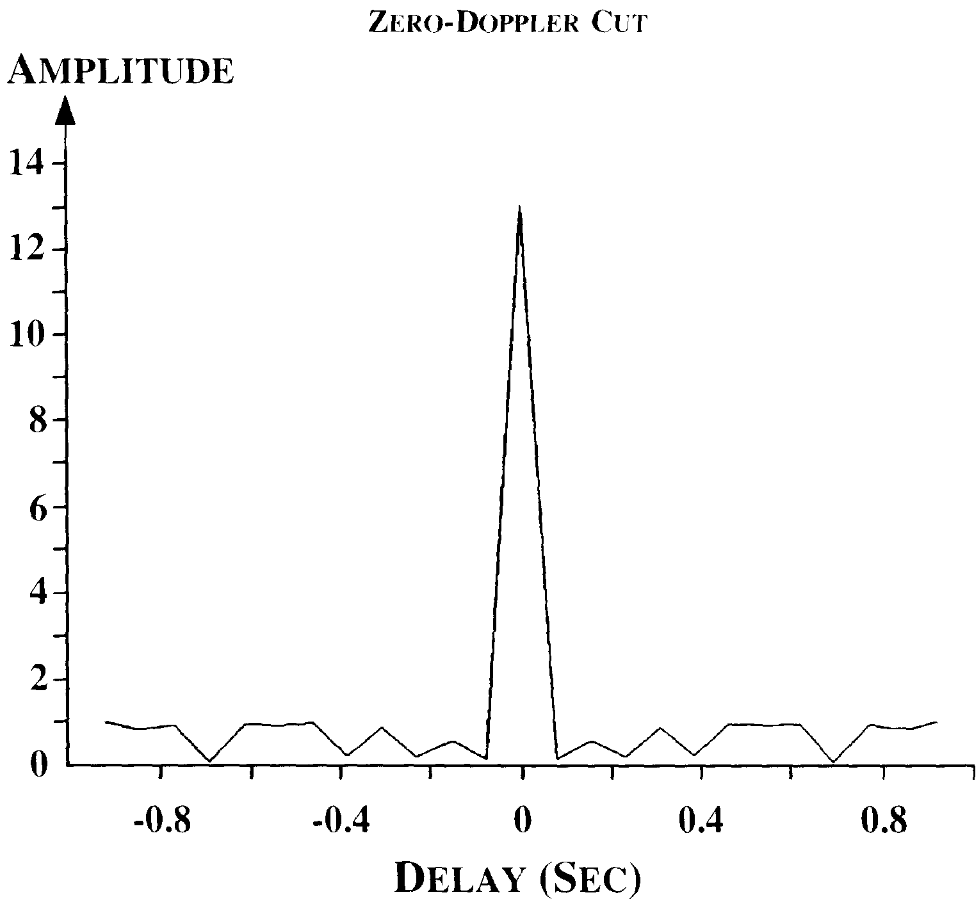



FIG.22 – AUTO CORRELATION OF 13 BIT GENERALIZED BARKER SEQUENCE FOUND BY GA

The practicalities of generating the continuous phase are not examined in detail, however, previous research[9] suggest that these would be difficult to generate. Moving to an alphabet of say 60 would be more practical. A genetic algorithm to search for these polyphase codes could be investigated as part of future research.

## Preliminary investigation conclusion

A genetic algorithm can be used to find bi-phase codes with minimum peak sidelobes. This was proved by finding the two best 25-bit bi-phase codes. The genetic algorithm was developed further to search for 64-bit codes and four new codes were discovered that had a better minimum peak sidelobe level than codes found by previous research using a different search method. Search of longer length codes was not pursued but the author is confident that genetic algorithms could be applied in this area.

A genetic algorithm was designed to find a 13-bit polyphase Generalized Barker Sequence. The principle of using genetic algorithms to search for polyphase codes has been proved, albeit on relatively short length codes. Based on the experience gained during the investigation of the bi-phase codes, it is the opinion of the author that if more time were available, a genetic algorithm could be designed to search for codes of length 45+. This could be the subject of further research later.

## PULSE DIVERSE WAVEFORM DESIGN

### Introduction

Previous research[13] suggested that a pulse diverse PSK FSK waveform could be designed to produce an ambiguity function that had minimal sidelobe levels in specified areas of range and Doppler. A genetic algorithm was used to design such a pulse but the phase increments and frequencies used were not realistic for current radar design. This section will outline the design of a practical PSK FSK pulse diverse waveform that can be designed to meet a specified ambiguity function and composite ambiguity function.

In this project, a variation to the FSK PSK waveform is proposed. It is called a Linear FM PSK waveform. A practical design of this new waveform is examined such that it meets the same specified ambiguity function and composite ambiguity function as the FSK PSK case.

### PSK FSK waveform design

The PSK FSK waveform[13] is a single pulse containing six individual chips where each chip is capable of having a different frequency and phase from the next. This is shown in (FIG.23).

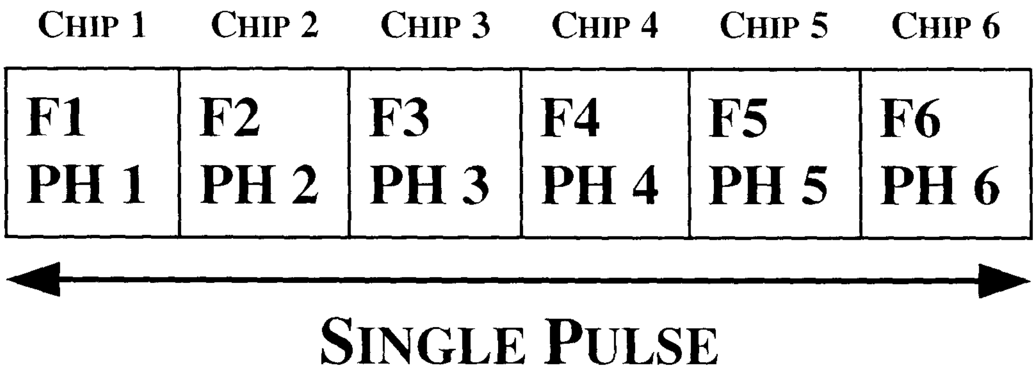



FIG.23 – GENERIC PSK FSK WAVEFORM DESIGN

The chips within the pulse diverse PSK FSK waveform in[13] were designed such that the frequency and phase increments are not practical for radar system design. For instance, the frequency range was between 10 Hz and 74 Hz and the phase was incremented in 1024 steps of 360° or steps of $\pi/512$ radians. These values are clearly impractical for modern radar design.

#### *Practical pulse design*

A radar operating at 5GHz may typically have a bandwidth of 10 % ~500 MHz.[3, p203] A typical uncompressed pulse length of 1.2μs equates to a range resolution of

180m. It was decided initially to design a pulse to approximately these parameters. A pulse length of 1.2µs would allow a chip width of 0.2µs (assuming six chips). The sampling frequency would have to be at least twice the highest frequency (Nyquist) and therefore at least 1GHz sample frequency was required for a maximum intermediate frequency of 500 MHz. It follows that each chip of 0.2µs would consist of 200 samples and the whole pulse of six chips would contain 1,200 samples. When this sampled signal was passed through the ambiguity function MATLAB programme[21] it took 2 minutes to compute an output and used up most of the computer memory such that it became very slow to run. To enable the FSK PSK and Linear FM PSK waveform comparison, the ambiguity function had to be repeated 1,000 times per genetic algorithm run. It was planned to do at least 90 good genetic algorithm runs in total for this investigation and this would have taken approximately three weeks. This was deemed unacceptable and a shorter pulse with a lower sampling frequency was required.

It was decided to reduce the size of the pulse and reduce bandwidth of the radar to 200 MHz. This in turn reduces the operating frequency to 3GHz. The practical frequency of each chip is in the range of 20 – 200 MHz and the practical phase was in 128 steps of 360° (2.1825°). The sampling frequency was chosen to be 500MHz and thus the sampling interval is 2ns. Each chip contained 51 samples making 306 samples for the six-chip pulse. The new length of the pulse is reduced from 1.2µs is 612ns. The pulse is shown graphically in (FIG.24).

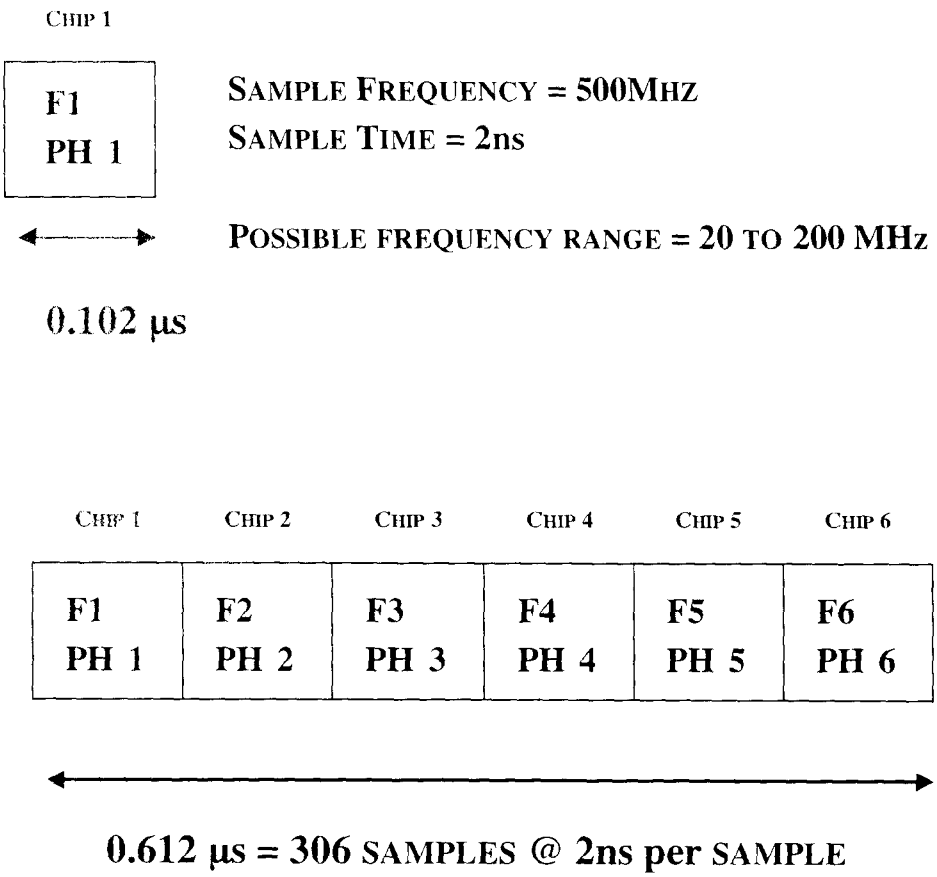



FIG.24 – PRACTICAL PSK FSK PULSE DESIGN

For a 3 GHz radar, a maximum range of 180km is assumed. The maximum unambiguous range is set to 225km (1.25 x max range) and therefore the PRF is set at 600Hz. The radar's parameters can be summarized in Table 3.

TABLE.3 – *Radar parameters for PSK FSK design*

| Parameter | Value |
|---|---|
| Frequency | 3GHz |
| Frequency chosen within the range within each chip | 20-200MHz |
| Sampling frequency | 500MHz (2ns) |
| Pulse width (uncompressed) | 0.612μs |
| Samples per pulse | 306 |
| Max Unambiguous Range | 225km |
| PRF | 600Hz |

The sampled FSK PSK waveform shown in FIG.24 can be generated easily within MATLAB

## Linear FM PSK waveform

In the section above, the frequency and phase in each of the six chips of FSK PSK pulse diverse waveform were constant throughout the duration of the chip. A variation of this could be that the frequency within the chip is altered throughout the chip duration by effectively applying a frequency sweep in each chip. The phase in each chip remains constant. This new pulse diverse waveform, introduced in this research, is referred to as a Linear FM PSK pulse diverse waveform. The ambiguity function of this new waveform is compared to that of the FSK PSK case to see which waveform offers a better performance. The comparison in performance is discussed later.

### *Practical Linear FM PSK waveform design*

The radar parameters are the same as those detailed in table 3, the only difference is that now there is a start and stop frequency of the individual chip. The start and stop frequencies can be any frequency between 20 and 200 MHz that facilitates a possible up or down frequency sweep.

The Linear FM PSK pulse diverse waveform used in this research is split into six chips as shown in (FIG.25). This sampled waveform can be generated easily by MATLAB.

CHIP 1

| |
|---|
| F1 START<br>F1 STOP<br>PH 1 |

SAMPLE FREQUENCY = 500MHZ
SAMPLE TIME = 2ns

←——→ POSSIBLE START AND STOP FREQUENCIES = 20 TO 200 MHZ

**0.102 μs**

| CHIP 1 | CHIP 2 | CHIP 3 | CHIP 4 | CHIP 5 | CHIP 6 |
|---|---|---|---|---|---|
| F1 START<br>F1 STOP<br>PH 1 | F2 START<br>F2 STOP<br>PH 2 | F3 START<br>F3 STOP<br>PH 3 | F4 START<br>F4 STOP<br>PH 4 | F5 START<br>F5 STOP<br>PH 5 | F6 START<br>F6 STOP<br>PH 6 |

←——————————————————→

**0.612 μS = 306 SAMPLES @ 2NS PER SAMPLE**

FIG.25 – LINEAR FM PSK WAVEFORM

*Ambiguity function of the waveforms*

A MATLAB programme[21] was used to generate the ambiguity function of the pulse diverse waveforms. Recalling that the pulse length is 0.612μs and the PRF is 600Hz, (Table 3) the size of the axis of the ambiguity function are ±0.612μs along the delay axis and ±300Hz along the Doppler axis. These axes are set by the ambiguity function programme and can not be altered.

The delay axis can be converted to a range axis of ± 92m using equation 3.

$$R = c\tau/2 \quad \text{— — — — — — — — — — — — — — —} \quad (3)$$

Where c=3x10$^8$ m/s, $\tau$ is pulse width in seconds, R is range in metres

The Doppler axis can be converted from Doppler frequency of ± 300Hz to a relative velocity axis of ± 15m/s using equation 4.

$$v = F_d\lambda/2 \quad \text{— — — — — — — — — — — — — — —} \quad (4)$$

Where v is in m/s, $f_d$ is in Hertz, $\lambda$ is in metres (0.1m)

It should be noted that the peak of the ambiguity function is at the origin. This is one of the properties of the ambiguity function discussed earlier.

*Composite ambiguity function of waveform sets*

The composite ambiguity function of a set of say six waveforms can be generated by summing each individual signal's ambiguity function. This principle was discussed earlier. The composite ambiguity function is simple to achieve in

MATLAB by modifying the ambiguity function programme. The range and velocity axes will be the same as those described above.

*Use of MATLAB*

As stated above, all the waveforms were designed using MATLAB. This computer language was chosen for two reasons:

(*a*) The Genetic Algorithm Toolbox[17] had been designed for MATLAB.

(*b*) A MATLAB function to produce an ambiguity function of a specified waveform had been designed.[21]

However, MATLAB is a slower language than, say, 'C' and this was a penalty during this project.

*Pulse design conclusion*

The waveforms that are to be investigated during this project can be generated by MATLAB. The sampled waveforms can be input to the ambiguity function programme and their individual ambiguity functions can be calculated and displayed in terms of velocity and range. Sidelobe levels within specified areas of the ambiguity function can be examined. It will be shown that sidelobe levels within specific areas of the ambiguity function can be minimized by designing the waveforms using a genetic algorithm.

## RADAR WAVEFORMS AND GENETIC ALGORITHM DESIGN

### Introduction

There is no mathematical route from a specified ambiguity function to a waveform that generates that particular ambiguity function. Taking the practical pulse diverse waveforms, one possible method would be to try every possible combination of frequency and phase in each chip and record the generated ambiguity function. The waveform whose ambiguity function gives the best fit to the specified shape could be selected. This is clearly impractical, as the number of possible permutations is excessive.

It has been shown that a genetic algorithm could be designed to search for bi-phase codes that when autocorrelated generated a minimum peak sidelobe level that is the same as some arbitrary desired level. Taking this one stage further, it is possible to design a genetic algorithm that would search for the phase and frequency parameters of pulse diverse waveforms that produce an ambiguity function that is minimized in some area. A similar argument applies when minimizing areas within a composite ambiguity function of a set of pulse diverse waveforms. This section outlines the selection of the masked areas and the design of the genetic algorithm that searches for the optimum waveform parameters.

### Mask design – Areas of minimization

As discussed earlier, the FSK PSK and Linear FM PSK pulse diverse waveforms produce ambiguity functions over the range ±15m/s in velocity and ±92m in range. Three masks of different sizes were designed for this research such that the peak levels within the masks should be reduced. The choice of area is somewhat arbitrary but if a range and velocity of clutter or interference were known then any mask could be designed accordingly.

The masks are shown in (FIGS 26, 27 and 28). The black area of the mask is the area where the sidelobes are to be minimized. The corresponding range and velocity for each area can be read from the plots.

Mask 1.

An arbitrary 'normal' mask.

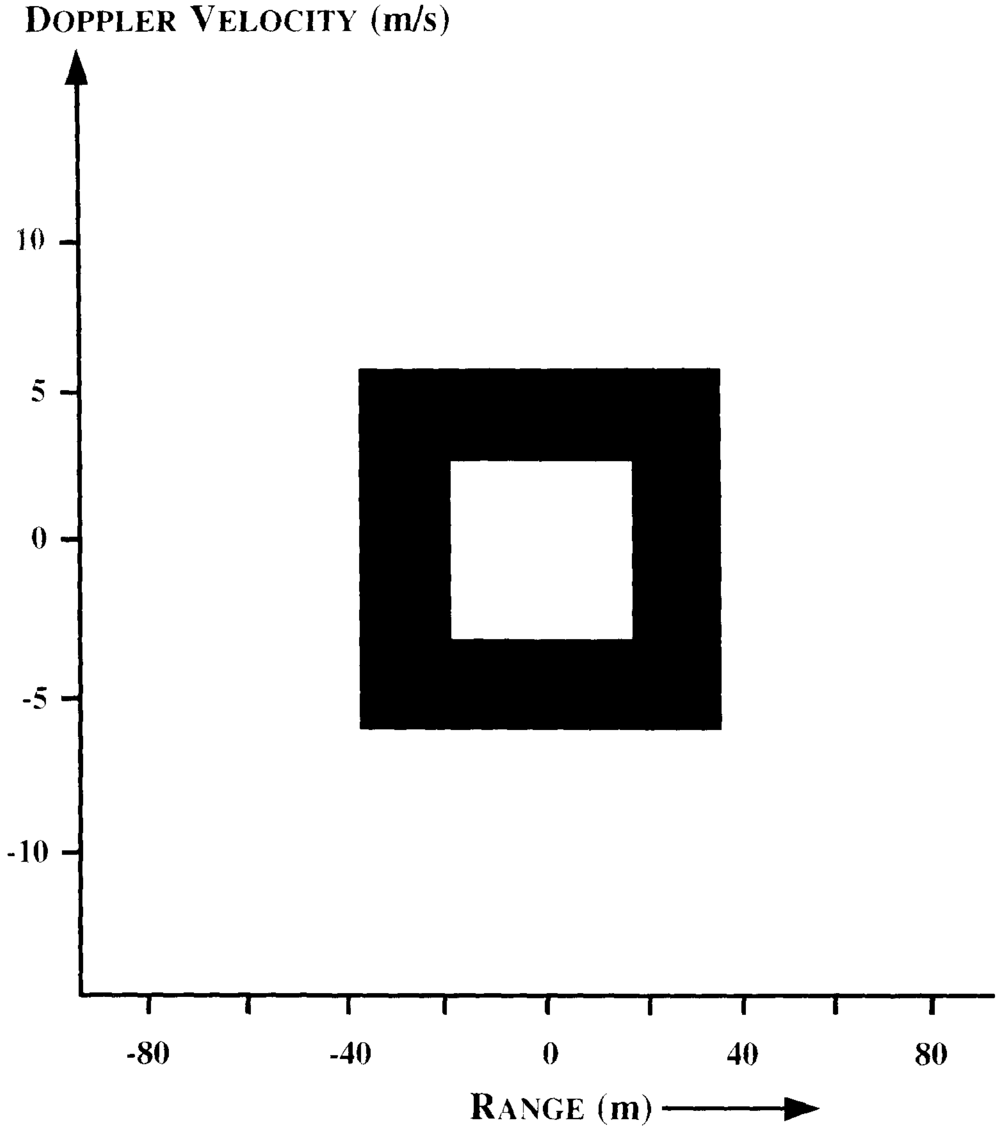



FIG.26 – MASK 1

Mask 2.

Based on Mask 1 but with the area in which the sidelobes are to be reduced is biased towards the centre peak.

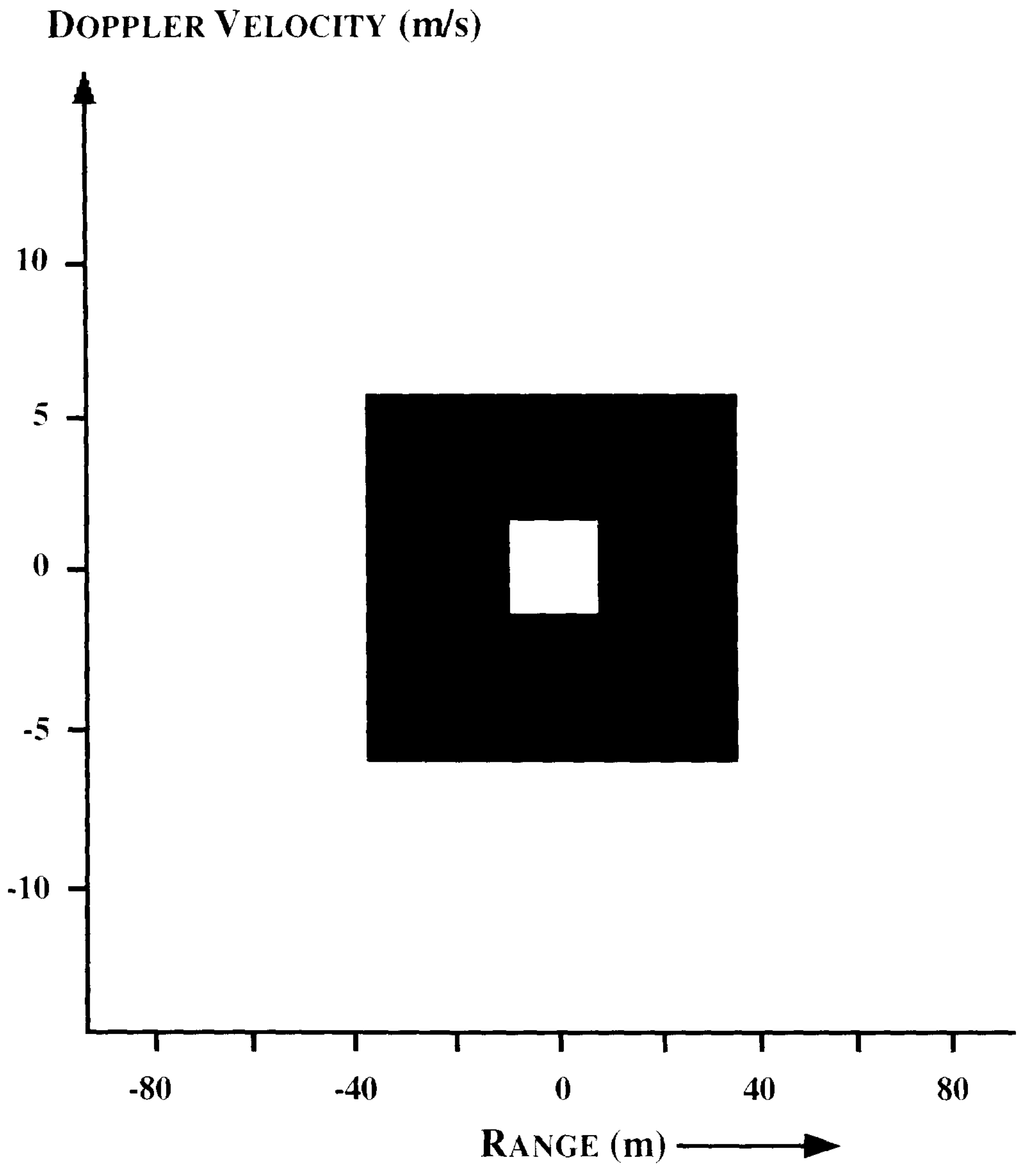



FIG.27 – MASK 2

Mask 3.

The same inner boundaries as Mask 2 but the area specified for sidelobe reduction is extended away from the central peak.

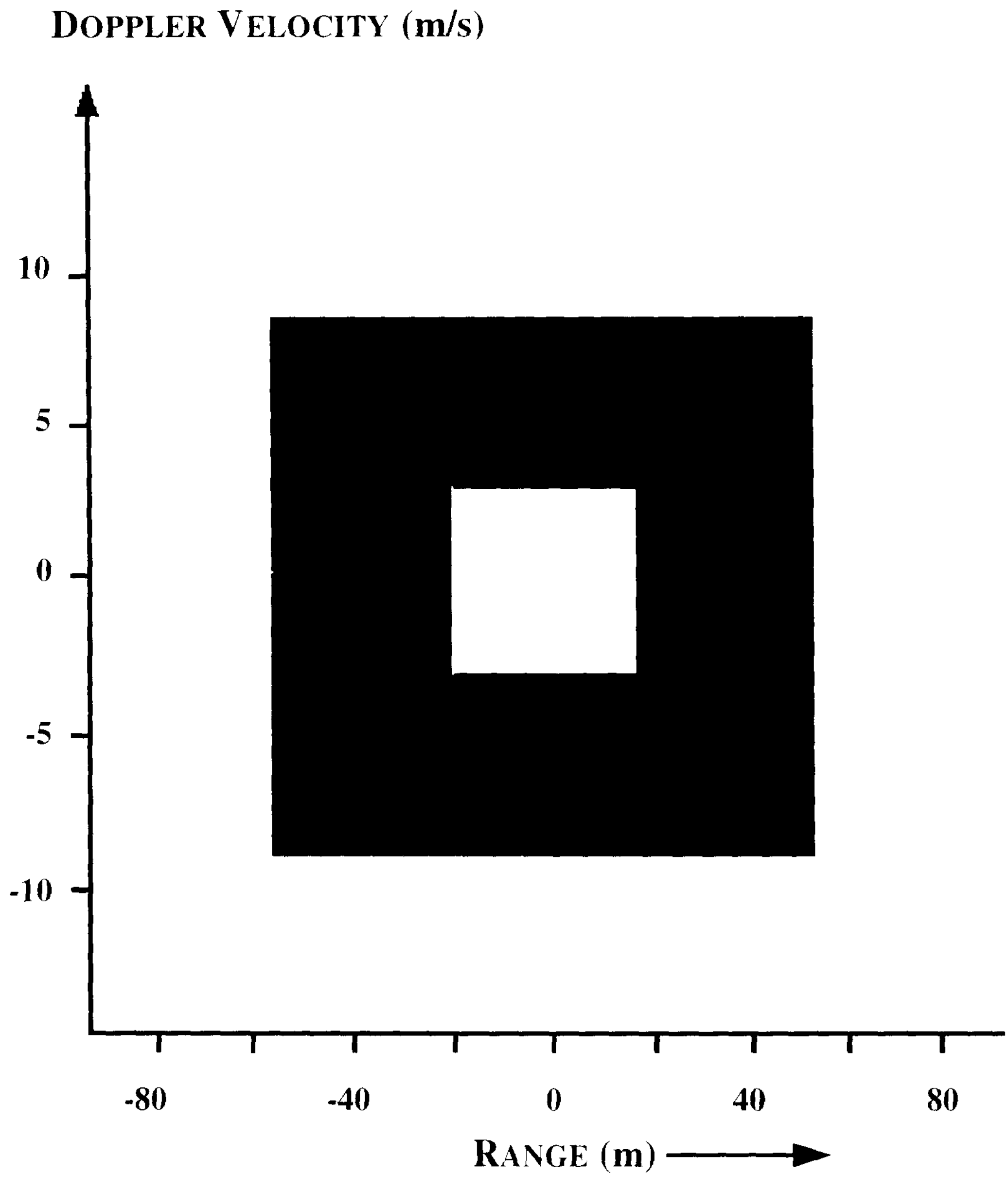


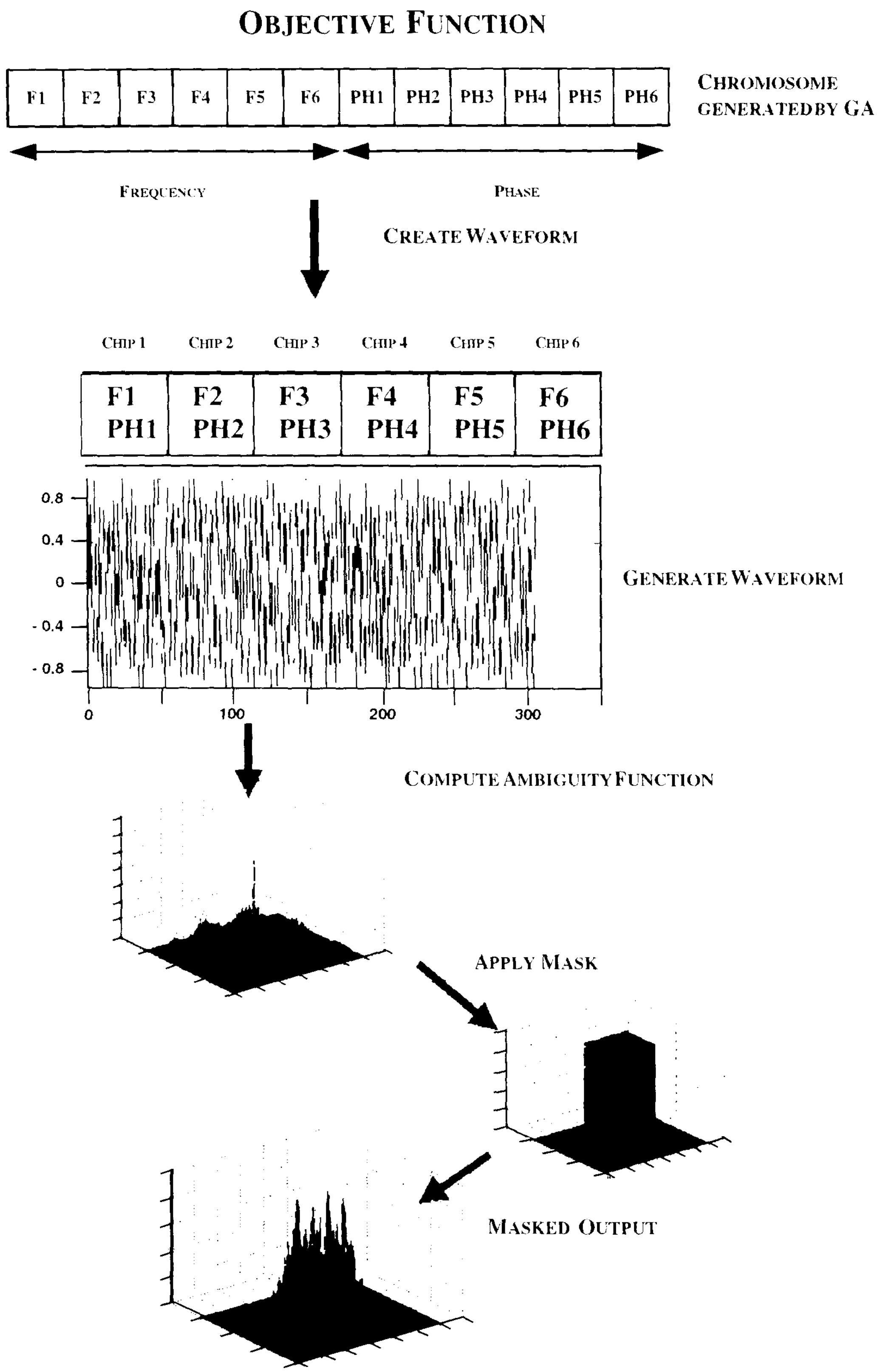

FIG.28 – MASK 3

The three masks are used by the genetic algorithm to search for the parameters of the pulse diverse waveform that will give the minimum peak sidelobe level in that area. It should be noted that areas outside the mask are deemed not important and thus will not be specifically minimized.

## Design of genetic algorithm for FSK PSK waveforms.

*Chromosome design for a FSK PSK waveform*

The practical FSK PSK pulse diverse waveform above has twelve variables; six frequencies and six phase variables. The chromosome comprises 12 genes. The chromosome design is shown at the top of (FIG.29).




COMPUTE MAXIMUM VALUE OF MASKED OUTPUT. THIS IS THE VALUE THAT NEEDS TO BE MINIMIZED BY THE GA

FIG.29 – STEPS OF OBJECTIVE FUNCTION FOR FSK PSK WAVEFORM

In accordance with the radar and pulse parameters outlined at Table 3, the frequency for each chip was chosen from the range 20 - 200 MHz and fixed for the duration of the chip. The phase in each chip is fixed and is an increment of 2.8125° (128 steps in 360°).

*FSK PSK waveform objective function design.*

The Objective Function is described in detail in FIG.298. The stages in the Objective Function are:

(*a*) Create a pulse diverse waveform from chromosome.

(*b*) Calculate ambiguity function of waveform.

(*c*) Apply mask to generate masked ambiguity function.

(*d*) Calculate the peak of masked ambiguity function.

(*e*) Record this level for chromosome fitness assignment.

Chromosomes that return a lower value of peak masked ambiguity function are desired. The frequency and phase values within the chromosomes will be those that will generate a FSK PSK waveform that have the minimum peak sidelobe within the masked area. The MATLAB code for the objective function is shown at Annex B.

The genetic algorithm will attempt to minimize the objective of minimum sidelobe level within the mask area only. The areas outside the mask are not of interest. Instead of using the peak output of the masked ambiguity function as the objective to minimize, a number of other objectives could have been used. These are described below.

(*a*) Minimizing the ratio of masked ambiguity function to the main peak of the ambiguity function.

(*b*) In addition to the masked output, the area outside the mask could have been examined with a view to also reducing and large peaks that may be undesirable.

(*c*) The volume of the masked output may have been minimized instead of the peak level.

All the above options are possible. Based on previous experimentation, each objective would probably have yielded a different final waveform. All of the above objectives could have been incorporated within the objective function with appropriate weightings attached to each objective. However, this approach would have needed time to design and the tuning of the individual weightings is normally carried on a trial and error basis. Alternatively a multi-objective genetic algorithm could have been designed and compared to a single objective genetic algorithm used in this project. Multi-objective genetic algorithms are not addressed in this article and this may be an area for further research.

## Selection of genetic algorithm parameters

The settings used by the genetic algorithm were used within the Genetic Algorithm Toolbox are shown in Table 4.

TABLE.4 – *FSK PSK waveform genetic algorithm parameters*

| Parameter | Value |
|---|---|
| Population | 100 |
| Number of Generations | 100 |
| Fitness Assignment | Ranking |
| Selection Method | Stochastic Universal Sampling |
| Crossover rate | 0.85 |
| Recombining Method | Multi-point Crossover at each gene |
| Mutation rate | 5% |
| Generation Gap | 0.9 |

*Composite Ambiguity Function of a FSK PSK waveform*

The composite ambiguity function of the FSK PSK pulse diverse waveform is compared to that of the Linear FM PSK pulse diverse waveform. It was decided that only Mask 2 (central peak bias) would be used for this comparison. The objective function that incorporates the composite ambiguity function is essentially similar to that described in FIG.29. The differences are that instead of one signal, the composite ambiguity function comprises six individual PSK FSK pulses. Therefore, the chromosome comprises seventy-two genes as opposed to twelve. The steps of the objective function are shown below and summarized in (FIG.30).

(*a*) Produce six individual PSK FSK waveforms from the chromosome produced by the genetic algorithm.

(*b*) Compute the output from the matched filter (MF) of each of these individual waveforms.

(*c*) Sum the individual output of matched filters to produce the composite ambiguity function.

(*d*) Apply the mask (Mask 2).

(*e*) Record the peak sidelobe level within this mask.

(*f*) Record this value for chromosome fitness assignment within the genetic algorithm.

The parameters used by the composite ambiguity genetic algorithm are the same as those outlined in Table 4.

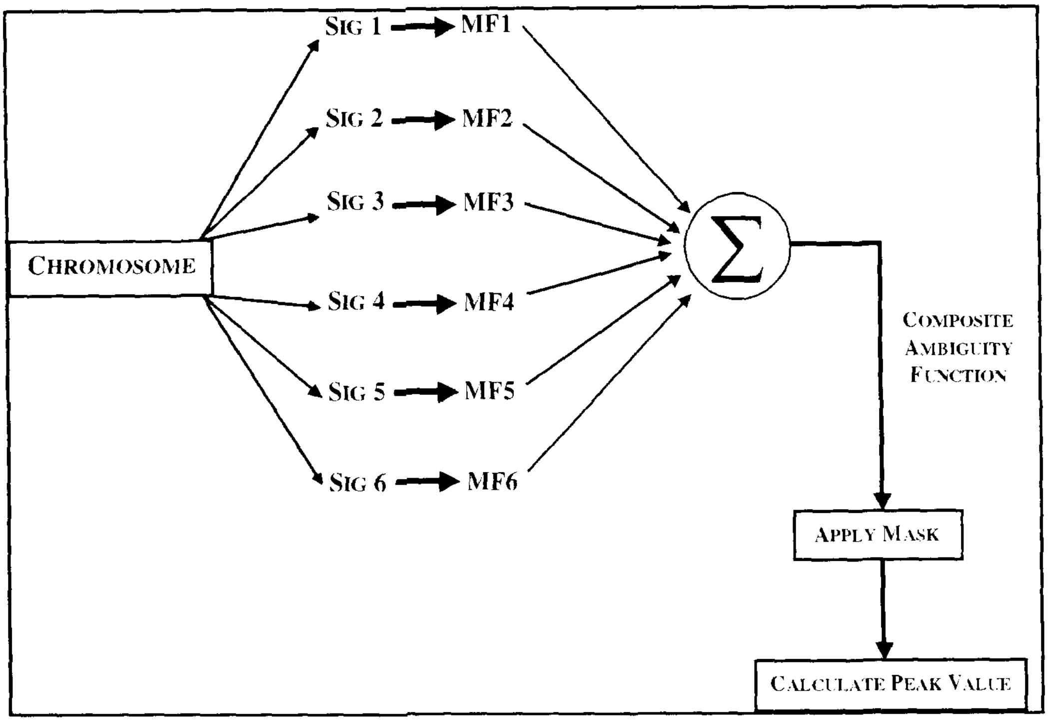



Fig.30 – Composite ambiguity objective function

*Genetic algorithm execution*

Three separate genetic algorithms were run each using a different mask. Each genetic algorithm was run ten times, as it is known from experience that the best solution may not be found on the first run. The total time per genetic algorithm is in the region of 20 hours making a total time of 60 hours.

The composite ambiguity function genetic algorithm with Mask 2 was run eight times. This took 48 hours to complete.

## Design of a genetic algorithm for a Linear FM PSK waveform

The design of this genetic algorithm is almost the same as for the PSK FSK pulse described above. The only difference is that now there is a start and stop frequency in each chip to allow a linear frequency sweep.

*Chromosome design of Linear FM PSK waveform*

The Linear FM PSK waveform described at Fig.24 has eighteen variables; 6 start frequencies, 6 phases and 6 stop frequencies. The chromosome comprises 18 genes. The chromosome design is shown at the top of (Fig.31). The start and stop frequency in each chip was chosen from the range 20 - 200 MHz. The phase was chosen as an increment of 2.8125° (128 steps in 360°).

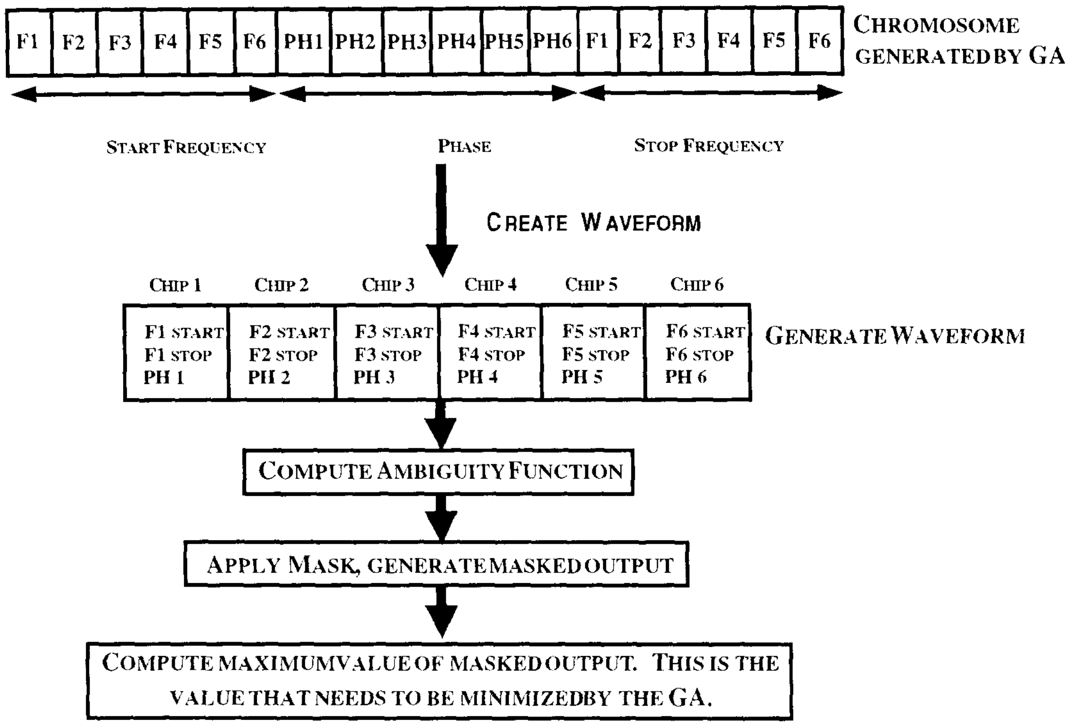



FIG.31 – OBJECTIVE FUNCTION FOR LINEAR FM PSK WAVEFORM

*Objection function design.*

The stages in the objective function were:

(*a*) From chromosome obtain start and stop frequency and phase of each chip.

(*b*) Create Linear FM PSK diverse waveform.

(*c*) Calculate ambiguity function of waveform.

(*d*) Apply mask to generate masked ambiguity function.

(*e*) Record the peak of masked ambiguity function.

(*f*) Record this level for chromosome fitness assignment.

Chromosomes that return a lower value of peak masked ambiguity function are desired.

## Design of composite ambiguity function for Linear FM PSK Waveform

The design of the composite ambiguity function for the Linear FM PSK waveform is similar to the FSK PSK case except for the different chromosome design highlighted in FIG.31.

*Genetic Algorithm execution*

Execution times for the Linear FM PSK waveform genetic algorithm and the composite ambiguity function are the same as those detailed previously.

# RESULTS

## Comparison of waveforms ambiguity functions

*Introduction*

Three masks were defined and FSK PSK waveforms and Linear PSK waveforms were designed using a genetic algorithm to minimize the peak in the masked floor of the resulting ambiguity function.

*Results*

The results are shown in Tables 5,6 and 7. The frequencies and phases that make up the pulse diverse waveforms are shown at Annex A.

TABLE.5 – *Comparison of waveforms using Mask 1*

| | WAVEFORM TYPE | | |
|---|---|---|---|
| PARAMETER | FSK PSK | LINEAR FM PSK | DIFFERENCE |
| Peak mask floor | 12.5 | 12.01 | 0.49 |
| Central peak of Ambiuity Function | 149.44 | 152.24 | - 2.8 |
| Ratio of peak masked floor to central peak (dB) | - 21.5 | - 22.06 | 0.56 |
| -6dB width of range at zero Doppler (m) | 18 | 8 | 10 |
| -6db width of velocity at zero range (m/s) | 0.2 | 0.2 | 0 |

TABLE.6 – *Comparison of waveforms using Mask 2*

| | WAVEFORM TYPE | | |
|---|---|---|---|
| PARAMETER | FSK PSK | LINEAR FM PSK | DIFFERENCE |
| Peak mask floor | 18.55 | 15.57 | 2.98 |
| Central peak of Ambiuity Function | 150.6 | 152.2 | - 1.6 |
| Ratio of peak masked floor to central peak (dB) | -18.1 | - 19.8 | 1.7 |
| -6dB width of range at zero Doppler (m) | 10 | 4 | 6 |
| -6db width of velocity at zero range (m/s) | 0.2 | 0.2 | 0 |

TABLE.7 – *Comparison of waveforms using Mask 3*

| | WAVEFORM TYPE | | |
|---|---|---|---|
| PARAMETER | FSK PSK | LINEAR FM PSK | DIFFERENCE |
| Peak mask floor | 14.99 | 15.3 | - 0.31 |
| Central peak of Ambiuity Function | 152.81 | 151.56 | 1.25 |
| Ratio of peak masked floor to central peak (dB) | - 20.1 | - 19.9 | - 0.2 |
| -6dB width of range at zero Doppler (m) | 14 | 8 | 6 |
| -6db width of velocity at zero range (m/s) | 0.2 | 0.2 | 0 |

*Comparison of peaked masked floor values*

The peak masked floor levels achieved by the two waveforms for each mask can be compared. The summary of the comparison is detailed below.

(*a*) The peak masked floor of Mask 1(Table 5) is similar for both types of waveform.

(*b*) The peak masked floor of Mask 2 (Table 6) is smaller when using linear FM PSK waveform than the value achieved by the FSK PSK waveform.

(*c*) The peak masked floor of Mask 3 (Table 7) is similar for both types of waveform.

These results suggest that the only significant differences in peaked mask floor levels are seen using Mask 2. However, the value of the masked floor peak is more usefully expressed as a ratio of the main peak of the ambiguity function. The reason for this is discussed below.

*Use of masked floor to main peak ratio*

The values of the peak levels in Table 6 show that the peak valued varied between 149.44 to 152.81. These values should be the same, given that all the signals contain the same number of samples. However, they do not contain the same amount of energy due to the effect of producing a sampled signal within MATLAB. The power in a signal is ½ $A^2$. This is proportional to the amount of energy (E) as in this case time is constant. Recalling the properties of the ambiguity function, the peak of the ambiguity function is $2E^2$. Waveforms of certain frequencies within the range 20-200MHz will not sample correctly at the sample frequency of 500MHz to produce signals of similar amplitudes.

This is best explained by plotting the two signals that generated the ambiguity functions with the minimum peak of 149.44 (FSK PSK pulse Table 5) and a maximum peak of 152.81.(FSK PSK pulse Table 7). These are plotted in (FIGs 32 and 33) respectively. Due to the restrictions using sampled signals the amplitude in some parts of the signal FIG.32 are smaller than the amplitudes shown in FIG.33. To overcome this in practice, one could exclude the frequencies in the range 20-200MHz that when sampled at 500MHz give lower amplitudes similar to those seen in FIG.32. Alternatively, increasing the sampling frequency could minimize this effect.

**MASK 1 - FSK PSK WAVEFORM - BEST SIGNAL FOUND BY GA**

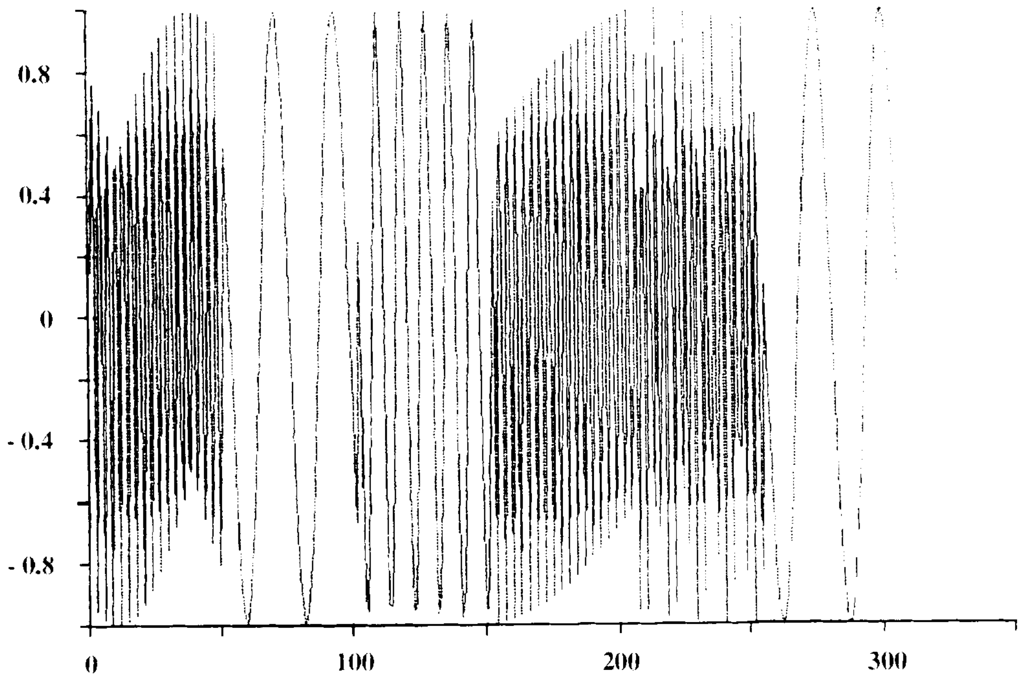



FIG.32 – PLOT OF SAMPLED FSK PSK SIGNAL FOR MASK 1

**FSK PSK PULSE MASK 3 - BEST SIGNAL FOUND BY GA**

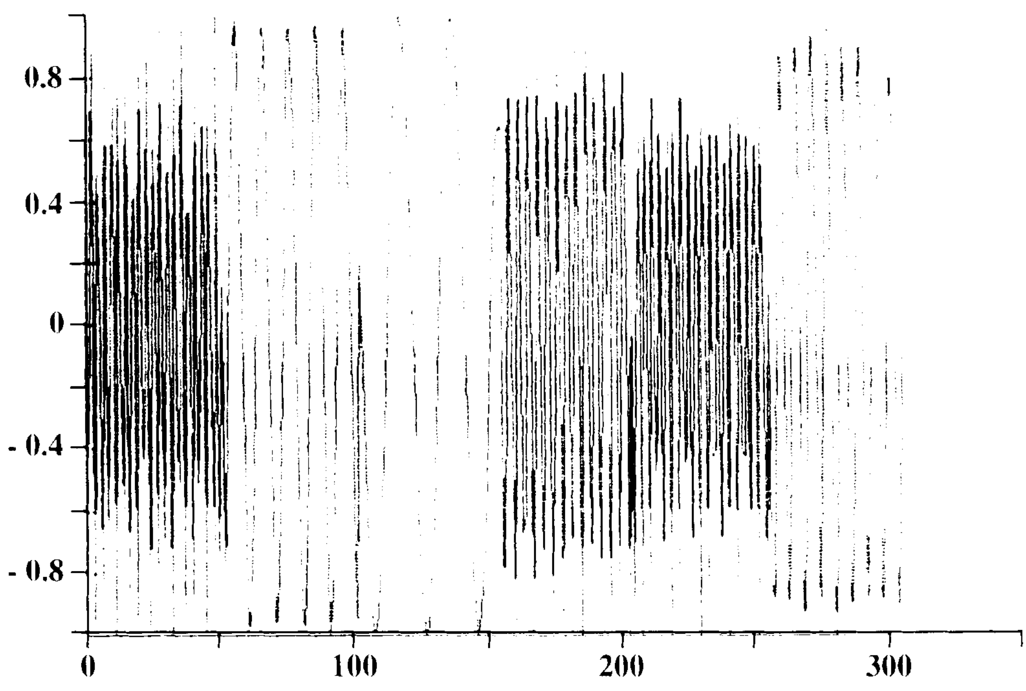



FIG.33 – SAMPLED FSK PSK SIGNAL FOR MASK 3

Because of the fluctuations in amplitude of the main peak, it would be better to compare the ratio of the masked floor with respect to the main peak.

*Masked floor level comparison*

For each individual mask, there is very little difference in the value of the masked floor ratio achieved by each waveform. A comparison can be made of all three separate masked floor levels ratios achieved in all three masks by both waveforms. The mean masked floor to central peak ratio on all three masks with both waveforms is −20.24dB and the standard deviation is 1.4dB. This implies that for both waveforms the masked floor to central peak ratio over all three masks was similar.

It appears that, when comparing the ratio of peak masked floor to central peak of all three masks on both waveforms, there is little benefit in using a Linear FM PSK pulse over a FSK PSK pulse. A practical design of a Linear FM PSK system would be more complex than the FSK PSK system. Based on the results from this project, the additional cost of designing a complex Linear FM PSK system will outweigh the benefits that it gives.

More chips within the pulse may yield different results but this is beyond the scope of this project. This is an area of further research.

*Comparison of ambiguity functions*

To illustrate the effect of the masked area and the clearance achieved by the genetic algorithm, the results of the waveforms ambiguity functions using mask 3 are shown in (FIGS.34A & 34B). The contours are set at 3, 10, 15, and 20 dB down from the main peak. The level of the masked floor is 20dB down from the main peak. There are no contours shown below the masked floor level. The mask shape is also shown.

**PSK FSK PULSE MASK3 - 3,10,15, ( MASK FLOOR) DB CONTOURS**

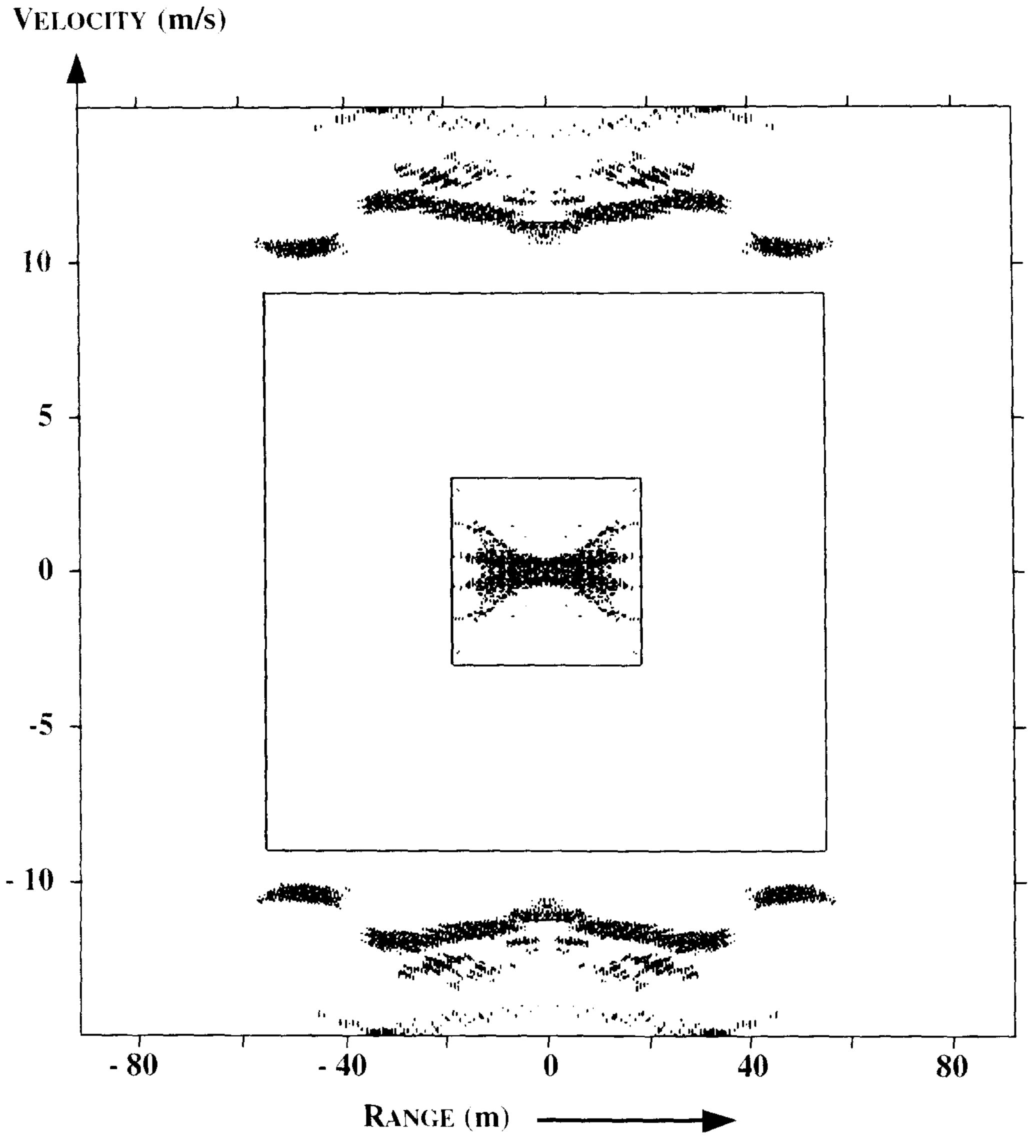



FIG.34A – FSK PSK WAVEFORM

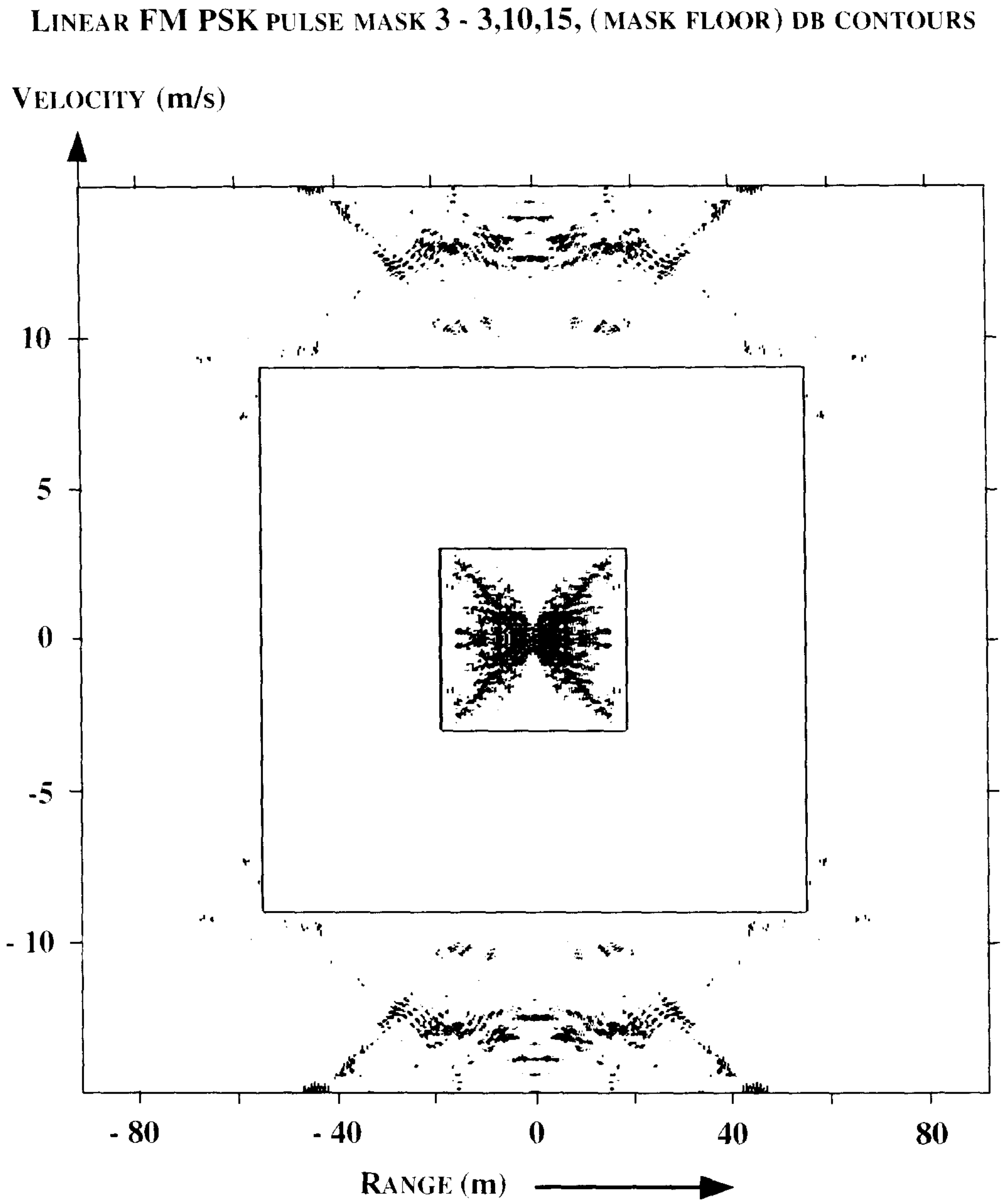



FIG.34B – LINEAR FM PSK WAVEFORM

*Comparison of Doppler and range discriminations*

The objective function in the genetic algorithm was designed such that it only minimized the peak floor level. It did not consider the discrimination properties of the two types of pulse diverse waveforms. However, there is an observation that is worth noting. For each mask, the –6dB width of the central peak in range and Doppler discrimination was recorded and the following is noted:

(*a*) The velocity discrimination at zero range of all masks for both types of waveforms are the same at 0.2m/s. An example of the velocity discrimination properties for the FSK PSK waveforms for mask 2 is shown in (FIG.35). The very narrow central spike indicates good discrimination properties.

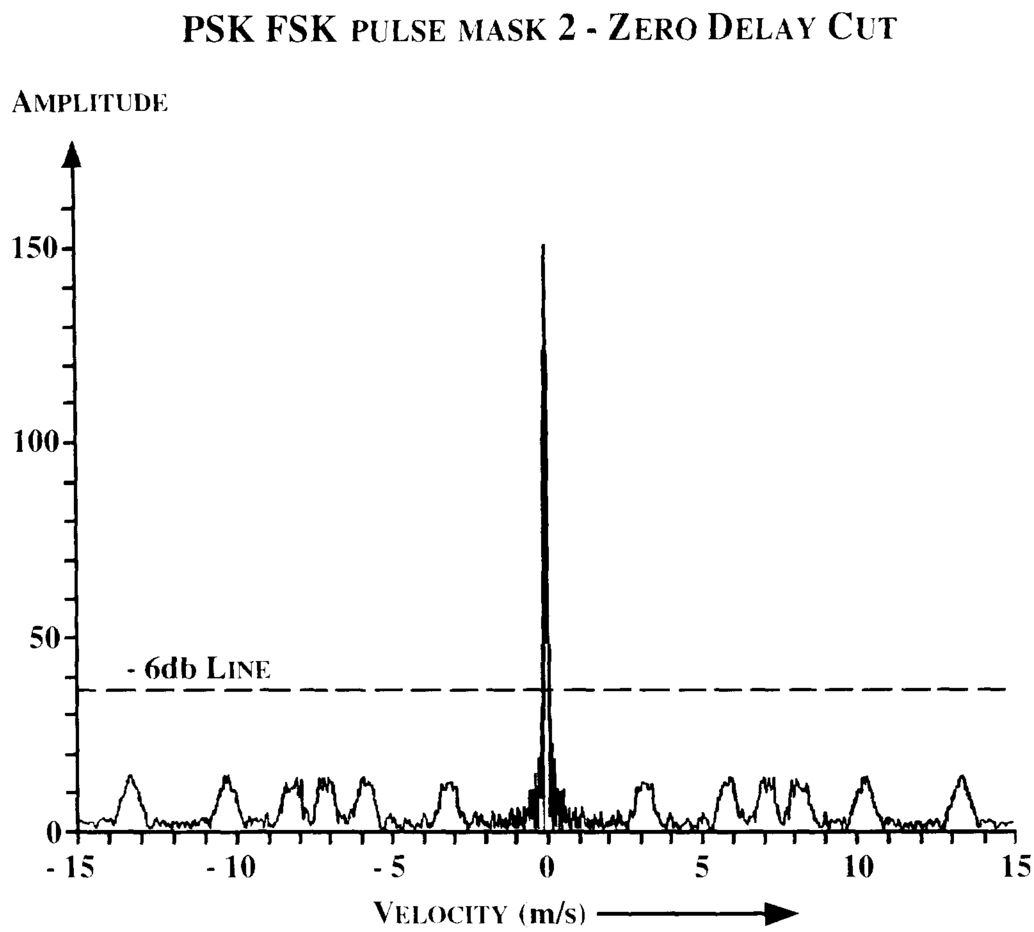



FIG.35 – FSK PSK PULSE MASK 2 – VELOCITY DISCRIMINATION PROPERTIES

(*b*) When comparing the performance of range discrimination at zero velocity, the Linear FM PSK pulse is better in all masks. The range discrimination properties of the FSK PSK waveform and the Linear FM PSK waveforms for mask 1 are shown in (FIGS 36 & 37) respectively. It can be seen that the width of the main peak is smaller for the Linear FM PSK pulse than the FSK PSK pulse. Similar results can be shown for masks 2 and 3.

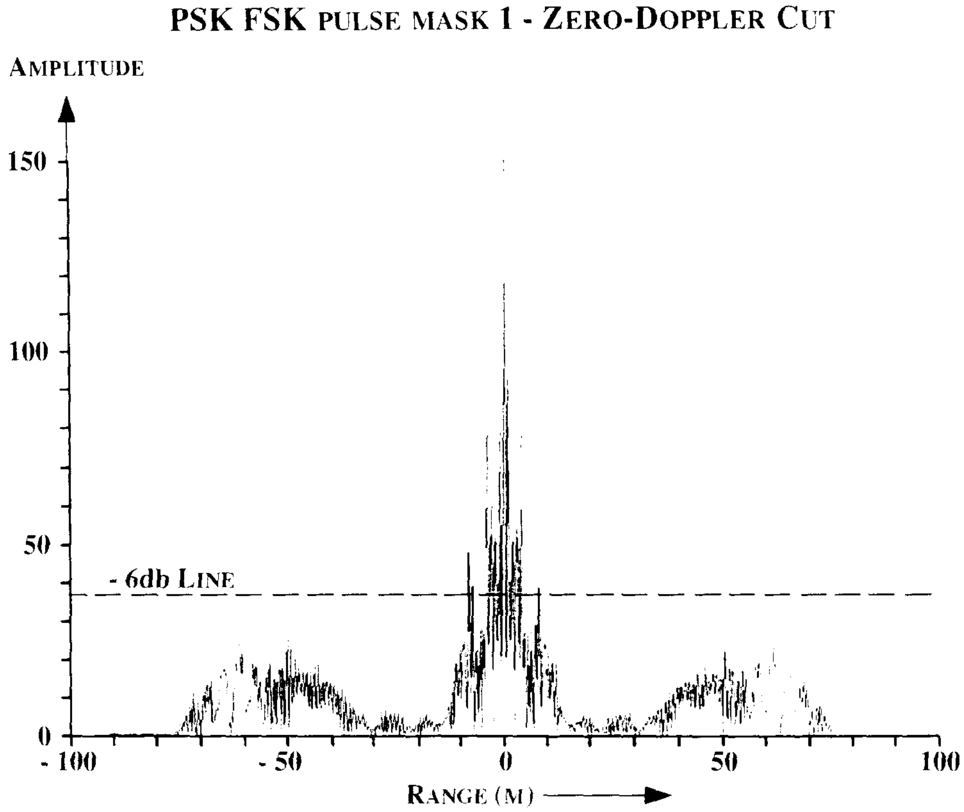



Fig.36 – FSK PSK waveform Range Discrimination (Mask 1)

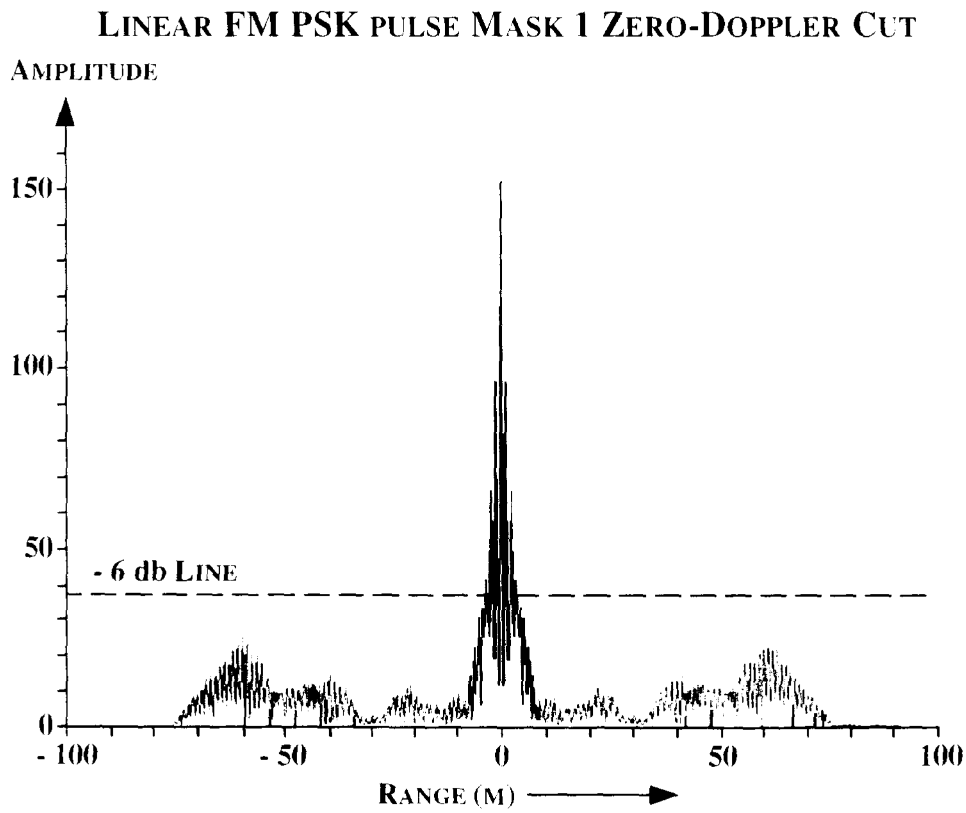



Fig.37 – Linear FM PSK waveform Range Discrimination (Mask 1)

## Composite ambiguity function comparison

*Introduction*

The performance of the two waveforms using the composite ambiguity function was compared using mask 2 only. The results are shown in Table 8. The frequencies and phases used to make these waveforms are shown in Annex A.

*Results*

The results are shown in Table 8 below.

TABLE.8 – *Composite Ambiguity Function results Mask 2*

| | WAVEFORM TYPE | | |
|---|---|---|---|
| PARAMETER | FSK PSK | LINEAR FM PSK | DIFFERENCE |
| Peak mask floor | 55.6 | 52.5 | 3.1 |
| Central peak of Ambiuity Function | 915.3 | 877 | 38.3 |
| Ratio of peak masked floor to central peak (dB) | -24.3 | -24.4 | 0.1 |
| -6dB width of range at zero Doppler (m) | 3 | 0.5 | 2.5 |
| -6db width of velocity at zero range (m/s) | 0.2 | 0.2 | 0 |

The composite ambiguity function is the magnitude of the coherent sum of the output at the matched filter of six individual signals. The central peak of the composite ambiguity function is approximately six times that of the individual ambiguity functions seen in Tables 5,6 and 7.

*Comparison of masked floor to peak ratio*

The masked floor to peak ratio of the two waveforms is almost identical at –24.3 dB and –24.4dB. There does not seem to be any advantage in using the more complex linear FM PSK waveform over the simpler FSK PSK waveform. This non-improvement effect was witnessed with the single ambiguity function. The contour plots of the two composite ambiguity functions are shown in (FIG.38) (FSK PSK ) and (FIG.39) (Linear FM PSK).

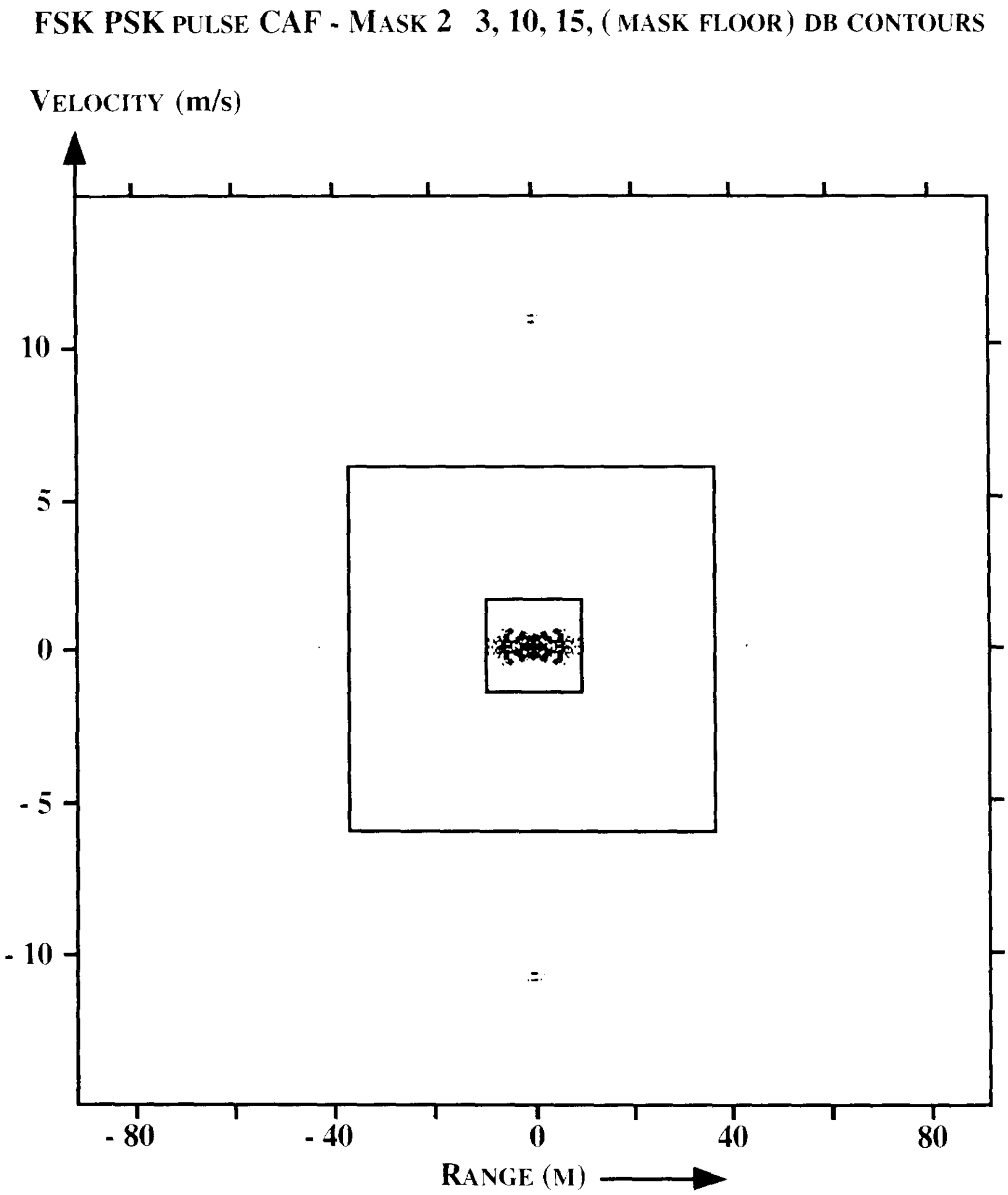



FIG.38 – FSK PSK PULSE COMPOSITE AMBIGUITY FUNCTION CONTOUR PLOT

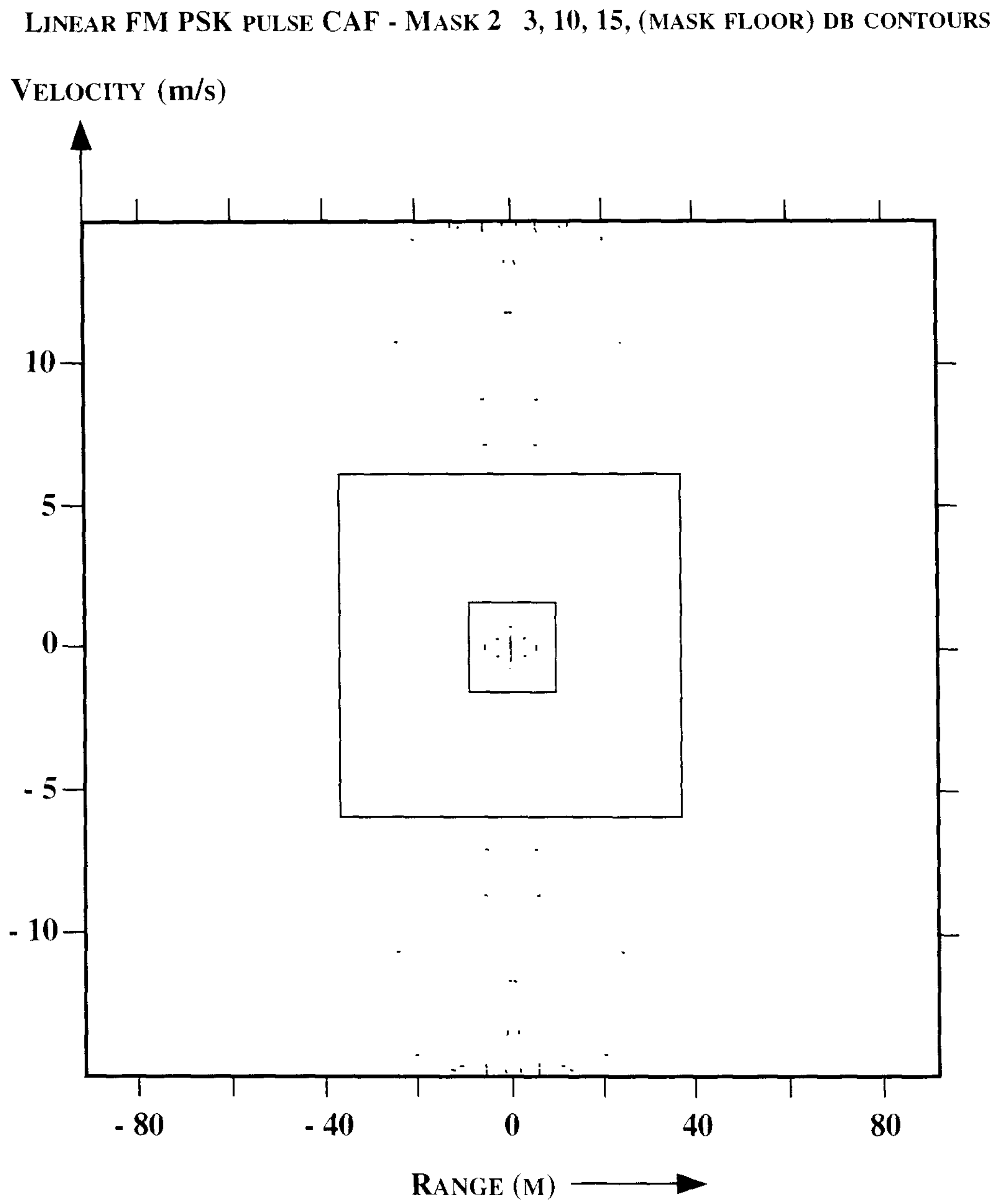



FIG.39 – LINEAR FM PSK PULSE COMPOSITE AMBIGUITY FUNCTION CONTOUR PLOT

*Comparison of velocity and range discrimination*

The velocity discrimination of both waveforms is at –6dB is 0.2m/s. This is similar to the results seen in Table 5 for the single ambiguity case. The range discrimination for the Linear FM PSK pulse at -6dB is 0.5m. This is superior to the FSK PSK pulse at 3m. The range discrimination for both waveforms' composite ambiguity functions are shown in (FIGS 40 and 41).

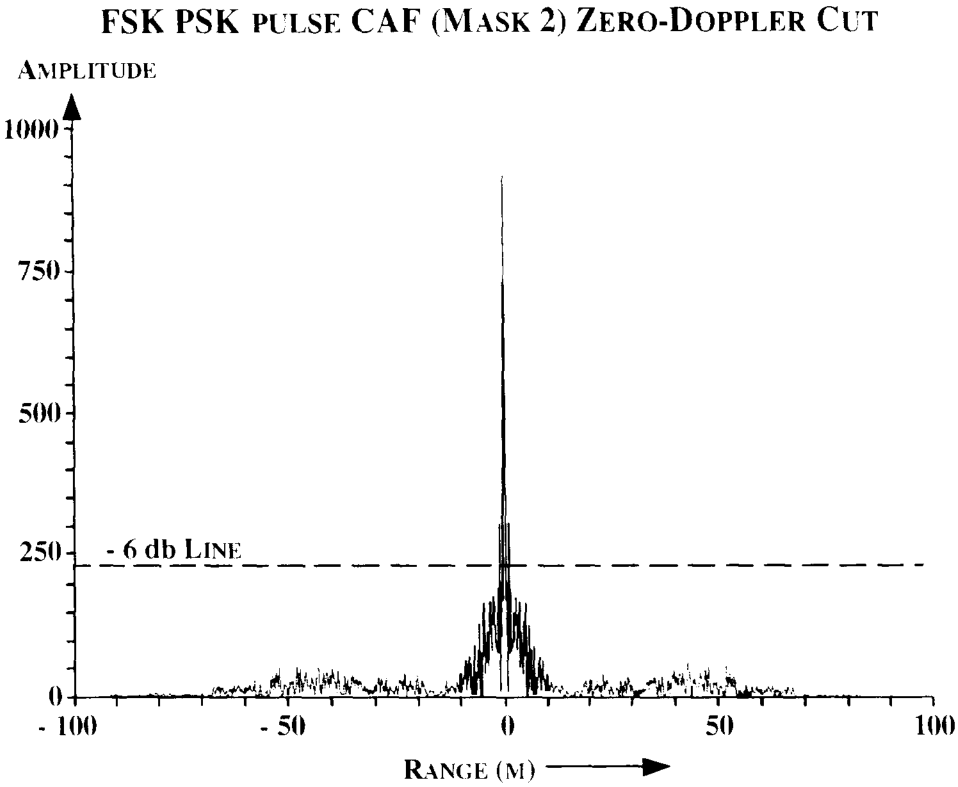



FIG.40 – FSK PSK PULSE COMPOSITE AMBIGUITY FUNCTION RANGE DISCRIMINATION

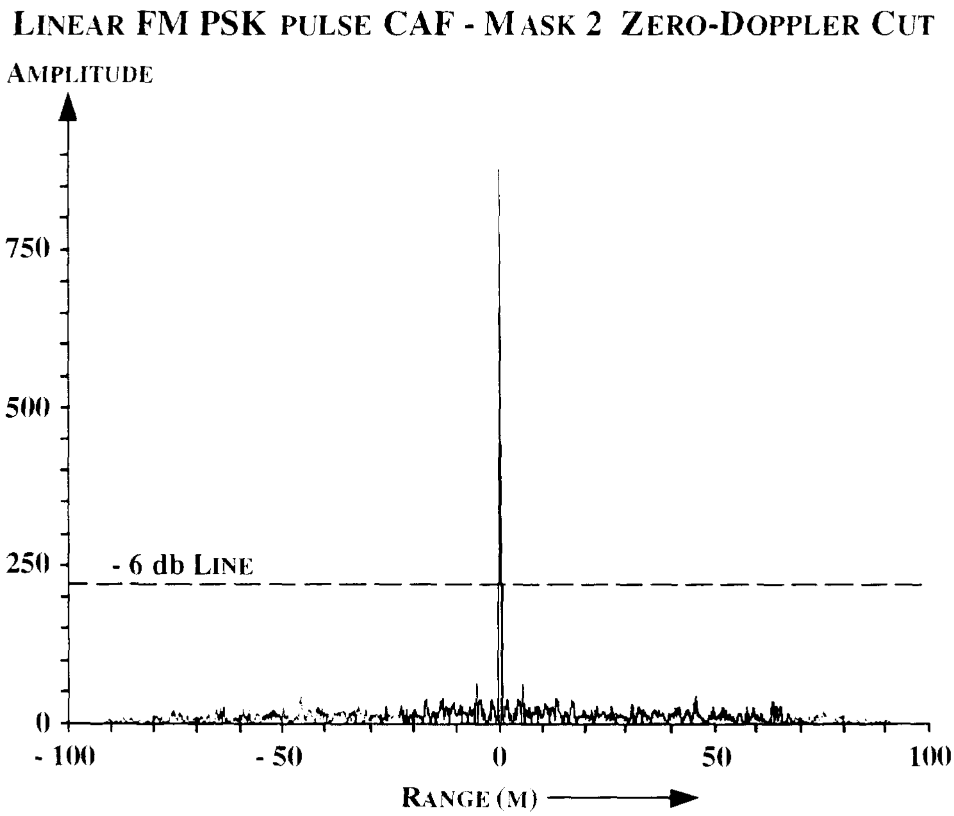



FIG.41 – LINEAR FM PSK PULSE COMPOSITE AMBIGUITY FUNCTION RANGE DISCRIMINATION

*Comparison of single ambiguity function and composite ambiguity functions*

The results of Tables 6 and 8 can be displayed together (Table 9) to determine the effect of using a composite ambiguity function over a single ambiguity function.

TABLE.9 – *Comparison of composite and single ambiguity functions*

| | AMBIGUITY FUNCTION | | COMPOSITE AMBIGUITY FUNCTION | |
|---|---|---|---|---|
| | **FSK PSK** | **LINEAR FM PSK** | **FSK PSK** | **LINEAR FM PSK** |
| Ratio of peak masked floor to central peak (dB) | -18.1 | -19.8 | -24.3 | -24.4 |
| -6dB width of range at zero Doppler (m) | 10 | 4 | 3 | 0.5 |
| -6dB width of velocity at zero range (m/s) | 0.2 | 0.2 | 0.2 | 0.2 |

The following conclusions can be drawn from Table 9:

(*a*) The FSK PSK waveforms' composite ambiguity function yields a better masked floor to central peak ratio than the single FSK PSK ambiguity function. This result for practical waveform design reinforces the results from previous theoretical studies.[13]

(*b*) The Linear FM PSK waveforms' composite ambiguity function yields a better masked floor to central peak ratio than the single Linear FM PSK ambiguity function. In the absence of any theoretical research on Linear FM PSK pulses, this result follows the trend of the FSK PSK pulse.

(*c*) The range discrimination of the composite ambiguity function of both types of waveform is better than the single ambiguity function.

(*d*) The velocity discrimination is the same for all waveforms and masks at 0.2m/s.

The practicalities of designing a radar that can incorporate the composite ambiguity function have not been addressed in this article however, the applicability of this method for fast moving targets has been investigated in previous research. The conclusion of this research proposes a practical implementation.[11]

*Comparison with 6-bit bi-phase codes*

The ratio of peak masked floor to central peak of this six-chip waveform can be compared with the performance of an optimum 6-bit bi-phase code. For the 6-bit bi-phase code, the best ratio is –9.54 dB. The ratio achieved by the FSK PSK and Linear FM PSK waveforms (Table 9) are much better than this.

## Effect of noise in a practical system

*Introduction*

During the generation of the ambiguity function, it has been assumed that both transmit and receive signals have been noise free. In the real world, the transmit and receive signals would be affected by noise. This noise may be generated by:

- Initial frequency and phase generation errors.
- Measurement errors at the output of the matched filter.

- Internal system noise.
- External noise such as frequency interference.
- Target generated noise.
- Effect of propagation on the different frequencies used in the waveforms.

The exact nature and effect of all this noise could be characterized by developing a noise model or collecting real data from real systems. Both of these are not in the scope of this article.

However, if a noise model is developed it is assumed that the noise affects different values of phase and frequency differently. It is possible to design a genetic algorithm to search for frequency and phase values of a waveform that are robust to the effect noise whilst still using the objective of minimizing the peak to masked floor ratio.

*Discussion of the effect of noise*

It is hypothesized that a different sequence of frequency and phase values that give similar ambiguity functions may be affected differently by the same type of noise. This implies that for pulse diverse waveforms there are sequences of phase and frequency that are more robust to noise. This hypothesis is formulated by considering the case of the 6-bit bi-phase codes that were discovered during the investigations earlier. There are fourteen 6-bit bi-phase codes that when autocorrelated give a minimum peak sidelobe level of 2 and the same RMS level in the sidelobes. These fourteen codes are shown in table 10.

TABLE.10 – *6-bit bi-phase codes with the same autocorrelation function properties*

| | |
|---|---|
| -1 -1 -1 -1 1 -1 | CODE 1 |
| -1 -1 -1 1 -1 1 | CODE 2 |
| -1 -1 -1 1 1 -1 | CODE 3 |
| -1 -1 1 1 -1 1 | CODE 4 |
| -1 1 -1 -1 -1 -1 | CODE 5 |
| -1 1 -1 -1 1 1 | CODE 6 |
| -1 1 -1 1 1 1 | CODE 7 |
| -1 1 1 -1 -1 -1 | CODE 8 |
| -1 -1 -1 1 -1 -1 | CODE 9 |
| -1 -1 1 -1 -1 -1 | CODE 10 |
| -1 -1 1 -1 1 1 | CODE 11 |
| -1 1 -1 -1 -1 1 | CODE 12 |
| -1 1 1 1 -1 1 | CODE 13 |
| -1 1 1 1 1 -1 | CODE 14 |

In this basic model, noise effects only one bit at a time. The effect of noise is to reverse the sign of the bit. The effect of noise can be calculated using the following algorithm. The results are summarized in Table 11.

(*a*) Start at code 1.

(*b*) First bit is flipped to simulate noise. The autocorrelation function of this noisy pulse is calculated and the minimum peak sidelobe level is recorded.

(*c*) The difference between the original minimum peak sidelobe level (2) and the new noisy signal's sidelobe level is recorded.

(*d*) The first bit is then returned to its original state and the noise is applied to the second bit, again reversing its value. This process is repeated for all the bits in the code.

(*d*) The differences calculated at (c) are summed and recorded. The lower the number, the more robust this code is to noise.

(*e*) Repeat steps (a) to (e) on remaining codes in turn.

TABLE 11 – *Effect of noise on each bit of a 6-bit code*

| | Min peak SLL of ACF when Bit 1 Changed | Min peak SLL of ACF when Bit 2 Changed | Min peak SLL of ACF when Bit 3 Changed | Min peak SLL of ACF when Bit 4 Changed | Min peak SLL of ACF when Bit 5 Changed | Min peak SLL of ACF when Bit 6 Changed | Measure of tolerenace (low is more tolerent) |
|---|---|---|---|---|---|---|---|
| CODE 1 | 2 | 3 | 3 | 2 | 5 | 3 | 6 |
| CODE 2 | 3 | 3 | 4 | 5 | 2 | 2 | 7 |
| CODE 3 | 3 | 5 | 2 | 3 | 3 | 2 | 6 |
| CODE 4 | 4 | 3 | 3 | 2 | 2 | 3 | 5 |
| CODE 5 | 2 | 2 | 5 | 4 | 3 | 3 | 7 |
| CODE 6 | 3 | 3 | 3 | 3 | 3 | 3 | 6 |
| CODE 7 | 3 | 2 | 2 | 3 | 3 | 4 | 5 |
| CODE 8 | 2 | 3 | 3 | 2 | 5 | 3 | 6 |
| CODE 9 | 3 | 3 | 4 | 5 | 2 | 2 | 7 |
| CODE 10 | 3 | 5 | 2 | 3 | 3 | 2 | 6 |
| CODE 11 | 4 | 3 | 3 | 2 | 2 | 3 | 5 |
| CODE 12 | 2 | 2 | 5 | 4 | 3 | 3 | 7 |
| CODE 13 | 3 | 3 | 3 | 3 | 4 | 3 | 7 |
| CODE 14 | 3 | 2 | 2 | 3 | 3 | 4 | 5 |

It can be seen from Table 11 that although all the 6-bit codes yield the same minimum peak sidelobe level of 2 and the same RMS value in the sidelobes, they are affected by noise in different extents. This experiment suggests that highlighted codes 4, 7, 11 and 14 are more tolerant to noise than the others. Based on these results, it is possible to hypothesise that there may be pulse diverse waveforms that are more robust to the effects of noise. If the noise is characterized and used by the genetic algorithm, it should be able to find robust solutions.

*Further research into effects of noise*

Future research in this area should investigate the effects of practical noise on the two types of waveform to determine which offers the more robust solution. A noise model could be developed or alternatively, the Prototype Generic Radar Model at Qinetiq (Formerly DERA) could be utilized. The Prototype Generic Radar Model,[22] that includes a noise model, can be used to characterize the performance of a waveform in different operating environments. The model is written in MATLAB and therefore the FSK PSK and Linear FM PSK waveforms designed in this project could be tested.[23]

## Conclusions

*Bi-phase code investigation*

- A genetic algorithm found the best 25-bit bi-phase codes and its search performance was much faster than the exhaustive search method.
- A genetic algorithm found two new 64-bit bi-phase codes that offer better performance than codes found during previous research.

*Generalized BARKER Sequence investigation*

A genetic algorithm has been designed to find 13-bit Generalised BARKER Sequences. The new sequences found were different to those found in previous research.

*FSK PSK and Linear FM waveform comparison*

Practical FSK PSK and Linear FM PSK waveforms can be generated in MATLAB. The ambiguity function and composite ambiguity function can be calculated. The sidelobe level in specified areas can be determined and the range and Doppler discrimination properties can be characterized.

A genetic algorithm can be designed to search for the FSK PSK and Linear FM PSK waveform parameters that generate a specified ambiguity function. Three predefined areas of the waveforms ambiguity function were minimized. The results show that in terms of masked area minimization there is no significant advantage gained by using the Linear FM PSK waveform over the FSK PSK waveform.

The Doppler and range discrimination capabilities of the FSK PSK waveform and the FSK PSK waveform were compared. In all cases the Linear FM PSK waveform offered better range and Doppler discrimination over the FSK PSK waveform. This feature is the only advantage of using the Linear FM PSK waveform.

The Doppler and range discrimination of the Linear FM PSK composite ambiguity function was superior to that of the FSK PSK composite ambiguity function.

The use of the composite ambiguity function improves the clutter rejection capabilities of the waveform sets when compared to the single ambiguity function.

*Areas of further research*

Using genetic algorithms, the search for optimum bi-phase codes could be extended to lengths greater than 64 bits.

The search for longer Generalized BARKER Sequences could be performed by a genetic algorithm based on the design described in the article. Longer sequences of length greater than 45 bits may be found using genetic algorithms.

The implementation of the genetic algorithm in 'C' instead of MATLAB would significantly reduce the algorithm run time. The investigation of longer and more complex waveforms will require a 'C' based algorithm.

The effect of significantly altering the parameters of the genetic algorithms used in this research was not examined. For example, different crossover and selection operators may have produced better results. The issue of using multi-objective genetic algorithms could be examined to determine its merits.

The Linear FM PSK pulse may offer improved clutter rejection capability in areas of the ambiguity function if a longer waveform and more chips were used in the waveform design. Longer waveforms could be designed in further research.

The waveforms designed could be incorporated into a radar model that could determine their usefulness in a simulated radar environment. If the effect of noise is characterized then new robust waveforms could be designed.

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23. Discussions with DR W. DAWBER (Naval Radar Scientist, Qinetiq ) in May 2001.

## Annex A

The frequencies and phases of the pulse diverse waveforms for each mask are shown below.

TABLE.12 – *Mask 1-frequency and phase values*

| **Linear FM PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
|---|---|---|---|
| Start Frequency (Hz) | 173176500 | 175294100 | 45411760 |
| Phase (Degrees) | 39.375 | 275.625 | 360 |
| Stop Frequency (Hz) | 179529400 | 89882350 | 80000000 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Start Frequency (Hz) | 184470600 | 174588200 | 68000000 |
| Phase (Degrees) | 323.4375 | 334.6875 | 98.4375 |
| Stop Frequency (Hz) | 189411800 | 180235300 | 40470590 |
| | | | |
| **FSK PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
| Frequency (Hz) | 169647100 | 22117650 | 56000000 |
| Phase (Degrees) | 174.375 | 75.9375 | 56.25 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Frequency (Hz) | 165411800 | 182352900 | 20000000 |
| Phase (Degrees) | 278.4375 | 129.375 | 84.375 |

TABLE.13 – *Mask 2-frequency and phase values*

| **Linear FM PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
|---|---|---|---|
| Start Frequency (Hz) | 103294100 | 63764710 | 178117600 |
| Phase (Degrees) | 289.6875 | 64.6875 | 258.75 |
| Stop Frequency (Hz) | 161882400 | 48235290 | 185176500 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Start Frequency (Hz) | 101882400 | 23529410 | 188705900 |
| Phase (Degrees) | 239.0625 | 312.1875 | 101.25 |
| Stop Frequency (Hz) | 196470600 | 53882350 | 157647100 |
| | | | |
| **FSK PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
| Frequency (Hz) | 190823500 | 48941180 | 26352940 |
| Phase (Degrees) | 247.5 | 213.75 | 78.75 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Frequency (Hz) | 139294100 | 181647100 | 85647060 |
| Phase (Degrees) | 309.375 | 151.875 | 84.375 |

TABLE.14 – *Mask 3-frequency and phase values*

| **Linear FM PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
|---|---|---|---|
| Start Frequency (Hz) | 195764700 | 24235290 | 91294120 |
| Phase (Degrees) | 354.375 | 250.3125 | 115.3125 |
| Stop Frequency (Hz) | 189411800 | 20000000 | 123764700 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Start Frequency (Hz) | 128000000 | 196470600 | 156941200 |
| Phase (Degrees) | 67.5 | 95.625 | 331.875 |
| Stop Frequency (Hz) | 166823500 | 109647100 | 122352900 |
| | | | |
| **FSK PSK waveform** | **CHIP 1** | **CHIP 2** | **CHIP 3** |
| Frequency (Hz) | 170352900 | 88470590 | 132235300 |
| Phase (Degrees) | 75.9375 | 39.375 | 78.75 |
| | **CHIP 4** | **CHIP 5** | **CHIP 6** |
| Frequency (Hz) | 156941200 | 149176500 | 43294120 |
| Phase (Degrees) | 281.25 | 326.25 | 151.875 |

TABLE.15 – *Mask 2-frequency and phase values for Linear FM PSK CAF*

| | | CHIP 1 | CHIP 2 | CHIP 3 |
|---|---|---|---|---|
| SIGNAL 1 | Start Frequency (Hz) | 46600110 | 193647100 | 91294120 |
| | Phase (Degrees) | 45 | 98.4375 | 115.3125 |
| | Stop Frequency (Hz) | 77176470 | 193647100 | 123764700 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 1 | Start Frequency (Hz) | 128000000 | 196470600 | 156941200 |
| | Phase (Degrees) | 67.5 | 95.625 | 331.875 |
| | Stop Frequency (Hz) | 166823500 | 109647100 | 122352900 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 2 | Start Frequency (Hz) | 49647060 | 165411800 | 184470600 |
| | Phase (Degrees) | 154.6875 | 337.5 | 300.9375 |
| | Stop Frequency (Hz) | 149176500 | 159764700 | 199294100 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 2 | Start Frequency (Hz) | 29176470 | 150588200 | 60941180 |
| | Phase (Degrees) | 149.0625 | 53.4375 | 255.9375 |
| | Stop Frequency (Hz) | 126588200 | 44000000 | 168941200 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 3 | Start Frequency (Hz) | 104000000 | 92000000 | 56000000 |
| | Phase (Degrees) | 199.6875 | 87.1875 | 194.0625 |
| | Stop Frequency (Hz) | 176705900 | 181647100 | 104705900 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 3 | Start Frequency (Hz) | 171058800 | 157647100 | 161882400 |
| | Phase (Degrees) | 208.0125 | 146.25 | 149.0625 |
| | Stop Frequency (Hz) | 22117650 | 46823530 | 38352940 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 4 | Start Frequency (Hz) | 124470500 | 35529410 | 41176470 |
| | Phase (Degrees) | 123.75 | 177.1875 | 118.125 |
| | Stop Frequency (Hz) | 154117600 | 101176500 | 185882400 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 4 | Start Frequency (Hz) | 76470590 | 182352900 | 163411800 |
| | Phase (Degrees) | 151.875 | 357.1875 | 39.375 |
| | Stop Frequency (Hz) | 162588200 | 164705900 | 152000000 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 5 | Start Frequency (Hz) | 123764700 | 35529410 | 155529400 |
| | Phase (Degrees) | 47.8125 | 264.375 | 286.875 |
| | Stop Frequency (Hz) | 112470600 | 101176500 | 116705900 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 5 | Start Frequency (Hz) | 195764700 | 197882400 | 42588240 |
| | Phase (Degrees) | 45 | 340.3125 | 312.1875 |
| | Stop Frequency (Hz) | 42588240 | 106117600 | 195058800 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 6 | Start Frequency (Hz) | 166823500 | 46117650 | 46823530 |
| | Phase (Degrees) | 241.875 | 300.9375 | 64.6875 |
| | Stop Frequency (Hz) | 116705900 | 135764700 | 29882350 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 6 | Start Frequency (Hz) | 25647060 | 158352900 | 20000000 |
| | Phase (Degrees) | 345.9375 | 230.625 | 255.9375 |
| | Stop Frequency (Hz) | 149882400 | 42588240 | 24235290 |

TABLE.16 – *Mask 2-frequency and phase values for FSK PSK CAF*

| | | CHIP 1 | CHIP 2 | CHIP 3 |
|---|---|---|---|---|
| Signal 1 | Frequency (Hz) | 42840770 | 49647060 | 96941180 |
| | Phase (Degrees) | 64.6875 | 137.8125 | 270 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| Signal 1 | Frequency (Hz) | 56705880 | 148470600 | 60941180 |
| | Phase (Degrees) | 216.5625 | 270 | 146.25 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| Signal 2 | Frequency (Hz) | 185882400 | 34823530 | 165411800 |
| | Phase (Degrees) | 81.5625 | 70.3125 | 101.25 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| Signal 2 | Start Frequency (Hz) | 188705900 | 95529410 | 78588240 |
| | Phase (Degrees) | 149.0625 | 188.4375 | 255.9375 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| Signal 3 | Frequency (Hz) | 178823500 | 39764710 | 189411800 |
| | Phase (Degrees) | 180 | 160.3125 | 123.75 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| Signal 3 | Frequency (Hz) | 161882400 | 88470590 | 137882400 |
| | Phase (Degrees) | 315 | 185.625 | 315 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 4 | Frequency (Hz) | 72235290 | 29176470163 | 90588240 |
| | Phase (Degrees) | 106.875 | 163.125 | 104.0625 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 4 | Frequency (Hz) | 195058800 | 87764710 | 154823500 |
| | Phase (Degrees) | 75.9375 | 270 | 208.125 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 5 | Frequency (Hz) | 76470590 | 122352900 | 34823530 |
| | Phase (Degrees) | 247.5 | 171.5625 | 132.1875 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 5 | Frequency (Hz) | 186588200 | 82823530 | 53882350 |
| | Phase (Degrees) | 331.875 | 81.5625 | 143.4375 |
| | | CHIP 1 | CHIP 2 | CHIP 3 |
| SIGNAL 6 | Frequency (Hz) | 89882350 | 39058820 | 157647100 |
| | Phase (Degrees) | 244.6875 | 253.125 | 106.875 |
| | | CHIP 4 | CHIP 5 | CHIP 6 |
| SIGNAL 6 | Frequency (Hz) | 90588240 | 145647100 | 161882400 |
| | Phase (Degrees) | 360 | 81.5625 | 247.5 |