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Bayesian techniques in remaining life calculations for welded joints of jacket structures

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SYNOPSIS

This paper reviews the available methods for performing Bayesian updating of the fatigue reliability of welded joints. These techniques are being investigated as part of a research and development programme, whose objective is the construction of an expert system which will provide operators with guidance on the planning and execution of inspection, repair and maintenance (IRM) strategies for fixed offshore platforms. The Bayesian updating methods discussed in this paper utilise structural system reliability analysis techniques, which are reviewed in detail.

The results of an example analysis are presented and used to demonstrate the potential benefits of the approach as a means of scheduling inspections.

INTRODUCTION

As the age of offshore structures increases it is becoming ever more important that it be possible for the results of weld inspections (carried out by the operators at regular intervals and at considerable expense,[1]) to be used to provide updated estimates of their residual safe lives. In general, two outcomes are possible as a result of the inspection of an individual weld; either a crack is observed and measured, or the weld is declared to be free of any detectable defect. Provided that the weld was appropriately designed in the first place, and provided also that it has not been overstressed as the result of some unforeseen loading, it is likely that the outcome of the inspection will be that the joint is found to be free of observable defects.

The typical responses to the two possible outcomes are to grind out or repair (either by welding or clamping) a detected crack, and to take no action where no crack is observed. These responses to the results of the inspection process are significantly sub-optimal, since they do not use the information obtained by inspection to update the initial estimates of the safe fatigue life of the weld in question. It is therefore a matter of urgency, in view of increasing platform age and the basic cost of underwater inspection, that a formalised theoretical framework be constructed within which the results of all weld inspections (both those in which cracks are detected and those in which they are not) can be used to derive posterior estimates of the safe lives of welded joints.

Platform lifetime assessment through analysis, inspection and maintenance (PLAIM) is a research and development programme currently being carried out with the objective of developing the databases, algorithms and knowledge bases necessary for the execution of validated fatigue crack growth/fracture analysis of welded tubular joints on fixed offshore platforms. To enable safety levels to be rationally quantified, and to facilitate the exploitation of expert knowledge both immediately and in the future, the algorithms that are developed will make use of reliability analysis techniques and will be encompassed within an expert system framework.

This paper briefly reviews the background to the problem, summarises the methodologies commonly pursued in this type of work, and gives an example of how these can be applied to provide the information on safety levels and inspection intervals following non-detection of a fatigue crack in a welded tubular joint. The approach adopted accounts in part for 'misses' that occur during inspection but not for 'spurious' detections, both of which form significant proportions of inspection results of such joints.[1]

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Background

The techniques of linear elastic fracture mechanics (LEFM) and of S-N analysis are commonly used in many branches of engineering for the prediction of safe fatigue lives. These analyses are usually approached from a deterministic point of view, ie an analysis is performed without explicitly accounting for the uncertainties associated with the input parameters, and the results are quantified in terms of a single parameter (either the safe fatigue life or a factor of safety on the applied stress range) with no indication of the degree of confidence that should be accorded to it. Furthermore, fatigue life calculations performed during design (whether they use LEFM or an S-N approach) typically use conservative values of the input data. In particular for offshore applications, the applied stress ranges are frequently over-predicted as a result of the use of conservative values of the drag and inertia coefficients (C_d and C_m) in the Morison equation, lack of consideration of directionality, over-prediction of current interaction, etc. This conservatism, allied with the uncertainties associated with such parameters as C and m (LEFM) and K and m (S-N analysis), means that it is frequently difficult to reconcile the analytical predictions of safe fatigue lives with the observed occurrence of fatigue cracks in the field.

The power to reconcile design prediction of fatigue lives with apparently contradictory field observations can be provided by a Bayesian analysis in which the prior predictions for safe fatigue lives (those made at the design stage) are updated, using inspection results, to give posterior fatigue life predictions. In recent years considerable effort has been expended in constructing and refining a basis for probabilistic LEFM and S-N analysis. Within this general field of probabilistic fracture mechanics several authors have concentrated on the derivation of such Bayesian updating techniques.

These typically use reliability analysis methodologies to derive posterior predictions of safe fatigue lives in terms of a reliability index (or alternatively a notional probability of failure), which is expressed as a function of the number of applied stress cycles rather than in the more familiar deterministic format of the number of cycles required to cause failure.

Some consideration has also been given to the use of fuzzy measures as a means of quantifying some of the uncertainties associated with fatigue crack growth, with the probability of failure expressed in terms of a fuzzy probability measure.[2] It is not yet established that this represents an improvement over a purely probabilistic approach, and is unlikely to gain widespread acceptance because of many of the conceptual problems that it introduces.

Approaches have been derived through which both LEFM and S-N analysis results can be expressed in an explicitly probabilistic format, and which allow inspection results to be used to update fatigue reliability estimates.

The following sections provide brief overviews of the relevant methodologies, ie LEFM, S-N analysis, a first order reliability method (FORM), and the analysis of parallel systems.

METHODOLOGIES

LEFM

In LEFM the relationship that is most commonly used for the prediction of fatigue crack growth is the Paris-Erdogan relationship, which defines the crack growth rate da/dN as:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where ΔK is the range in the stress intensity factor during a load cycle, a is the crack length (or depth), and C and m are constants.

The stress intensity factor range, ΔK, is given by:

$$\Delta K = \Delta\sigma\, Y(a)\sqrt{\pi a} \tag{2}$$

where $\Delta\sigma$ is the remotely applied stress range and Y(a) is the geometry function.

Failure is assumed to occur when the crack length a reaches some critical size a_c. Various criteria, corresponding for example to through-thickness cracking, general yielding or tearing of the remaining ligament, or the onset of brittle fracture, can be used to determine a_c. The actual choice of failure criterion may depend on the material properties, the geometry, and the applied stress range but is more likely to be dictated by certification and similar operator requirements.

S-N analysis

In S-N analysis a power law relationship is assumed between the stress range and the number of cycles required to cause failure as in equation 3:

$$N(\Delta\sigma) = \frac{K}{(\Delta\sigma)^m} \tag{3}$$

where K is a constant. This simple form can be directly applied in the case of constant amplitude loading to determine joint life. In the far more usual case of variable amplitude loading, life is determined through the summation of the damage (equivalent to crack growth) arising during each loading cycle. ΔD_i the incremental fatigue damage caused by a stress cycle of range $\Delta\sigma_i$ is given by equation 4:

$$\Delta D_i = \frac{1}{N(\Delta\sigma_i)} = \frac{(\Delta\sigma_i)^m}{K} \tag{4}$$

Failure is assumed to happen when the total damage D exceeds 1.0, although Thorpe and Sharp,[3] indicate that 1.7 is more appropriate for joints tested in air.

First order reliability method

First order reliability methods have gained acceptance in the structural engineering community over the past few years and have been widely publicised.[4–8]

The probability of failure of a structure or structural component can be expressed as:

$$P_f = \int_{x \in Q} f_x(x)dx \tag{5}$$

where **x** represents the random vector of uncertain quantities (basic variables) influencing the performance of the structure or component under consideration, and Q represents the failure subset of the basic variable space.

It is conventional in structural reliability analysis to rewrite equation 5 as:

$$P_f = \int_{M<0} f_x(x)\, dx \tag{6}$$

$$\text{where } M = g(\mathbf{x}) \tag{7}$$

$$\text{and } g(\mathbf{x}) < 0 \text{ for all } \mathbf{x} \in Q \tag{8}$$

M is termed the safety margin for the problem and is negative when evaluated at any point in the failure region Q of the basic variable space. The function g(**x**) is termed the g-function.

Consider an n-dimensional random variable space in which each of the basic variables U_1 to U_n are mutually independently normally distributed with zero means and unit standard deviations. Let this basic variable space be termed the U-space to distinguish it from the original basic variable space (now termed the X-space). The probability of failure can be expressed as:

$$P_f = \int_{u \in Qu} \phi(\mathbf{u})\,d\mathbf{u} \tag{9}$$

where $\phi()$ represents the n-dimensional joint standardised normal probability density function.

By expressing the probability of failure in the form of equation 9 (ie as the integral of $\phi(\mathbf{u})$ over some region of the U-space) it is possible to express any component structural reliability analysis in a standard form. The general problem of evaluating equation 5 is therefore replaced by the standard problem of evaluating equation 9. In order to evaluate the g-function in terms of the U-space basic variables it is necessary to define a mapping that can be used to transform between the U- and X- spaces. A suitable choice of transformation is the Rosenblatt transformation,[9,10] defined by:

$$u_1 = \Phi^{-1}\left[F_1(x_1)\right]$$

$$u_i = \Phi^{-1}\left[F_i(x_i|x_1,\ldots,x_{i-1})\right] \text{for } i = 2 \text{ to } n \tag{10}$$

where Φ is the standard normal cumulative distribution function. The conditional cumulative distribution functions in equation 10 can be obtained from the joint probability density functions as follows:

Since by definition

$$f(x_i|x_1,\ldots,x_{i-1}) = \frac{f(x_1,\ldots,x_i)}{f(x_1,\ldots,x_{i-1})} \tag{11}$$

the required conditional cumulative distribution function is given by:

$$F(x_i|x_1,\ldots,x_{i-1}) = \frac{\int_{s_i=-\infty}^{s_i=x_i} f(x_1,\ldots,x_i)\,ds_i}{f(x_1,\ldots,x_{i-1})} \tag{12}$$

The required transformation from U- to X-space is the inverse of that given in equation 10. This can be obtained by sequentially inverting the one dimensional transformations, giving:

$$x_1 = F_1^{-1}\left[\Phi(u_1)\right]$$

$$x_i = F_i^{-1}\left[\Phi(u_i|x_1,\ldots,x_{i-1})\right] \text{for } i = 2 \text{ to } n \tag{13}$$

Because all the U-space variables are mutually independently normally distributed, it follows that a linear function of them is also a normally distributed random variable (by the Central Limit Theorem); it therefore follows that if M can be expressed as a linear function of the U-space variables then it will be normally distributed. If M' is a linearisation of g(**u**) at some arbitrary point such that M' is given by:

$$M' = a_0 + \sum a_i u_i \text{ for } i = 1 \text{ to } n \tag{14}$$

then the mean and standard deviation of M', $\mu_{M'}$ and $\sigma_{M'}$, are given by:

$$\mu_{M'} = a_0 \tag{15}$$

$$\sigma_{M'} = \left\{\sum a_i^2\right\}^{0.5} \text{ for } i = 1 \text{ to } n \tag{16}$$

Defining a quantity β to be the inverse of the coefficient of variation of M' gives:

$$\beta = \frac{\mu_{M'}}{\sigma_{M'}} \tag{17}$$

If the linearisation of g(**u**) is performed at the closest point on the boundary g(**u**)=0 to the origin,[11] then the quantity β is termed the Reliability Index. The probability of failure, P_f, is given by the probability that M is negative. If the origin is in $\bar{Q}$ (the safe subset of the variable space) then

$$P_f = P[M<0] = \Phi(-\beta) \tag{18}$$

If the origin is in Q (the failure subset of the variable space) then:

$$P_f = P[M<0] = \Phi(\beta) \tag{19}$$

Defining a_i as $\partial(g(\mathbf{u}))/\partial u_i$ it can be shown that if $\mathbf{u}^*$ is an initial estimate of a suitable linearisation point for g(**u**) then $\mathbf{u}^* + \delta\mathbf{u}^*$ will be a better estimate where:

$$u_j^* + \delta u_j^* = \frac{a_j}{\sum_{i=1}^{i=n} a_i^2}\left\{\sum_{i=1}^{i=n} a_i u_i - g(\mathbf{u}^*)\right\} \text{for } j = 1 \text{ to } n \tag{20}$$

Once a converged solution to equation 20 has been obtained the reliability index β is given by:

$$\beta = \sqrt{\sum_{i=1}^{i=n} (u_i^*)^2} \tag{21}$$

and the failure probability P_f is given either by equation 18 or equation 19.

Reliability analysis of parallel systems

A parallel system is one in which all the component failure events are required to occur before the system is considered to have failed; for a k-component parallel system in which the U-space component failure boundaries ($g_i(\mathbf{u})=0$) are linear the system failure probability is given by:

$$P_{f,s} = \Phi_k(-\beta, \rho) \qquad (22)$$

where $\Phi_{k()}$ represents the k-variate standard normal cumulative distribution function, β is the vector of component reliability indices, and ρ is the correlation matrix for the linearised component failure boundaries.

For large values of k the direct evaluation of the system failure probability using equation 22 becomes uneconomic; moreover if the surfaces defined by $g_i(\mathbf{u})=0$ are not linear unacceptable errors may be introduced. Several different approaches are available for evaluating equation 22 which avoid these problems. The system failure probability can be expressed as:

$$P_{f,s} = P[E_1 \cap E_2 \cap \cdots \cap E_k] \qquad (23)$$

where E_1 to E_k represent the k component failure events for the system. Using the definition of conditional probabilities this can be expanded to give:

$$P_{f,s} = P[E_1 | E_2 \cap \cdots \cap E_k] \times P[E_1] \qquad (24)$$

By the repeated application of equation 24 the system failure probability can be expressed as:

$$P_{f,s} = P[E_1 | E_2 \cap \cdots \cap E_k] \times P[E_2 | E_3 \cap \cdots \cap E_k]$$

$$\times \ldots \times P[E_{k-1} | E_k] \times P[E_k] \qquad (25)$$

Equation 25 can be evaluated by determining the probability of failure for event E_k, and then transforming the failure subset Q_k into a new U-space U', see Melchers,[12] and Tang and Melchers.[13] $P[E_{k-1} | E_k]$ is then found by evaluating the probability of failure for event E_{k-1} in the conditional U-space U'. This procedure is then repeated until all the terms on the right-hand-side of equation 25 have been evaluated.

Fatigue reliability updating

LEFM based

The current state-of-the-art in respect of Bayesian updating of safe fatigue life estimates is represented by the approach due to Madsen *et al.*[14] In addition to the safety margin M used to differentiate between Q and $\bar{Q}$ (the failure and non-failure subsets of the basic variable space) two sets of event margins M_i, i=1 to r, and M_j, j=1 to s are defined. The M_i event margins represent the outcomes of inspections at which no cracks are detected, while the M_j event margins represent inspections at which cracks are detected and measured.

The failure margin M is defined as:

$$M = a_c - a(N) \qquad (26)$$

where a(N) is the (predicted) crack length after N cycles of loading and a_c is an appropriate critical crack length.

The event margin M_i (corresponding to no crack being detected) is defined by:

$$M_i = a(N_i) - a_{di} \leq 0 \qquad (27)$$

and the event margin M_j (corresponding to a crack being detected and measured) is given by:

$$M_j = a_j - a(N_j) = 0 \qquad (28)$$

where a_{di} is the size of the largest non-detectable crack, a_j is the measured crack length, and N_i and N_j are the number of cycles corresponding to the inspections.

The prior probability of failure is given by $P[M \leq 0]$ and the posterior probability of failure is given by:

$$P[M \leq 0 | M_1 \leq 0 \cap \cdots \cap M_r \leq 0 \cap M_{r+1} = \cdots = M_{r+s} = 0]$$

$$= \frac{P[M \leq 0 \cap M_1 \leq 0 \cap \cdots \cap M_r \leq 0 \cap M_{r+1} = \cdots = M_{r+s} = 0]}{P[M_1 \leq 0 \cap \cdots \cap M_r \leq 0 \cap M_{r+1} = \cdots = M_{r+s} = 0]} \qquad (29)$$

The equality constraints, due to the equality in the event margin definition in equation 28, can be removed from equation 29 by expressing the updated probability in the form:

$$\frac{\dfrac{\partial^s P[M \leq 0 \cap M_1 \leq 0 \cap \cdots \cap M_r \leq 0 \cap M_{r+1} \leq x_{r+1} \cap \cdots \cap M_{r+s} \leq x_{r+s}]}{\partial x_{r+1} \cdots \partial x_{r+s}}}{\dfrac{\partial^s P[M_1 \leq 0 \cap \cdots \cap M_r \leq 0 \cap M_{r+1} \leq x_{r+1} \cap \cdots \cap M_{r+s} \leq x_{r+s}]}{\partial x_{r+1} \cdots \partial x_{r+s}}} \qquad (30)$$

where the partial derivatives are evaluated at $x_{r+1}=\ldots=x_{r+s}=0$. Equation 30 is given by Madsen *et al.*[14]

Madsen *et al* use a first order reliability method to evaluate the individual probabilities of occurrence of the different events, and system reliability techniques to determine the probability of the coincidental occurrence of two or more events as a means of obtaining updated estimates of probabilities of failure incorporating the information derived from inspections.

S-N based

The fatigue analysis used in obtaining updated estimates of fatigue life need not be based on LEFM. Busby and Carr,[15] have reported results obtained using an S-N approach in which they have updated predictions of the number of through-thickness cracks as a function of time using inspection results. They have adopted the modification by Stahl and Geyer,[16] to Wirsching's model,[17,18] for T_2, the time to through-thickness cracking. T_2 is assumed to be given by:

$$T_2 = B_1 \times B_2 \times B_3 \times T_n \qquad (31)$$

where T_n is the nominal T-curve fatigue life, and B_1, B_2, and B_3 are lognormally distributed random variables representing environmental and Miner's rule uncertainties, modelling uncertainties, and uncertainties in the S-N curve and stress concentration factors respectively.

S-N analysis does not explicitly involve the actual crack length at any time other than at failure (when the crack length is by definition equal to the material thickness for through-thickness cracking). In order to circumvent this problem Busby and Carr have assumed a constant crack growth rate as a basis for predicting the depth of a crack at any time. They have also introduced relationships defining the residual fatigue life of welds after repair (whether by welding or grinding). For joints repaired by welding the residual fatigue life is assumed to be equal to the original nominal fatigue life. For joints repaired by grinding the residual fatigue life is found by multiplying the

Table I: Results of Niu's analysis (for T joints under axial load with equal chord and brace diameters)

Quantity	*Mean*	*Standard deviation*
p	0.669	0.0193
q	–0.300	0.0363
$\frac{2c_{crit}}{1}$	0.219	0.056

Table II: Remaining input data for example calculations (units of MPa and m)

Quantity	*Distribution*	*Mean*	*Standard deviation*
Stress range	normal	100.0	20.0
Initial crack length	exponential	0.008	
Undetected crack length	exponential	0.016	
m	normal	2.92	0.3
Ln(C)	normal	–25.55	0.4

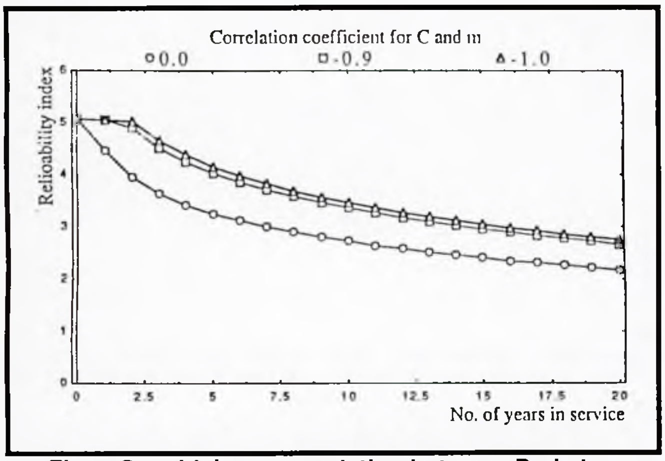

Fig 1: Sensitivity to correlation between Paris law parameters

original nominal fatigue life by a grind factor K, which is a function of the grind depth and the material thickness.

Comparison

Madsen *et al* give results for the propagation of a through-thickness crack in a flat plate in which the prior reliability indices are almost exactly linearly related to the log of the number of load cycles. This implies that the prior fatigue life distribution is lognormal, which is the usual assumption made in the derivation of S-N curves from the results of fatigue tests.

Skjong,[19] has demonstrated the application of the methods proposed by Madsen *et al* to a K-joint in a fixed offshore structure. He has shown that the prior distribution of the predicted fatigue life is the same irrespective of whether LEFM or S-N analysis is used.

It is in the updating of the fatigue life estimates that the LEFM approach can be shown to be superior to S-N analysis. Because LEFM explicitly considers the crack dimension, whereas S-N analysis does not, a greater amount of information can be derived from the updating process if LEFM is used. Moreover, when using LEFM it is possible to explicitly model such parameters as initial and critical crack lengths, variability of material properties, and crack detection and measurement errors. The ability to investigate the importance of the uncertainties associated with such parameters is essential when planning inspection and repair strategies.

EXAMPLE CALCULATIONS

Reanalysis of experimental data

One aspect of the PLAIM project is the reanalysis of fatigue crack growth data for tubular joints, with the objective of deriving parametric models for stress intensity factors and critical crack lengths that can be used directly in a probabilistic fracture mechanics analysis. The results of this reanalysis, Niu,[20] are reported elsewhere. Niu has used the experimental data to infer geometry functions of the form:

$$Y(c) = p\left(\frac{2c}{1}\right)^{q} \tag{32}$$

where 1 is half the footprint length for the weld.

For the case of T joints under axial load (with equal chord and brace diameters) Niu's analysis gives the results shown in Table I where c_{crit} represents the measured half-length of the surface crack corresponding to through-thickness cracking. Q and P have been assumed to be normally distributed and $\frac{2c_{crit}}{1}$ has been modelled as being lognormally distributed.

Remaining input data

The remaining input data for the example calculations is summarised in Table II (units of MPa and m).

The diameters of the chord and brace have been assumed to be 0.9m, giving a value of 1 of 1.72m. Ln(C) and m have been modelled as (negatively) correlated variables.

Results

The results are presented in Figs 1–3 in the form of plots of reliability index against the number of years of service. A loading rate of 3 x 10^5 cycles/year has been assumed.

Figure 1 shows the sensitivity of the computed reliability estimates to the level of correlation assumed between Ln(C) and m, correlation coefficients of 0.0, –0.9 and –1.0 having been used. The largest range in β is for the calculated reliability indices after 2 years. When Ln(C) and m are fully negatively correlated β is 5.02 whereas when Ln(C) and m are assumed independent β is 3.95. The corresponding probabilities of failure are 3 x 10^{-7} and 4 x 10^{-5} respectively, ie a difference of more than two orders of magnitude. A third set of results is also shown corresponding to a correlation coefficient of –0.9. This is the value proposed in Ref 14. Although this may seem to be high other sources, eg Slatcher,[21] have proposed that a value as large as –0.95 may be appropriate. The value of –0.9 has been retained for the remainder of the example calculations.

Figure 1 shows that for the input data used here the reliability index has an upper limit of approximately 5.1. This corresponds to failure occurring in the absence of any fatigue crack growth (as a result of the critical crack length being less than or equal to the initial crack length). Intuitively it may seem appropriate to dismiss such an occurrence as an 'impossibility'; however, given the constraint that no load cycles have occurred (ie after 0 years in service), this intuitively 'impossible' failure mechanism is the only one available. It must therefore be a feature of any analysis of this type that somewhere within the range of service life being considered there will be a change of failure mechanism from one which does not involve any fatigue crack growth to one which does. In the case of the curves in Fig 1, for correlation coefficients of –0.9 and –1.0 this change occurs just before 2 years of service, while for the correlation coefficient of 0.0 the change occurs almost immediately after the beginning of the service life. It should also be noted that a reliability index of 5.1 corresponds to the very low notional probability of failure of 1.7 x 10^{-7}.

Figure 2 demonstrates the results of applying the Bayesian updating procedure based on the assumption that inspections are carried out at 5 year intervals, and that no cracks are detected. It should be noted that the upper limit of 5.1 on the prior estimates of the reliability index does not apply to the updated estimates. This is because the subset of the basic variable space, within which the critical crack length is less than or equal to the initial crack length, does not intersect the subset corresponding to an inspection in which no cracks are detected.

Each inspection without crack detection causes an increase in the calculated reliability index. However the magnitude of the reliability index immediately after an inspection reduces with increasing service life. It should moreover be understood that the probability of not detecting a crack at an inspection is also reducing with time (although this is not evident in the results since non-detection has been assumed). The implication of this is that even frequent inspection cannot be used to justify an indefinite extension of the service life of a component that is potentially susceptible to fatigue cracking.

Figure 3 shows how the updating technique can be used to schedule inspections when it is deemed necessary to maintain some known minimum level of safety (in this case a reliability index of 3.5) throughout the service life. In the example given here inspections after 8 and 14 years would (if no cracks were detected) be sufficient to justify a 20 year service life.

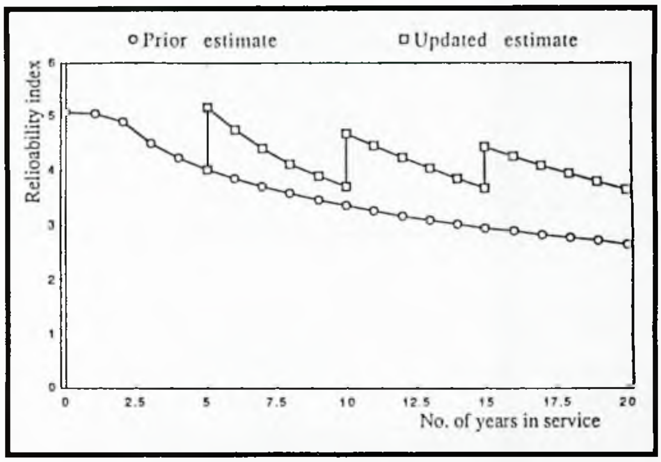

Fig 2: Reliability estimates updated by inspections at 5 year intervals

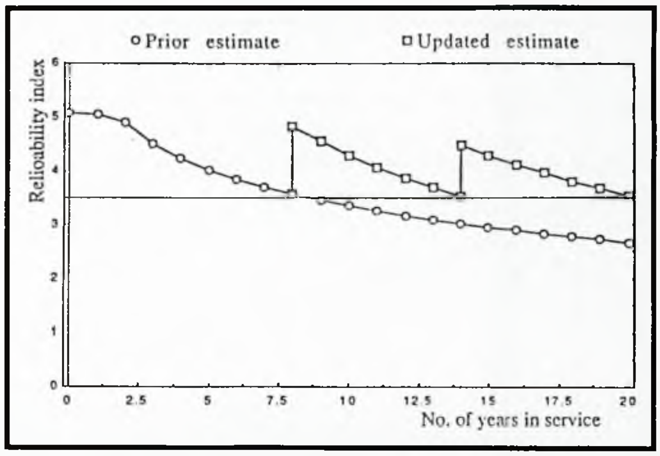

Fig 3: Inspections scheduled to maintain reliability index of 3.5

CONCLUSION

The use of Bayesian updating in fatigue calculations has been demonstrated. The approach allows the uncertainties associated with crack growth, detection, and remaining life estimation to be rationally accounted for. It allows inspection strategies which will provide predetermined levels of safety to be planned. In certain circumstances the inspection intervals that are derived will be greater than the present 5 year interval required for recertification. If exploited this could lead to savings in inspection costs and reductions in the hazards experienced by diver-inspectors.

LEFM based methods appear to have a greater potential for remaining life calculations than S-N based methods. However, this is because of the crack length *vs* load cycles relationship implicit in LEFM and not because an LEFM analysis provides a qualitatively 'better' prediction of fatigue life than an S-N analysis. The advantages of using an S-N based approach would be that S-N analysis is well understood in the offshore industry, and that many of the calculation steps necessary for determining prior estimates of the fatigue reliability are required to be carried out during the design and certification of jackets. It is anticipated that the reanalysis of tubular joint fatigue test data being carried out by Niu may provide the crack length *vs* load cycles relationship that will allow the S-N based approach to provide the same level of updating as the LEFM based method. However, the LEFM approach will still retain the advantage that it makes possible the extrapolation of the limited experimental data that is available beyond the constraints imposed by the environment in which the experiments were performed (eg the effect of sea-water on crack growth can be accounted for by modifications to the Paris law parameters).

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