Trans.I.Mar.E., Vol. 100, pp. 241–248

Thermodynamic modelling of Stirling engines — work at Cambridge University

D. H. Rix, Ph.D., C.Eng., M.I.Mech.E.
Cambridge University

SYNOPSIS

This paper describes how thermodynamic simulation is seen as central to the detail and concept design of Stirling engines. After briefly reviewing existing simulation techniques, the formulation of a novel simulation based on a Lagrangian reference system is described. This is part of a joint programme of both theoretical and experimental work. The experimental programme, which is based on a purpose-built laboratory engine, falls into two parts. First, tests have been conducted by motoring with expansion and compression spaces at the same temperature. Secondly, tests are currently underway where a substantial temperature difference exists between them. Some early results are presented, including comparisons with predictions from the simulation. Finally some indications are given as to future research objectives.

INTRODUCTION

The Stirling engine has always been regarded with some degree of disdain because of its complex and difficult to understand working principles. These basic problems of comprehension bedeviled the early history of the Stirling engine and it was not until the Philips organization applied their scientific and analytical skills that specific power output and efficiency began to rise to the high levels achieved by engines today.

For example, if it is desired to increase power, one obvious approach is to increase the surface area of the heat exchangers, so as to increase the nett heat flow through the engine. If this is done by lengthening the heat exchanger tubes, engine dead volume and working fluid pumping losses will increase, both of which will have the effect of reducing power. Hence the optimum length of the heat exchanger tubes will be at the point where the gains cease to outweigh the increasing losses.

For marine engines, overall efficiency is likely to be of prime importance, hence the design will need careful optimization to satisfy this criteria. In certain applications cost may be critical, in which case the design will need to be optimized to minimize the deleterious effect of simplification and cheap materials.

Theoretical modelling or simulation is therefore the obvious tool to use if optimization is not to involve the building and testing of a very large number of differing prototypes. In what follows the various levels of complexity at which this may be done will be described.

Complex 'third-order' models are of inherently greater value and accuracy than simpler models because they are based on fundamental physical principles rather than on empirical correction factors. Although some doubts have been expressed about their value in the past, it has always been my belief that their failings were due to a lack of serious development, in conjunction with detailed experimental measurements from real engines, and also that, in solving the equations used to describe the behaviour of the working fluid, fundamental physical principles were violated. The simulation developments which will be described seek to remedy these shortcomings.

David Rix served a student apprenticeship with Petter Diesels and then went on to work in the Design Office. After becoming involved in the U.K. Stirling Engine Consortium, he transferred to Cambridge University in 1980 in order to concentrate on Stirling engine research. Currently he is a Royal Society University Research Fellow within the Department.

The experimental part of this work has involved the construction of a special laboratory machine, which may be considered as a motored engine. In constructing this, careful attention was paid to achieving a design which operated under realistic conditions, i.e. in terms of mean pressure, velocity, temperature and working fluid. Also it had to be amenable to modelling, with such features as equal length heat exchanger tubes, as well as provide for the ready collection of data, i.e. proper provision for instrumentation. Above all, it had to be reliable, without problems such as lubricant contamination of the heat transfer surfaces, and easily constructed by a typical University workshop.

A motored engine, as distinct from one running under its own power, was used to reduce the number of experimental variables, in that it was possible to replace flame heating of the 'hot end' of the engine by a liquid heat-transfer medium, first water and then oil. The fundamental equations are as applicable to a motored engine as they are to one which is self-sustaining.

The experimental work was divided into two distinct categories: that performed with the expansion and compression spaces at approximately ambient temperature, using water as the heat-transfer medium, and that done with a large temperature gradient between the expansion and compression spaces, using oil as the heat transfer medium. This also provided an opportunity to apply the simulation to a real design exercise.

One important but unexpected discovery was the generation of distorted pressure–time traces under certain conditions, which led to the idea of characterizing the internal losses by non-dimensional groups. It will be demonstrated how these rationalize the apparently anomalous behaviour of certain working fluids.

The choice of variables which may be measured and so compared with predictions of the simulation is limited. Apart from gross operating parameters such as speed and mean pressure, the two used were heat transfer, in the compression and expansion spaces, and pressure–time variation at different points in the engine. Given a simulation validated in this way, third-order models are of additional merit in that they can then supply reasonably accurate information about local working fluid properties and metal temperatures.

NOMENCLATURE

a	speed of sound	(m/s)
D	characteristic dimension	(m)
M	molecular weight	(kg/kmol)
N	rotational speed	(rev./s)
P	pressure	(Pa)
Q	heat flow/cycle	(J)
R	gas constant	(J/kmol·K)
T	temperature	(K)
v	velocity	(m/s)
z	pumping power	(W)
γ	ratio of specific heats	
μ	viscosity	(kg/m·s)
ρ	density	(kg/m^3)

Suffices

C	compression space
E	expansion space
j	jth mass

SIMULATION MODELS

Simulation models vary widely in complexity and applicability, depending on the underlying simplifications incorporated into them. One classification system commonly used ranks them as first-order, second-order and third-order methods[1].

First-order methods are the most basic, giving a single value for power output based on the fundamental design parameters such as mean pressure, speed and displacement, i.e. the Beale equation[1]:

Power output (W)	=	0.005	x	Swept volume (cm^3)	x	Speed (rev./s)	x	Mean pressure (bar)

Consequently such methods are of little use to the designer.

Second-order methods start from a simplified 'idealized' analysis, i.e. no friction, infinite heat transfer rates, etc., so as to give basic values for power output and heat input. These are then modified by the addition of various energy loss terms, usually calculated in a semi-empirical fashion. Such models have been used for many years, most notably by Philips, since they are quick and undemanding in computer resource terms. However, accurate assessment of the loss terms demands an enormous investment in experimental effort.

In Philips' case it involved hundreds of man-years of work. Even then the results are only applicable to designs of an essentially common family, although it must be admitted that such empirical data do cover for certain non-uniformities which defy even the most complex analytical models.

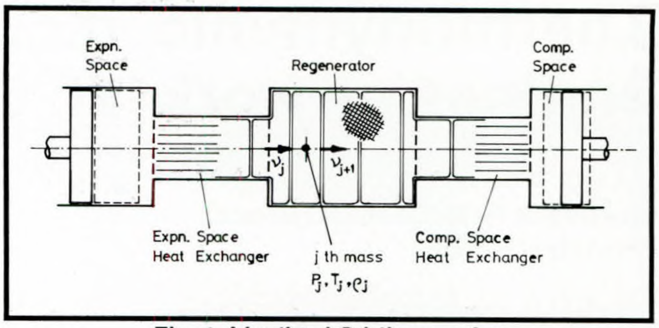

Fig. 1. Idealized Stirling engine

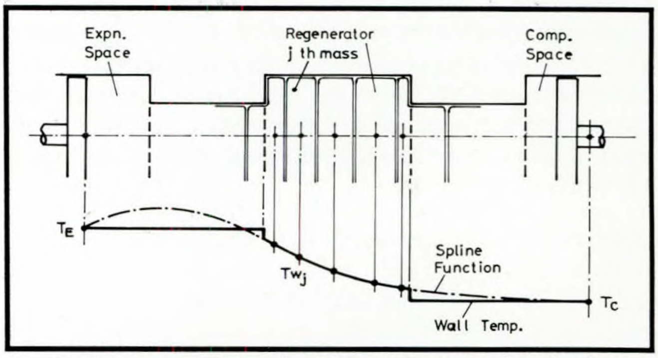

Fig. 2. Representation of wall temperature distribution in an idealized Stirling engine by spline function

The energy loss terms which need to be considered include such things as:

1. fluid friction losses in the heat exchangers;
2. non-isothermal behaviour;
3. static heat conduction through the engine walls;
4. 'shuttle' conduction caused by the hot displacer piston alternately moving from the hot to the cold end of the cylinder;
5. displacer/cylinder wall ('appendix') gap pumping loss.

Attempts have been made to produce more general correlations of these losses in terms of variables such as velocity and temperature, *viz.* by Martini[2], but it is unlikely that the idealized analysis gives sufficiently accurate estimations of velocity and temperature to enable such correlations to be applied with any real accuracy. Although it is convenient to have the various energy losses separated out, attempts to reduce any one of these may be in error, since they are often interrelated. This was illustrated by the earlier example of heat transfer and pumping loss.

Given these limitations, Stirling engine analysts have turned to models based on the fundamental physical principles of conservation of mass, momentum and energy, so-called third-order models. In these the working fluid within the engine is divided into various sub-masses, the conservation equations are applied to each and then solved simultaneously to yield local and instantaneous values of variables such as velocity, pressure and temperature. By repeating this procedure for incremental advances of time or crank angle, values of gross parameters such as power or heat input may be summed for a complete cycle.

These models have only become possible with the advent of large powerful computers, and whilst it is sometimes argued that this undermines their usefulness, the widespread and successful use of finite element methods in stress analysis suggests that computing power should not be a limitation.

There are pitfalls, not least in the use of numerical solution techniques, to solve the conservation equations. Also since the models which have been developed to date have almost invariably been one-dimensional, it is still necessary to rely on empirical correlations to calculate heat transfer, fluid friction loss, etc. None the less, it is still believed that satisfactory and generally applicable models can be developed. Quite apart from the work at Cambridge University, the results achieved by NASA-Lewis[3] seem to bear this out.

THE SIMULATION

Formulation of the conservation equations

The following is offered as a summary of the simulation which has been developed in the course of this programme of work. Full details are given in ref. 4. It is currently formulated for a standard 2-piston crankdrive engine, as shown diagrammatically in Fig. 1, a so-called 'alpha' configuration. This also shows the working fluid divided into a number of sub-divisions with locally averaged properties ρ, v, T applying to each.

Where the simulation breaks new ground in relation to other published third-order simulations[1] is in its use of a Lagrangian co-ordinate system. Rather than the sub-divisions being fixed in both position and time, with the working fluid flowing through them, the sub-divisions oscillate with the fluid particles and contain a fixed mass of working fluid, typically M_j.

There are naturally good reasons why the first approach, using an Eulerian co-ordinate system, has been adopted elsewhere. The most important of these is because a typical Stirling engine has several abrupt changes in flow cross-sectional area which are most readily dealt with in a stationary sub-division system. However if this difficulty is accepted, the Lagrangian formulation avoids two otherwise serious shortcomings.

First, if the conservation equations are solved by numerical techniques, 'numerical diffusion' is a potential problem. In practical terms this can be likened to axial heat transfer occurring at a rate determined by the size of the time, or crank-angle, increments used in the repeated solution of the equations. Instead the Lagrangian formulation allows heat transfer to occur, as in practice, at a rate determined by working fluid velocity.

Secondly, in modelling the compression and expansion space swept volumes, it is no longer necessary to treat the working fluid as stationary, with uniform properties throughout and adiabatic behaviour. Rather heat transfer is directly related to velocity, albeit only the axial component of velocity in a one-dimensional formulation.

Another potential failing of all one-dimensional formulations is the need to use empirical correlations for heat transfer, friction losses, etc. Stirling engine analysts have invariably been forced to use steady flow correlations in what is otherwise a reversing flow situation. There are no alternative data available.

It is now believed that this is not a serious limitation, except in the case of low Reynold's number flow through the regenerator, a problem which will be discussed later.

One final simplification to note concerns the momentum equation, viz.:

$$\text{Sum of forces on sub-mass} = \text{Rate of change of momentum}$$

The rate of change of momentum term is assumed to be zero, or in practical terms, the effect of pressure waves is assumed to be insignificant. This seems to be a reasonable assumption since Stirling engines operate at low Mach numbers. Also it avoids the Lagrangian formulation falling prey to the criticism that pressure waves propagate at sonic velocity and not at an arbitrary rate set by the integration scheme, i.e. at particle velocity.

An attempt was made by Organ[5] at Cambridge University to address this problem using the method of characteristics, though it has not been subsequently pursued.

Linearization and solution

The conservation equations as applied to the sub-mass system of Fig. 1 form a non-linear set. The method chosen to solve them involves linearization by expanding the variables about their values at the end of the previous time step. This in effect means that rather than formulating and solving the equations in terms of ρ_j, v_j, etc., they are solved in terms of $(\rho_j - \rho_{\text{old value } j})$, $(v_j - v_{\text{old value } j})$, etc.

The resultant set of equations is then solved by a standard implicit technique. The solution proceeds by advancing to the next time step. Implicit solutions have rarely been attempted by other Stirling engine analysts, in spite of their propensity to allow larger incremental time steps and greater solution stability.

Modelling of wall temperatures

One of the greatest difficulties experienced in developing this Lagrangian formulation concerned modelling the regenerator wall, or matrix, temperature distribution. The method finally adopted was by the use of a 'spline' temperature distribution (see Fig. 2). This is adjusted at the end of each time step to account for the nett heat transfer which has occurred during that period.

Elsewhere, as shown in Fig. 2, the wall temperatures are assumed to be constant at either T_E or T_C. It is necessary to start with an assumed temperature distribution within both the working fluid and the regenerator matrix. Then, rather like the warm-up process in a real engine, the simulation proceeds through a number of transient cycles until the nett heat transfer to the regenerator/cycle and the nett change in working fluid internal energy/cycle are both zero.

A technique is employed to increase the rate at which this happens, but it may still take 50–100 cycles. The 'forcing' rate is apparently limited by solution instability. This is an obvious area to tackle in future development work with a view to reducing computer run time.

TEST MACHINE

The test machine (see Fig. 3) is of the same 'alpha' two-piston configuration as the schematic machine considered previously. As Fig. 3 shows, the machine is effectively upside-down in relation to conventional practice, a feature adopted to allow for possible fluidized bed heating. The design parameters chosen were:

Total swept volume	116 cm^3
Maximum operating speed	2000 rev./min
Maximum working pressure	7 MPa
Maximum metal temperature	650 °C
Working fluid	Air, He, etc.

The crankdrive represents a logical solution to achieving the twin aims of simple construction and a high degree of reliability (at least 100 h between rebuilds). The crankcase is pressured at mean working fluid pressure and all bearings are dry running, or grease lubricated, to avoid potential oil contamination. A Ross-type crank mechanism is employed to give the requisite 90° (approx.) phase angle between expansion and compression space pistons from a single crank throw, thus requiring only the most simple form of crankshaft. This mechanism gives minimal piston side thrust and so avoids the need for piston cross-heads. The crankshaft is provided with a proprietary rotary face seal to enable the crankcase to be pressurized.

Further general features of the design include modular construction, with parts such as the crankcase, cylinder liners and crankshaft main bearing housing all separate and readily changeable. Special tools are an integral part of the design to allow speedy and accurate dismantling and assembly. This has proved a particularly valuable feature as has the almost non-existent need for sealing and retaining compounds in assembly. A full description of the detailed design of the machine is given in ref. 6.

EXPERIMENTAL WORK

$T_E = T_C$

Initial testing of the machine was performed by motoring it with the expansion and compression spaces kept at temperatures approximately equal to ambient. The measurements made, for comparison with the predictions of the simulation, were the pressure–time variations in the machine (expansion space, compression space, either side of the regenerator) and the rate of heat transfer to or from the expansion and compression space heat exchangers. The compression space heat exchanger was surrounded by a water jacket and the expansion space heat exchanger was mounted in a free-standing water bath. Hence by recording the flow rate and temperature rise of the water in each, it was possible to calculate the heat transfer. Note that the designations expansion and compression space are used here in the sense of this being a heat pump and not a prime mover when they would be reversed.

Various working fluids were tried in the machine (helium, air, neon, carbon dioxide and argon) and it was noted that the pressure–time traces on the expansion side of the machine became distorted when using the heavier gases. This was a particularly striking phenomenon, since pressure traces taken on the compression side of the machine remained largely unaffected (see Fig. 4c). It is caused by 'choking' of the regenerator, not in the conventional sense of sonic flow, but because of the rapidly rising flow resistance as the Reynold's number decreases ($Re = v\rho d/\mu$). In Fig. 4(a), where the mean pressure is lowest and hence the Reynold's numbers are lowest, the pressure in the expansion and compression spaces is varying exactly in-phase with the corresponding piston motion. In other words negligible flow is passing through the regenerator and the expansion and compression space pressure variations are independent of each other.

As the mean pressure is increased (see Figs. 4b and 4c), the Reynold's number increases sufficiently over parts of the cycle to let an increasing amount of flow to occur through the regenerator. Hence although the pressure traces become nearer to being in-phase with the nett volume variation of the pistons, indicating that flow is occurring through the regenerator, the distortion of the expansion space pressure trace shows that choking of the regenerator is sometimes occurring, i.e. when flow velocities and hence Reynold's numbers fall to low values in the regenerator.

Dimensional analysis was applied in a most revealing fashion. It was shown that the apparently different behaviour of the various working fluids could be reconciled by the non-dimensional relationship:

$$\frac{Za}{PND^3} = \text{function}\left(\frac{P}{N\mu}, ND\sqrt{\frac{M}{RT}}, \ldots + \text{ other groups}\right)$$

The left-hand side is non-dimensional and indicates pumping power whilst the two most significant non-dimensional groups shown on the right-hand side are analogous to Reynold's number and Mach number.

One practical confirmation of this is shown in Fig. 5, where it can be seen that there is a noticeable similarity in the pressure–time traces taken for different working fluids under widely differing conditions of speed and mean pressure, but at constant values of the groups $P/N\mu$ and $ND\surd(M/PT)$.

The results were extensively compared with predictions from the simulation, which led to various revisions in its formulation. Not least it was found that the exact form of the momentum equation was critical if pressure–time trace asymmetry was to be correctly predicted. Less satisfactorily, it was found that the standard heat transfer and friction factor correlations used for the regenerator matrix gave extremely erroneous results when choking flow was present.

In the case of the friction factor, multiplication by a

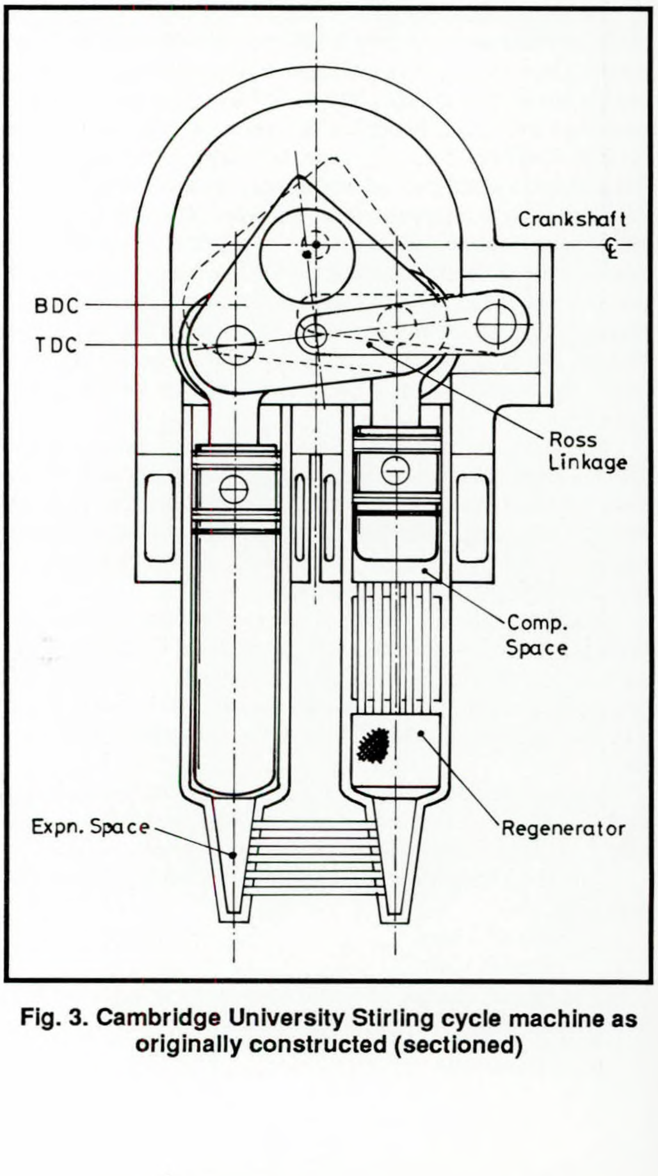

Fig. 3. Cambridge University Stirling cycle machine as originally constructed (sectioned)

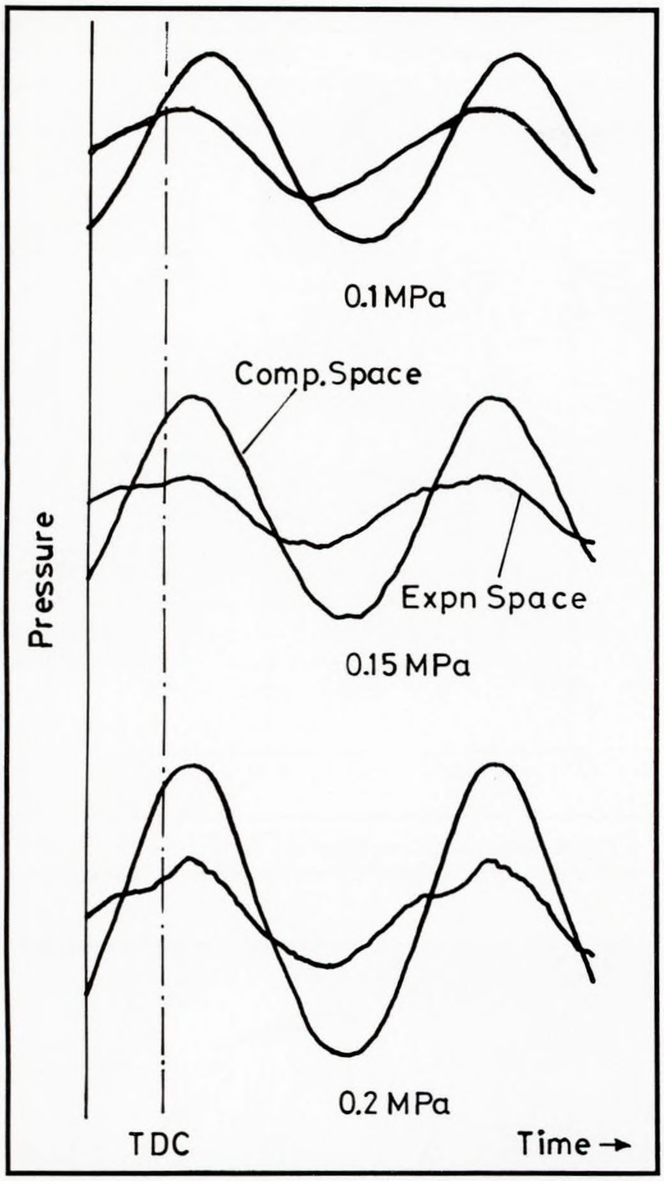

Fig. 4. Pressure traces for expansion and compression spaces showing the effect of pressure variation on choking flow development

correction factor brought the measured and predicted results into good agreement (see Fig. 6). In the case of heat transfer, such an expedient did not prove satisfactory. However, it was notable that choking flow in the regenerator had a remarkably beneficial effect on heat output[4]. Contrary to conventional wisdom, the machine transferred more heat when using air as working fluid than when using helium under otherwise identical conditions of speed, pressure and temperature.

It was concluded that these anomalies in the simulation should not be taken as inherent defects, but rather as defining its limitations. For instance it is obviously important to avoid the choking flow regime by suitable choice of operating conditions and suitable design of the heat exchangers. An understanding of these limitations is vital if the simulation is to have general applicability. Coincidentally it did encourage a searching examination of both the simulation formulation and the experimental heat flow measurements[4]. This resulted in the adoption of the spline function representation of the regenerator matrix temperature distribution and changes in the water

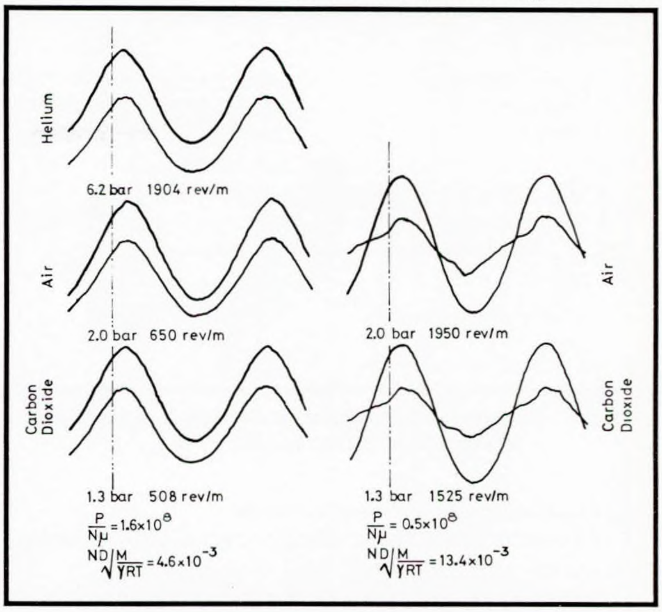

Fig. 5. Pressure traces for different working fluids demonstrating the similarity between them for constant values of ($P/N\mu$) and ($ND\sqrt{M/\gamma RT}$)

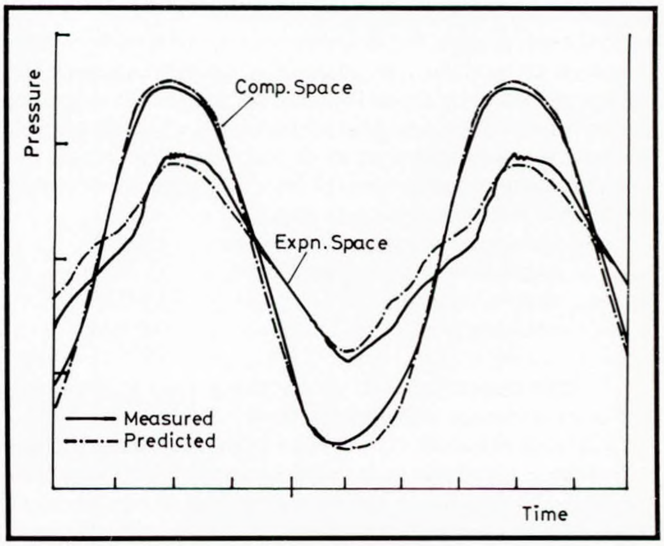

Fig. 6. Comparison of measured and predicted pressure traces (air, 5.1 MPa, 2000 rev./min)

circulation system. It also caused a calculation to be made of the static heat conduction losses, which as hoped were found to be small.

$T_C > T_E$

Engine and test system redesign. Following the success of the work described in the previous section, it seemed logical that the next step should be to add a substantial temperature gradient across the regenerator and so create a new set of conditions against which to develop and validate the simulation. The main change that this required was to the external heat-transfer medium, since to reach a temperature of several hundred degrees with water would have required pressurization. Liquid metals were considered, but hot oil was chosen for the following reasons, although its maximum operation temperature was limited to approx. 300 °C.

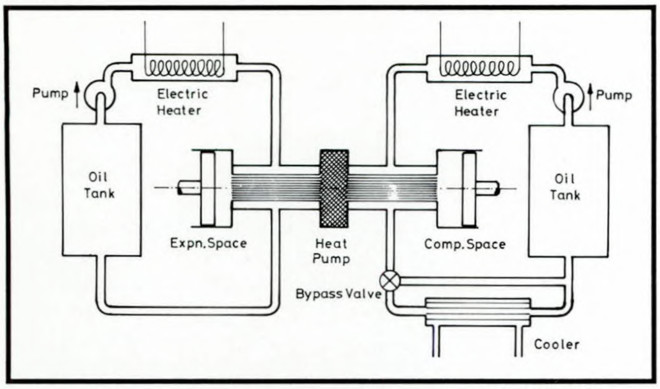

Fig. 7. Schematic arrangement of high-temperature Stirling heat pump facility ($T_C > T_E$)

1. Relatively safe and easy to handle.
2. Commercial hot oil circulators were available for heating the oil.
3. Near-identical systems could be used for both expansion and compression ends of the machine, allowing experimental flexibility, i.e. reversal of the machine, etc.

A schematic diagram of the oil heating system is shown in Fig. 7.

It was also decided that the existing heat exchangers were unsuitable for conversion, especially since they were far from an optimum design. For instance they contained very large dead volumes and they presented unacceptably large paths for static wall heat conduction from the hot to the cold side of the engine. In designing new heat exchangers it would be possible to apply the simulation so as to optimize their design, i.e. achieve maximum heat transfer from the compression space. The design parameters chosen were:

compression space temperature	250 °C
expansion space temperature	75 °C
mean pressure	5 MPa
working fluid	Helium
speed	1900 rev./min

Helium was selected as the working fluid to avoid any potential problems with choking flow.

Fig. 8 shows some typical results from the design optimization process, namely how the heat transfer/cycle in the compression and expansion spaces varies with the length of the tubes in the respective heat exchangers, all other variables such as tube number and size remaining constant. It will be seen that the tube lengths corresponding to maximum heat transfer from the compression space are 100 mm for the expansion space heat exchanger and 300 mm for the compression space heat exchanger. In both cases these values are also not far from being the optimum for heat transfer to the expansion space, although it was found that in other cases, such as the optimization of the regenerator, this was not necessarily so. Obviously design optimization in this way is an iterative process, since one design variable cannot be changed without affecting the others. However, it has usually been found that the second iteration gives sufficiently accurate answers.

Turning to the detailed design of these new heat exchangers, the principles followed were:

1. minimum connecting dead volumes;
2. minimum wall conduction losses, i.e. thin-wall construction;
3. tappings for pressure gauges and miniature pressure transducers.

The resultant designs can be seen in Fig. 9, where the two heat exchangers and the regenerator are shown assembled on

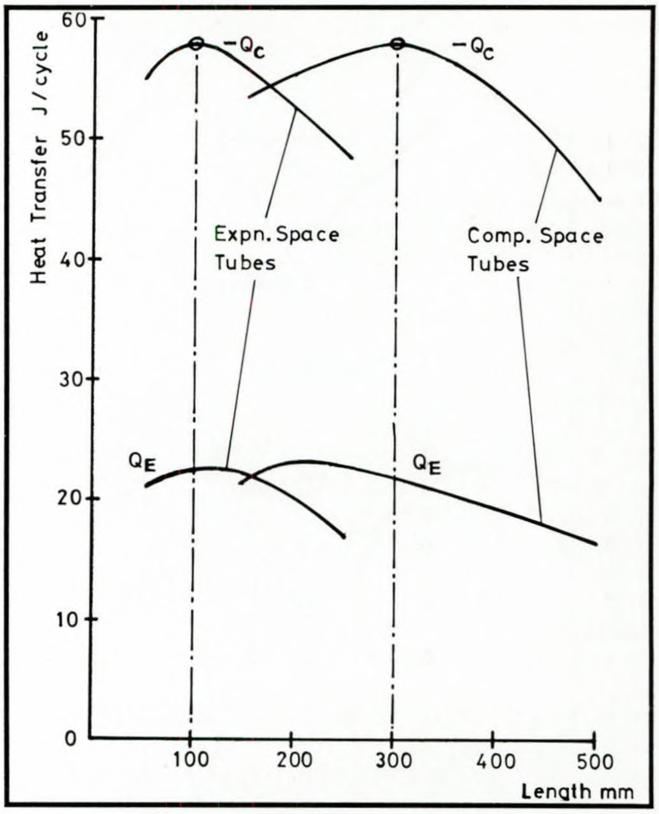

Fig. 8. Design optimization curves for revised heat exchanger tube lengths (other variables held constant)

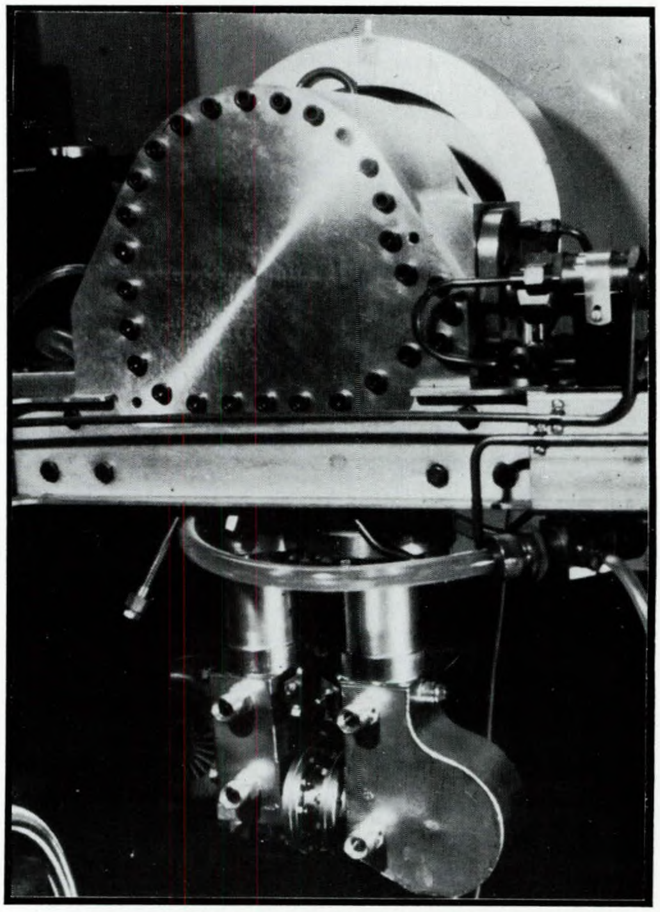

Fig. 9. Cambridge University Stirling cycle machine rebuilt as a high-temperature heat pump

the engine crankcase. The tube 'bundles' are surrounded by sheet metal jackets through which the oil heat-transfer medium is circulated.

Comparison of predicted and measured heat transfers. The revised engine has been tested over a wide range of pressures, speeds and temperature differences (i.e. $T_C - T_E$). Data have been collected with regard to heat transfer in the compression and expansion spaces, pressure–time variations throughout the machine and external wall temperatures. Considerable effort has been devoted to getting an accurate heat balance for the machine, *viz*.:

$$\text{Power input} = Q_c + Q_{cyl} + Q_{lub} + Q_{conv} + \ldots - Q_E + \text{Error}$$

This involved recalibration of thermocouples and the engine dynamometer, etc. In general, unaccounted heat transfers are less than 4% of the power input to the engine, rising to about 10% at low levels of power input.

In comparing predicted and measured results, one problem concerned wall conduction and convective heat losses, items which are not yet included in the simulation. The best approximation which could be obtained of these was to measure heat transfer in the expansion and compression spaces with the engine stationary and subtract or add these to the measured results. Although it is appreciated that the temperature gradient across the regenerator is different from that when it is being motored, heat conduction across the regenerator is small enough to make the difference insignificant. In fact it was found that the main heat losses were convective ones from the engine structure and though the obvious solution was to apply lagging, the large number of thermocouple leads and the possibility of heat-transfer oil saturating the lagging rendered this impractical.

Figs. 10 and 11 show typical comparisons between measured and predicted heat transfers for different mean pressures and speeds, respectively. The temperature difference in both cases is 100 °C. Agreement between the two sets of results is generally quite good, other than at low speed. It had been previously found with this engine that cyclic irregularity was significant at low speeds, i.e. the variation in angular velocity, from the mean value, over a given revolution. It is expected that this would significantly affect heat transfer. It is in order then that heat transfers are always over-predicted, since there are a number of other minor losses which are not included in the simulation, i.e. piston ring leakage, piston annular gap loss, piston-to-wall 'shuttle' conduction losses, etc.

Serious differences between measured and predicted results were, however, found when low heat-transfer oil flow rates or temperatures much less than 100 °C were used. The problem appeared to be the creation of non-uniform heat-transfer conditions between the oil and heat exchanger tubes. At low flow rates the oil fails to penetrate the centres of the tube bundles, particularly in the compression end heat exchanger.

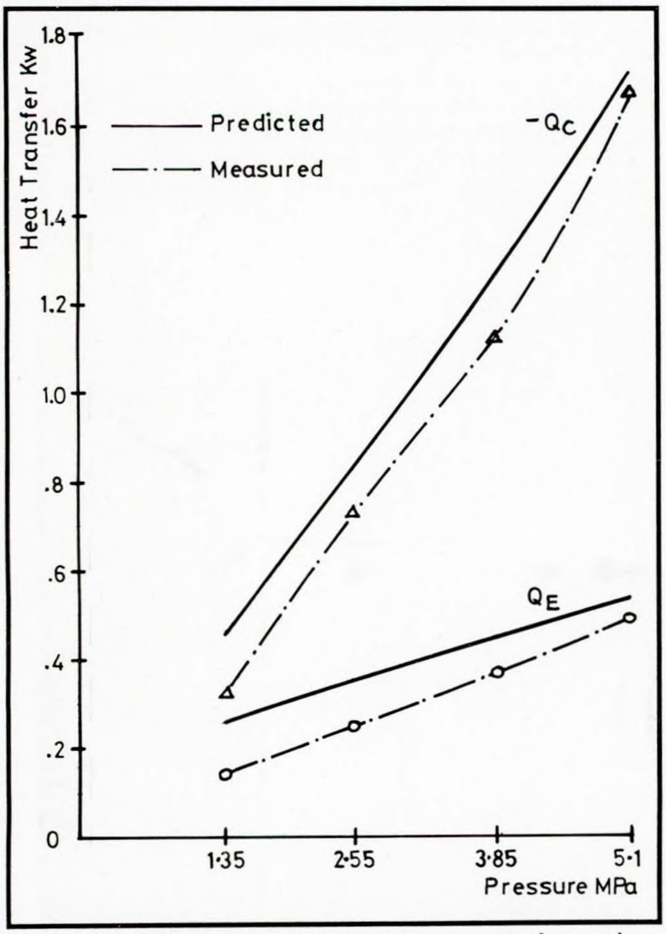

Fig. 10. Measured and predicted compression and expansion space heat transfers at constant speed (950 rev./min, ΔT = 100 °C)

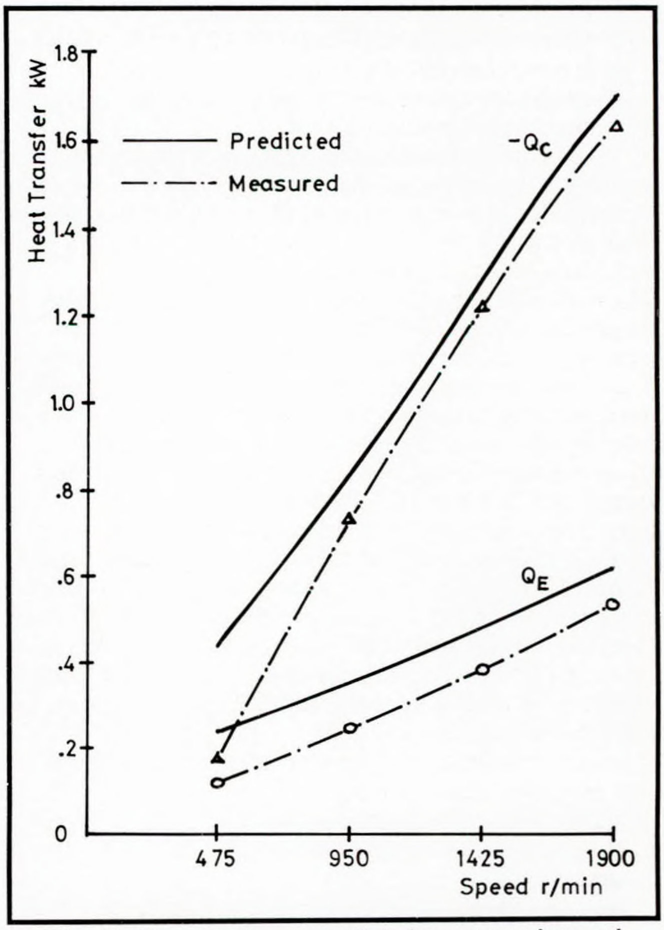

Fig. 11. Measured and predicted compression and expansion space heat transfers at constant pressure (2.55 MPa, ΔT = 100 °C)

This effect is compounded by the heat-transfer oil becoming more viscous as the temperature falls. This is particularly true at the expansion end where quite low temperatures may be reached. It was perhaps significant that, under certain conditions, ice would form at each end of the hot oil jacket around the heat exchanger tubes. This does not represent an error in the simulation so much as once again defining its limitation.

CONCLUSIONS

There is a real need for simulation of the Stirling engine if an optimum design is to be achieved. Existing models have tended to be flawed by either their empirical nature or, in the case of more complex models, violation of fundamental physical principles. An improved third-order simulation has been outlined which, by virtue of its adoption of a Lagrangian reference system, seeks to take due regard of these failings. Furthermore, a continual process of cross-checking the predicted results with those actually measured has steadily eliminated errors and weaknesses in the formulation.

For this purpose it was necessary to construct a special laboratory engine which could be motored to provide the necessary high quality data, as well as operate over a wide range of conditions. This has proved successful, thus justifying the design philosophy behind it.

Good agreement has been demonstrated between predicted and measured heat transfers and pressure–time variations for:

1. two different heat exchanger geometries;
2. different working fluids;
3. different pressures up to 5 MPa;
4. different speeds up to 2000 rev./min;
5. zero temperature difference ($T_C = T_E$);
6. temperature differences up to 150 °C ($T_C > T_E$);
7. conditions where low Reynold's number regenerator flow choking exists.

The resultant simulation is now a very useful design tool and this has been demonstrated by applying it to the design optimization of new heat exchangers. Undoubtedly there is still room for improvement, and it is intended that attention be given to the modelling of wall temperatures in the expansion and compression spaces using the same technique applied to the regenerator.

Undoubtedly there will also be new types of simulation developed in the future. For instance, also at Cambridge University, Organ[7] is developing a simulation based on the use of entropy generation rate to simplify the treatment of the conservation equations. Finally it is intended to extend the use of simulation in the design of Stirling engines to the combustion system, using the rapidly developing techniques of combustion modelling.

ACKNOWLEDGEMENTS

This work was carried out with the assistance of the Science and Engineering Research Council, The Royal Society and Darwin College, Cambridge.

REFERENCES

1. C. D. West, Principles and Applications of Stirling Engines, Van Nostrand Reinhold (1986).
2. W. Martini, Stirling Engine Design Manual, DOE/NASA/3152-78/1 NASA CR-135382 (April 1978).
3. S. Geng, Calibration and Comparison of the NASA-Lewis Free Piston Stirling Engine Model Predictions with RE-1000 Test Data, vol. 4, *Proc. 22nd IECEC*, Philadelphia (Aug. 1987).
4. D. H. Rix, The Further Development and Application of an Advanced Stirling Cycle Simulation, vol. 4, *Proc. 22nd IECEC*, Philadelphia (Aug. 1987).
5. A. J. Organ, Gas Dynamics of Stirling Cycle Machines, *Proc. I. Mech. E. Conf.*, Reading (March 1982).
6. D. H. Rix, The Design of a Laboratory Stirling Cycle Machines, *Proc. I. Mech. E. Conf.*, Reading (March 1982).
7. A. J. Organ, Thermodynamic Design of Stirling Cycle Machines, *Proc. I. Mech. E.*, vol. 201, no. c2 (1987).

Discussion

L. C. CHAN (Undergraduate student of Hong Kong Polytechnic): First I would like to express my appreciation of the achievement behind the papers. In addition I have some questions for the authors.

1. Single-acting and double-acting alpha-configuration engines have been developed and manufactured by various companies. Will the configuration still be in the main trend of commercial Stirling engine development for the coming years or will new configurations, such as free piston take its place? (**Commander Reader's** and **Professor Walker's** papers).

2. It was pointed out that one disadvantage of one-dimensional third-order simulation is the reliance on correlations to calculate heat transfer, fluid friction losses, etc. Is there any implication that three-dimensional simulation can completely eliminate the experimental work related to empirical correlations? (**Dr. Rix's** paper).

3. The rise of third-order simulation was not reputed as an efficient tool for design optimization. Will the new simulation technique produce a breakthrough in its efficiency in terms of computational time and data presentation? Also, will the new simulation be capable of adopting different engine configurations without much modification? (**Dr. Rix's** paper).

4. Studies of heat transfer through porous materials, especially under oscillatory flows, are not, at present, mature. It is therefore doubtful whether there is a general mathematical model suitable for computer simulation purposes. (**Dr. Rix's** paper).

A. FOWLER (Lecturer, University of Newcastle): The authors have collectively presented us with some highly informative and very interesting insights into the current state of the art with respect to Stirling engine technology, and I would like to congratulate and thank them for their efforts.

Being particularly interested in control systems and dynamic aspects associated with underwater power sources, a number of questions arise within this context.

Could the authors comment first upon the method of achieving speed control in response to changing load? The sequence of events following load changes in a diesel engine, for instance, is well established, with the speed-sensitive governor controlling fuel rack position and hence instantaneous torque to achieve a very fast response system which can ensure minimal steady-state and transient droop, even with rapid and large swings in applied load. However, the need to match oxygen flow to fuel during transients becomes potentially more problematic in the case of a closed-cycle engine[1]. What is the equivalent sequence of events in the Stirling system and are there any dynamic effects such as thermal inertia in the heater/regenerator loop which slug the response or otherwise complicate the control systems? If so, how are these features accommodated?

Turning to a related issue, that of engine mechanical reversibility: a novel reversing mechanism is mentioned in **Professor Walker's** paper based on a valve for changing the sequence of cylinder linking in the four-square engine. Could such a device incorporate a fully modulating principle to achieve progressive torque variation over the complete range from full-ahead to stop and through to reverse? Such a system would in some respects appear analogous to the Stephenson-link of steam reciprocator fame and would, if practical, constitute a potentially most efficacious facility within the itinerary of the Stirling engine's characteristics.

Also, with respect to control of combustion conditions, it is noted that an equivalence ratio of approx. 1.1 is quoted, which is similar to that which would be expected in good boiler practice. However, in the Stirling engine combustion chamber, fuel is burning at high pressure in a continuous oxygen-rich flame with some internal exhaust recirculation, and it might have been supposed that overall oxygen consumption would be nearer stoichiometric. Could the authors comment on the problems and limitations in attainment of good combustion with minimal oxygen consumption in the Stirling combustor?

Furthermore, could the authors quote typical gas constituencies at entry to the combustion flame and in the engine exhaust? How is the mass flow rate of recirculated gas controlled (the thermic ballast system) and is this achieved through inherent regulation by the oxygen eductor system, or is an active control component required?

It would also be interesting to receive details of specific operational parameters, such as specific fuel and oxygen consumption (gross and nett is applicable considering any auxiliary loads); also the salient temperatures and pressures achieved in practice, around the Stirling *P–V* diagram.

The particularly flat part-load characteristics of the Stirling is a notable feature and would also provide a useful addition to the presented data.

Dr. Rix has presented some interesting details of this engine simulation concentrating on the Lagrangian formulation. Manadon have also done much work in the simulation field, and perhaps **Commander Reader** could outline for comparison the basic philosophy of his models in terms of subsystems identified, variables processed and areas of application. Have any dynamic models been developed and applied for control system studies, for example?

The question of oxygen storage is critical to all thermodynamic subsea systems, and United Stirling's LOX system is probably a world leader. Could the authors benefit us with their experiences concerning potential problems of safety, unwanted boil-off (when the plant is shut down) and attainment of required boil-off rate when the plant is running? What is the 'all-in' specific weight and volume of the LOX system, including any auxiliary plant?

Finally, although probably incidental to the main function of the collective presentations, it was fascinating to learn of the extensive use of 'reversed' Stirlings in refrigeration applications. Perhaps the authors could summarize the respective potential advantages (and indeed disadvantages) of such systems compared with more orthodox techniques, including perhaps a reference to specific potential marine applications.

Reference

1. Development of Control Systems for an Underwater Power Source, Control '85 conference publication 252,, I.E.E., vol. 1 (1985).

E. H. COOKE-YARBOROUGH (Cooke-Yarborough Technology): Towards the end of his paper, **Commander Reader** mentioned other marine applications of Stirling engines, including power for remote sites and for offshore platforms.

Low-power Stirling engines have already been used in these applications[1]. These engines use a metal diaphragm, instead of a piston, to provide mechanical power output to drive a special alternator. This avoids the problems of seal wear and of friction.

One such engine, delivering 25 watts, was the main power source for the U.K. National Data Buoy. Using propane for fuel, it powered the buoy at sea for 21 months between 1978 and 1980.

Another similar engine delivered 60 watts to power an unmanned major lighthouse off the Irish coast for two periods, each of 12 months.

These engines can have almost infinite operating lives. One of them, powered by the heat of radioactive isotopes[2], operated at the Harwell Laboratory from November 1974 until it was decommissioned in May 1987, after $13^1/_2$ years continuous operation.

The use of such Stirling engines can seriously be considered in situations where it would not be possible to use solar power or wind power. This could arise, for example, if operation is required under water. A chemical heat source should be able to power one of these engines underwater for many weeks. Continuous operation under water for a period of many years is possible if radioisotopes are used to provide the heat.

References

1. E. H. Cooke-Yarborough, Small Stirling-cycle Power Sources in Marine Applications, Oceans '80 Conference Record, I.E.E.E. Publication Number 80CH1572-7 (September 1980).
2. E. H. Cooke-Yarborough & F. W. Yeats, Efficient Thermomechanical Generation of Electricity from the Heat of Radioisotopes, Harwell Laboratory Report AERE-R8036 (May 1975).

A. BURNETT (President and Chief Executive, Offshore and Marine International Services Associates): I would like to thank the authors for a very interesting paper, particularly in view of the increased interest over the last few years in subsea development programmes including the need for power down to 1000 m or more water depth of oil/gas and mineral deposits, below or on the sea bed in various parts of the world.

This paper raises a number of points of interest which will now be addressed.

In **Professors Nilsson**'s paper, mention is made of the need to add exhaust gas compression units to the plant when submerged below say 600 m water depth. It is well known that mineral nodules exist in quantity on the sea bed at water depths below 600 m. What will be the cost penalty of the exhaust gas compression system for such mineral recovery purposes? Will it affect, to any great extent, the cost-effectiveness of the commercial operation of autonomous offshore submarine units fitted with the Stirling engine?

It is welcome to note that low noise and reduced vibration features are now embodied in the Stirling engine for offshore use. These features, whilst being of considerable interest to military submarines, must also have advantages for offshore use. Would it be possible to highlight a few of these?

It is not clear from the paper the range (miles/km) limitation of the Saga I submarine nor how it is intended to maintain the submarine itself. Will it be necessary to remove the Stirling from the submarine for overhaul purposes, or can the engine routine maintenance work be carried out *in situ*?

Now that helium conservation systems are available for divers under saturation, whether deployed from diving bells or autonomous offshore submarine units, how will this affect the storage capacity of helium in bottles around the offshore submarine hull as well as the range and the possibility of utilizing some available helium storage space for other purposes?

It would be interesting if a further paper could be presented when the forthcoming Saga I offshore trials have been both completed and fully analysed. We wish all success with the forthcoming offshore trials scheduled for this year.

Authors' replies

G. T. Reader: First of all let me thank all those who have taken time to pose interesting questions and widen the discussion.

Mr. Chan asks about the commercial development of the Stirling engine and the type of configuration that will be used. The alpha-type engine produces more specific power per unit volume and for many applications where power density is crucial, e.g. in underwater power plants, this configuration will be preferred. However in other applications other configurations will be more suitable. Thus free-piston engines and kinematic engines of the beta and gamma types are being commercially developed. Mr. Chan is referred to the publications given in refs. 1 and 2 below which will give a better overview.

Dr. Fowler has raised many interesting points. He states that load-speed control of the diesel is well-established and asks if this is so for the Stirling. The answer is yes. The above references discuss the control techniques available such as mean pressure modulation, phase angle variation and so on. The topic is well covered in the numerous NASA and United Stirling publications available. This may seem somewhat of a glib answer but it is not intended that way, for a short answer suitable for this type of written discussion could confuse rather than enlighten. Regarding the computer models, the basic philosophy I have found to be the most useful is that called the 'de-coupled' approach. With this the system is divided into its various major components, e.g. heat input device, engine unit and transmission system. For the first and last items it is possible to use an empirically based factor for combustor efficiency and mechanical efficiency respectively. It is possible to model these in greater analytical depth but generic factors are usually used in concept studies and semi-empirical factors used in design and simulation. With regard to the engine unit, with the de-coupled approach the thermodynamics of the engine are considered initially which is separated into a number of discrete volumes. The thermodynamic processes are idealised such that the cylinders are considered to be adiabatic or isothermal and the heat exchangers isothermal. The non-steady flow energy equation is item applied where appropriate and the concept of conditional enthalpy at space boundaries invoked. When a stabilized solution has been achieved the data obtained is then used to calculate the system losses and check the stressing of the heater and cooler heat exchangers. Once this process is complete the results are used to calculate the input parameters to the simulation. If there is not agreement with those given, adjustments are made to the specification of the initial cylinder conditions and the simulation process is repeated until an acceptable solution is obtained. The analysis is steady-state and although I have not as yet attempted a transient analysis directly, others have. However, the literature is not especially enlightening in this area and for the same reasons as that it is not for the closed-cycle diesel.

I would like to thank **Mr. Cooke-Yarborough** for highlighting an omission on my part. The engine he mentions, the TMG, proved to be successful in its application and is still worthy of attention. I understand that these devices are still available but for the future both weight and cost would need to be reduced to ensure commercial success.

References

1. G. T. Reader & C. Hooper, Stirling Engines, E. F. Spon, London (1983).
2. G. Walker, Stirling Engines, Oxford University Press (1980).

G. Walker: I appreciate the interest of questioners on the topic of Stirling engines. In reply one is tempted to go on at length but I will heed the well-established precept; brevity above all else.

Mr. Chan's speculations on the course of future development of Stirling engines are as good as mine. Reference to the basic books shows there are many different ways to go about things. My opinion is that insufficient work has yet been done to indicate clearly the 'best' solution. Our work at Calgary on large engines is directed to compact engines with reasonable power density and high efficiency, that are easy to start and reverse and are of good reliability with little or no maintenance requirement, and are of modest cost. These 'motherhood' targets are equally applicable to many other forms of engine and, needless to say, we have far to go to achieve our objectives.

Dr. Fowler asks about mechanical reversibility. The reversing device illustrated in my paper is intended to specifically fix the rotation of the engine in the clockwise direction. Power modulation may be achieved by the engine control system, of which half-a-dozen types have been demonstrated.

Dr. Fowler mentions the use of Stirling refrigerating machines. This is presently the only well-established application of Stirling engines and is concentrated on the field of the miniature cryogenic refrigeration units used for infra-red night vision and missile guidance systems. I believe there is a larger future for Stirling refrigeration systems in the area of Freon-free refrigeration for many applications including air conditioning.

Much information of interest about Stirling refrigerators can be found in the following reference[1].

Reference

1. G. Walker, Cryocoolers, Vols. I and II, Chapter 3, Plenum Publishing Corp., New York, NY (1983).

H. Nilsson: In response to **A. Fowler**:

Speed control – For the United Stirling underwater engine, speed is kept constant by running the generator in parallel with the batteries. The power output from the engine is set by a given fuel flow.

Fuel/oxygen flow control – At a given fuel flow the oxygen flow is adjusted to establish a complete well controlled combustion, which means that there is a slight excess of oxygen, over the stoichiometric value.

Gas constituencies – Major fractions in the exhaust gas are carbon dioxide, water vapour and a small amount of oxygen.

Exhaust gas control – The flow rate is determined by the nozzle area. Extensive development work has resulted in the current design.

Fuel consumption – Current status is approximately 1 kg of O_2/kWh.

Oxygen storage – Extensive safety analysis has been performed for the LOX systems. The SAGA system is furthermore certified by Lloyd's.

The boil-off, due to heat leakage, is very low, and the current LOX systems can remain shut down for weeks without the pressure increase, caused by the heat leakage, being unacceptable.

The LOX system of SAGA has a volume of approximately 5 m^3. The size of the Swedish Navy system is non-available information.

In response to **A. Burnett**: A system with exhaust gas compression, say to 600 m, is an open system, with an exhaust disposal to the open sea.

At increasing depth/pressure the CO_2 in the exhaust becomes more and more difficult to handle, as the triple-point for CO_2 approaches.

A very deep diving system, as required for mineral recovery, has hence to be a closed system, with an inboard storage, say cryogenic, of the exhausts. Alternatively, dissolving the exhaust gases is a solution.

D. H. Rix: In reply to **L. C. Chan,** no implication is intended that three-dimensional simulation can eliminate experimental work related to the empirical correlations of a one-dimensional model. All that can be said is that the more sophisticated simulation would provide a better framework to operate within.

Whether or not third-order simulation is an efficient tool for design optimization is entirely dependent on the computing resources available. Most commercial finite element stress analysis work requires considerably greater computing power than that available.

As far as adaptation to different engine configurations is concerned, although this is quite possible with all the different simulations, inevitably, the more detailed the model, the more work will be entailed.

It is agreed that there is a need for much more information about oscillating flow through the regenerator and heat exchangers of a Stirling engine. However, the problem is more complicated than one of simple oscillating flow, because of the simultaneous pressure variation. When this is added to the fundamental inconsistencies of third-order Eulerian formulation simulation, experimental validation of the simulation can become a matter of chance. Hence the reason for developing a Lagrangian formulation.