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Longitudinal Vibration of Marine Propeller Shafting

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Synopsis

Longitudinal vibration of the propeller shafting has been the cause of seizure, serious scoring and excessive wear in the flexible couplings of Naval vessels and has in some cases given rise to unacceptable vibration of the bridge structure. Other defects such as difficulty in retaining stern gland packing, wear and loosening of stern tube bushes and loosening of rivets in thrust block seats have been attributed to the same cause. In three- and four-shafted ships the vibration is aggravated when turning because the blades of the inner propeller on the outside of the turn cut into the slip stream of the outermost wing propeller. It has been necessary on this account, in the majority of big ships, to reduce the power on the outermost wing shaft when turning at high speed, a procedure which introduces difficulties in control and reduces manœuvrability.

The present paper describes how theory suggested that the vibration was magnified by resonance between the natural frequency of the system and the impulses arising from the number of blades on the propeller, and predicted that the amplitude would be reduced to an acceptable figure by changing the number of blades. Details are given of the vibration trials carried out to confirm the theory, and it is seen that the predicted improvement is, in fact, achieved. The trial results are worked up to give fundamental data as to thrust block flexibility, entrained water, thrust variation, damping factor, and the effect of turning. Using this information complete calculations are set out for a particular line of shafting, showing the effect of two possible thrust block positions and of using three- or five-bladed propellers. A second example treats the case of a very long shaft and shows that there is a need for further data. Using the worked examples as a basis for discussion, general principles are suggested by the application of which trouble may be cured in existing ships and avoided in new construction.

Introduction

During the last five years a considerable amount of work, both theoretical and experimental, has been devoted to this problem. Experimental work has included the measurement of vibration during sea trials of ten ships and a static deflection test of a thrust block on shore under full thrust load. Theoretical work has consisted of analysing trial results to obtain fundamental data and then using that data to predict the behaviour of the shafting in new designs or the effect of modifications proposed for existing ships. Marine engineers will appreciate that when a ship, and more especially a warship, is in service, there is a considerable lapse of time between the expression of a theory and its translation into fact; for example, over two years passed between the first proposal to fit a five bladed propeller and its sea trials. Partly for this reason there are still gaps to be filled in and this paper must, therefore, be regarded in some aspects as an interim report. The author hopes, however, that even with these shortcomings, it will prove both interesting and useful.

Early History

In 1937 H.M.S. Warspite completed an extensive reconstruction which had included replacement of the original direct drive machinery by modern geared turbines. The new engines proved satisfactory in all normal respects until steering trials commenced, when the claw type flexible couplings between the turbines and pinions of the inner shafts suffered serious damage The damage consisted of scoring, rapid wear and even seizure of the mating claw faces. The first action was to increase supply of oil to the couplings, but further trials showed little improvement and is was concluded that excessive loads were being applied by vibration. After a most comprehensive investigation it was eventually discovered that a heavy longitudinal vibration occurred in the inner shaft on the outside of a turn and that this vibration involved rapid fore and aft sliding of the coupling claw faces. It was concluded that the cause of the vibration lay in the fact that with the stern of the ship swinging round the inner propeller was working partly in the wake of the outer screw, the latter being located 24 feet further forward. This condition is illustrated in Fig. 2.

Further trials were then carried out with the object of discovering how to avoid the condition arising. It was eventually concluded that the only effective method was to eliminate the slip stream of the wing propeller on the outside of the turn by almost closing the main throttle valve of that set of machinery when turning at high speeds. This procedure was found to reduce the amplitude of vibration by one half, from an average maximum of ±·050in. to ±·025in.

In addition to the flexible coupling damage, there was heavy hull vibration in the aft portion of the ship, and serious leaks devel-

FIG. 1.—*Worn coupling face*

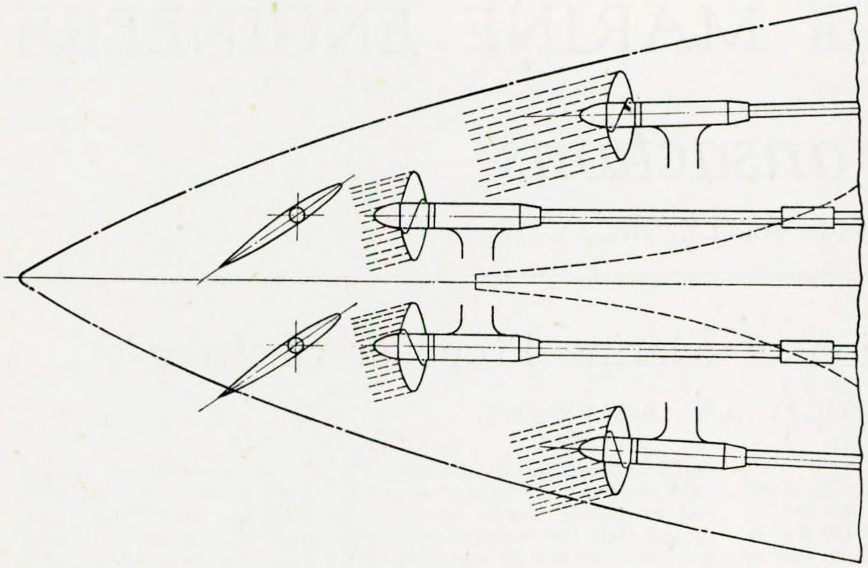

FIG. 2.—*Propeller interaction.*

Calculations were made of the natural frequencies of torsional lateral and longitudinal vibration and the latter seemed the most probable, although it was necessary to make an unexpectedly large allowance for flexibility in the thrust block and seat in order to match the observed speed. Subsequent enquiry revealed that the thrust blocks of the inner shafts did in fact become extremely lively at the speed in question and that loosening of bolts, keys and rivets was a regular occurrence.

At this stage the records from Warspite were re-examined, and the longitudinal natural frequency was calculated, making similar allowances for thrust block flexibility. The calculations gave a natural frequency of about 900 cycles per minute, corresponding to a critical speed of 300 r.p.m., i.e. full power, with 3-bladed propellers. From these results, it was concluded that resonance between propeller impulses and the natural frequency was largely responsible for building up high amplitudes of vibration. It was also concluded that the propeller thrust variation existed on straight course due to the blades passing through the comparatively dead water near the hull, and was much accentuated during turns if the blades also encountered, at the opposite end of a diameter, the fast running water in the slipstream

oped in the region of the propellers. The "easing" procedure described above prevented the recurrence of both troubles.

Following this experience Queen Elizabeth and Valiant, then being re-engined, were fitted with oversized main thrust blocks, and at the same time the "easing" rules were applied to all battleships except the twin screw Rodney and Nelson.

Later History

The trials of Warspite were not followed up by any further investigation, presumably because the easing procedure was so successful, but in new construction some consideration was given to the fore and aft distance between wing and inner propellers. It is evident that if all propellers could be placed abreast, as in Fig. 3, the trouble could not arise, but such an arrangement would be most undesirable in a warship from the point of view of action damage. For this reason the only possible improvement appeared to lie in increasing the distance and in more recent ships this is of the order of 40 to 50 feet

The author's attention was first drawn to the problem in 1943, during discussion with another department regarding an apparent "critical speed" in H.M.S. Furious at about two-thirds of full power revolutions.

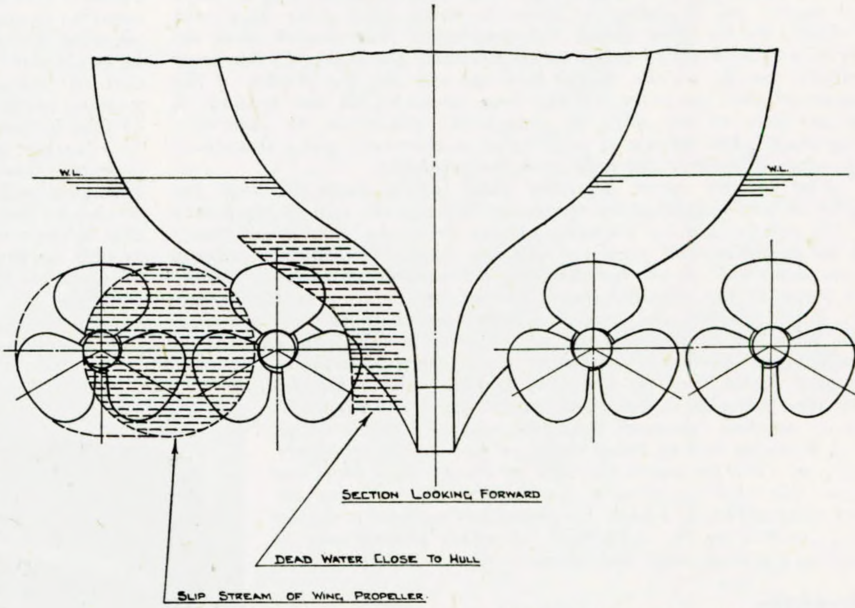

FIG. 4.—*Dead water and slipstream. Vessel turning to starboard.*

from the wing screw, as shown in Fig. 4.

Complaints were received at about this time from the Illustrious class, 3-shafted ships, regarding excessive vibration and difficulty in keeping packing in the centre shaft stern gland. More recently these ships have suffered from loosening of stern tube bushes, loose rivets in thrust block seats and unduly rapid wear in the flexible couplings. Here again the speeds reported agreed with the calculated longitudinal critical of the centre shaft.

A further fillip was given to investigation by protests against the operational inconvenience of the easing procedure laid down for battleships, which made it necessary to sound the warning bell to the appropriate wing engine room before each turn, and also upset the operation of the boilers in that unit. Another less serious complaint concerned the resulting slight increase of turning circle diameter and time to turn.

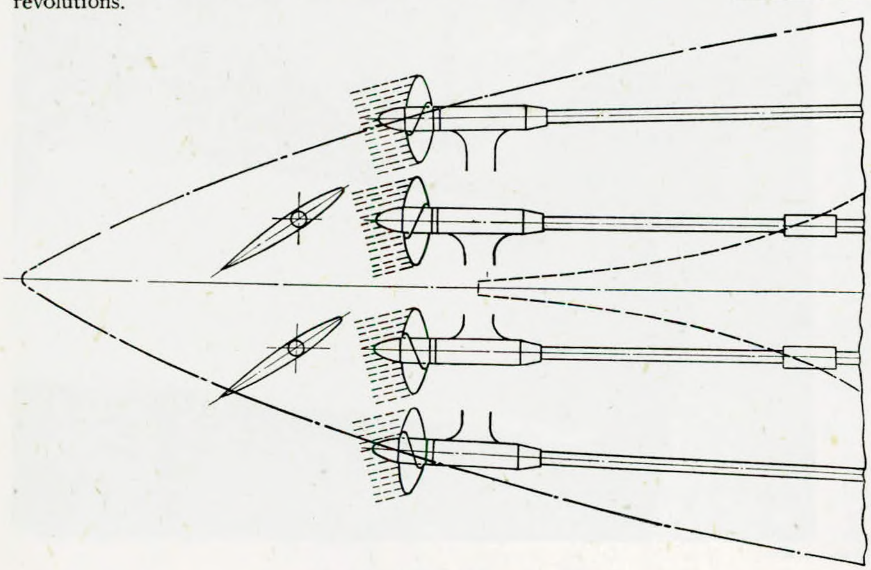

FIG. 3.—*Propellers all abreast.*

Theoretical Possibilities of Improvement

The conclusions so far reached, on admittedly slender evidence, suggested that resonance was an important factor in building up high amplitudes and that it might be possible to reduce the vibration to acceptable limits if it could be avoided. Two possible methods presented themselves: moving the thrust block further aft and so raising the critical above full speed, or increasing the number of propeller blades and running through the critical at a lower power where it might be harmless. The first method was attractive in that it avoided any loss of efficiency which might arise from a propeller with more than three blades, but could not readily be applied to existing ships. It was approved in certain new construction, but unfortunately the vessels concerned are among those cancelled at the close of the war. The possibilities and limitations of such a re-arrangement are discussed more fully in a later section of the paper. The second method was obviously the simplest for ships in service and appeared promising provided that such a propeller could be made to work without excessive cavitation.

Sea Trials

It was obviously desirable that further experimental evidence should be collected before embarking upon the design and manufacture of a propeller, and arrangements were accordingly made to carry out vibration measurements in H.M.S. Formidable, with a view to ordering a five-bladed propeller for the centre shaft if the results confirmed the importance of resonance. Similar trials were also arranged for new aircraft carriers then nearing completion and it was hoped that sufficient fundamental data would be collected to allow reliable prediction of the performance of any arrangement of shafting and number of propeller blades. The data required consisted principally of the following—flexibility of thrust block and seat, weight of entrained water to add to the propeller, propeller thrust variation and damping constant, and the effect of turns.

With these objects in view, arrangements were made to measure the amplitude of vibration of the main gear wheel, the thrust block casing, and the shaft in the gland compartment. In addition the forward movement of the main gearwheel relative to the gearcase and the forward movement of the shaft in the gland compartment were recorded. The programme included measurements of amplitude and frequency at a planned series of speeds on a straight course, turns at the critical speed and at full power with a series of different rudder angles and with the outermost wing shaft both "eased" and "uneased", and the measurement of amplitude at various points on the thrust block casing and seat at a constant speed. More recently direct measurement of the forward movements, and also the vibration measurements in the gland compartment, have been omitted for reasons which will appear later.

In all, trials have been carried out in one quadruple screw battleship, two quadruple screw carriers, two triple screw carriers, three twin screw carriers, one quadruple screw cruiser and one twin screw destroyer. With the exception of the twin screw carriers which have liner type bossings and the centre shafts of the triples, all the propeller shafts have been carried in "A" brackets. So far only one five-bladed propeller and two four-bladed propellers have been tested, the remainder being three-bladed. The results from another pair of fives should become available next year and those from further fives and fours in two or three years time.

Instruments

The particular range of frequency concerned, from 300 to 1,250

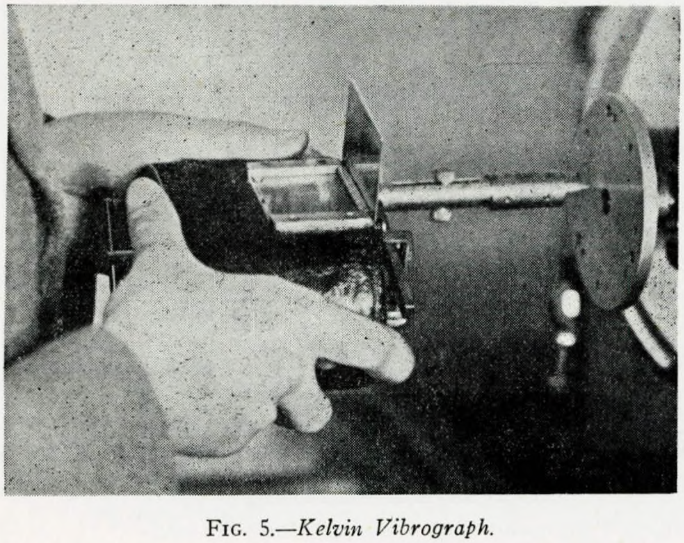

FIG. 5.—*Kelvin Vibrograph.*

FIG. 6.—*General Radio Vibration Meter.*

vibrations per minute, is not over well provided for in respect of measuring instruments. For instance, the inductive type electrical pickups of the Sperry M.I.T. set, which has proved such a valuable tool in investigating vibrations in marine reduction gears, do not respond satisfactorily to such low frequencies.

This difficulty has led to the use of three entirely distinct types of instrument. The first includes the Cambridge Vibrograph and the Kelvin Vibrograph, both of which are hand held semi-seismic instruments which produce a scratched record, the former on a celluloid strip and the latter on pink waxed paper. Each automatically time marks the record, the former from a separate clock and the latter in a less constant manner from the rotation of its own paper drum. In each the scratching stylus is connected to a spring loaded probe which is held against the vibrating object with sufficient pressure to bring the stylus to a central position on the strip. Both instruments are satisfactory for their part of the job which is to provide a visible record from which frequency and shaft speed can be checked relative to each other. They are not altogether satisfactory for amplitude measurements because although the human body is an excellent damper, the calibration does appear to vary with different individuals and perhaps not surprisingly to be affected at times by the fact that the operator is standing on a vibrating base. Neither is convenient for use on a rotating shaft, such as the forward end of the gearwheel, because the spring pressure is not sufficient to hold a push rod in firm contact.

The second type of instrument used is the General Radio Vibration Meter, this has a crystal pickup contained in a case which can be held firmly against a push rod bearing on the forward end of the gearwheel shaft. The crystal pickup responds to acceleration and the resulting small electric impulses are amplified and integrated twice to give an r.m.s. vibration amplitude reading on a galvanometer. The instrument does not in the ordinary way give any indication of frequency, being designed for use with a low frequency analyser which has not yet become available in this country, but it can by the operation of selector switches read acceleration or velocity instead of amplitude and, for a purely sinusoidal vibration, frequency can be deduced from the relative values of these quantities. It has, however, been used during these trials purely to measure amplitude and with the exception of Warspite is responsible for all the amplitude figures quoted. Very great patience is required of the operator because the instrument has annoyingly unstable tendencies and he must watch and wait until he considers it has settled down before taking a reading. Similar patience, a steady hand and physical endurance are required

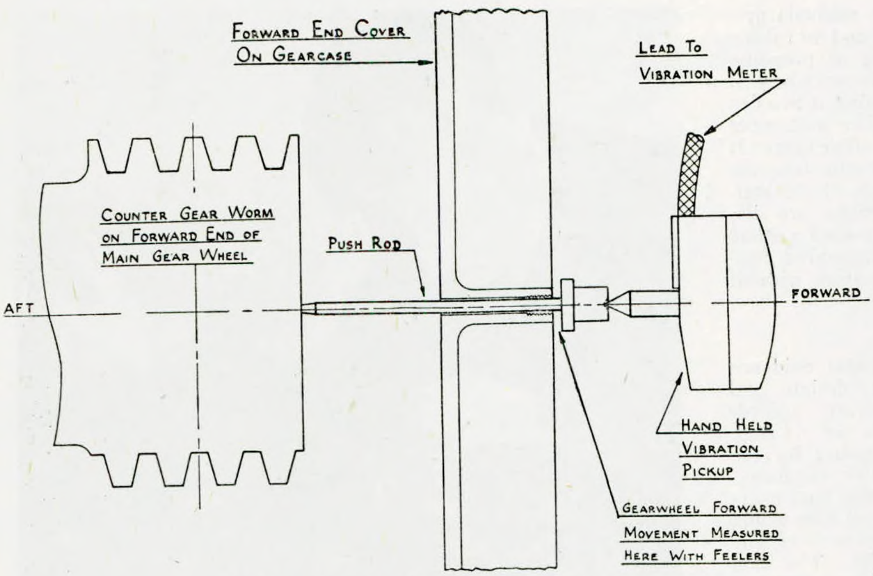

FIG. 7.—*Push rod and vibration pickup.*

of the individual who holds the pickup, often in a position of acute discomfort. An important point to note is that despite rubber feet the instrument picks up vibration through its case and must, therefore, always be held on the operator's knee. This point is illustrated in Fig. 6 which shows calibration in progress, a check which is applied before and after every trial.

The third instrument used was a vernier gauge made for the purpose of measuring amplitude in the gland compartment. A section of the shaft was painted dull black and a circumferential line scribed through the paint to the bright surface while turning slowly and in the absence of vibration. When longitudinal vibration occurred the fine bright line expanded into a broader bright band and the vernier gauge, mounted on a bracket close to the shaft, was used to measure the width of the band.

The forward movement of the main gearwheel relative to the gearcase was measured with feelers and block gauges. A hole was drilled through the forward end cover in way of the end of the shaft, and a push rod inserted and held against it as shown in Fig. 7. Measurements were made between the head of the pin and the outside of the end cover.

Results of Sea Trials

The readings taken during the first sea trials, on an inner shaft of a quadruple screw carrier, showed clearly that there was in fact a longitudinal critical speed, in this case slightly below full power, and that the frequency of vibration was as expected equal to three times the speed of revolution. A plot of vibration amplitude at the thrust block against r.p.m. was found to conform roughly to the shape of the resonance curve for a single degree of freedom system with a dynamic magnifier at resonance of ten. This meant that the propeller thrust variation, plus and minus some three tons, was being magnified tenfold by resonance to an alternating force of thirty tons, and confirmed the importance of resonance in building up large amplitudes.

The effect of turns was investigated at full power and the results obtained were generally similar to those from Warspite. It was found that when turning to starboard with equal power on all shafts the amplitude of vibration of the port inner was increased four to five times while for a similar turn with the port outer eased to 50lb./sq. in. steam pressure the multiplier was only two.

The multiplier of four, in conjunction with the dynamic magnifier of ten, was increasing the three ton thrust variation to the considerable figure of ∓ 120 tons, approximately equal to the full thrust load.

The trials in H.M.S. Formidable followed some three months later and the results were generally similar except that the amplitudes on straight course were somewhat greater, as might be expected for a centre propeller behind structure, and the effect of turns rather less. In this case the critical speed was lower and turns were carried out both at full power and at the critical with peculiar and misleading results, in that the amplitude of vibration was found to be the same in both cases. This led the author to conclude that the effect of turning increased very rapidly with speed and later to hope that a new ship with a critical well below full power would prove satisfactory with three-bladed propellers, an illusion rudely shattered by her first sea trials. It is now considered that the similarity of amplitude in the two cases must be attributed either to the rapid change of shaft speed occurring during turns, or to the breakdown of the vibrating system when thrust reversal occurs. The maximum amplitude observed in most ships has been approximately that required to cause the pressure on the thrust pads to vary from zero to double the steady thrust, but the vessel mentioned above proved a complete exception to this rule: the thrust collar jumped the full axial clearance between ahead and astern pads, and the pinions were thrown the full axial clearance of their teeth, moving over one quarter of an inch.

Fig. 8 shows the amplitude of vibration of the main gearwheel in Formidable plotted against r.p.m., and the dotted curve below is the prediction made at that time for the probable performance of a five-bladed propeller. It should be appreciated that somewhere to the right there is, theoretically at any rate, an alarming second critical whose position depends on the very elusive relationship between thrust block flexibility and propeller entrained water. Fear of this uncertainly placed second critical would have led to the choice of four rather than five blades, but it was considered that the former would give an even greater thrust variation than three blades if used behind the "sternpost" structure in way of this centre shaft.

The five-bladed propeller shown in Fig. 9 was in fact tried in H.M.S. Illustrious (a sister ship of Formidable) two years later and it will be seen that the results were even better than the original prediction. The improvement when turning was equally striking and the movement of the thrust block is now barely perceptible under the worst conditions.

Similar improvement has been found on fitting four-bladed propellers to the twin screw carriers. These vessels have liner type bossings instead of the usual Naval "A" brackets, and it was anticipated that this feature would increase the propeller thrust variation. Trials were accordingly arranged in an early ship of the class and as expected a critical speed was found in the

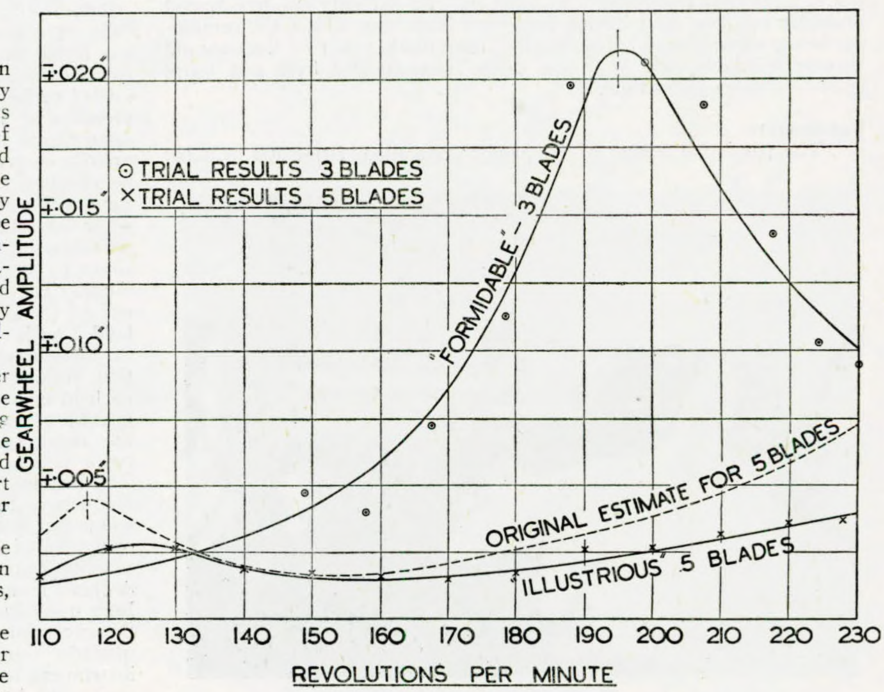

FIG. 8.—*Trial results—three- and five-bladed propellers.*

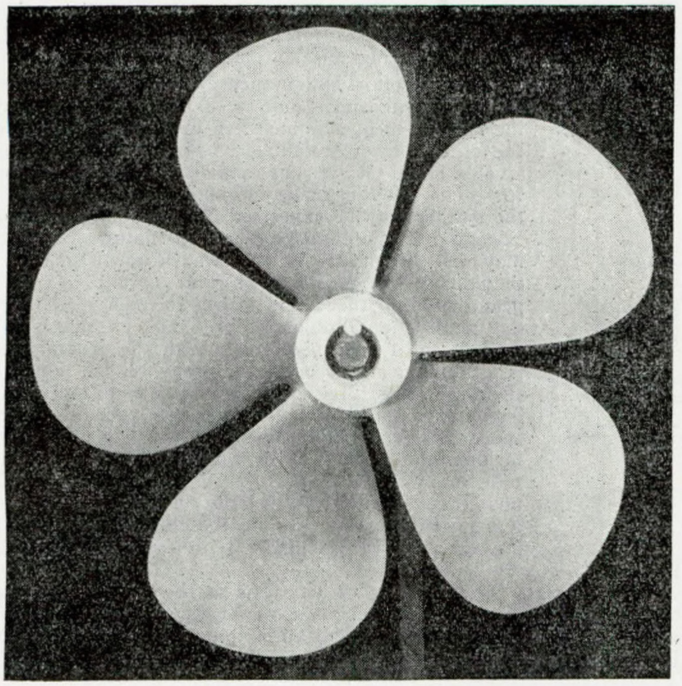

FIG. 9(a).—*Five-bla ed propeller model.*

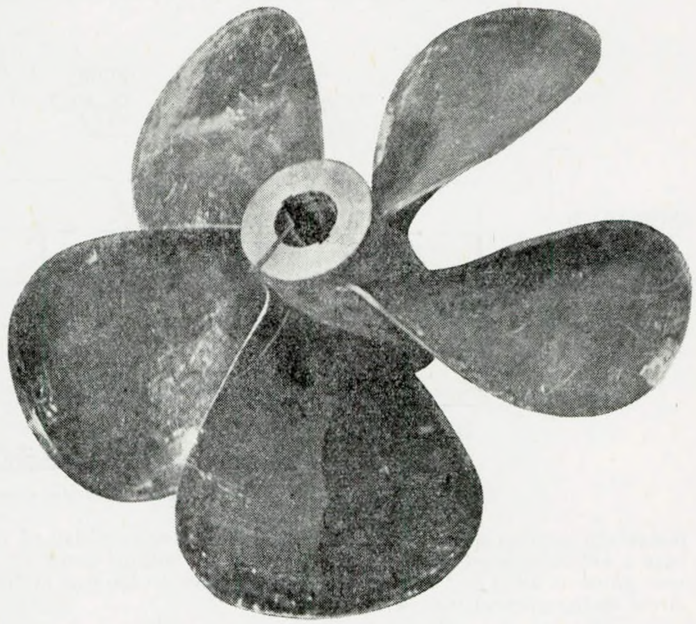

FIG. 9(b).—*Five-bladed propeller.*

long starboard shaft. The shorter port shaft proved to have its critical above full power. As might be expected the effect of turning was not serious with twin screws and it was not expected that the amplitudes observed would cause any damage. There was, however, considerable vibration of the hull and bridge structure at the critical speed of the starboard shaft and partly for this reason two starboard four-bladed propellers were ordered. These propellers have now been fitted with satisfactory results, as shown in Fig. 10.

The trials in the four shafted cruiser and the twin screw destroyer were arranged with the object of finding out whether it was necessary to give any thought to longitudinal vibration in new construction ships of those classes. It was concluded that longitudinal criticals should be avoided by design in the former but might be accepted in the latter, provided that the critical speed was well below full power.

Other trials have yielded more comprehensive data on various aspects of the problem, including the effect of speed and rudder angles on the multiplier for turns, which is discussed in more detail in a latter section of the paper.

Deflection Test on a Thrust Block

The first sea trials indicated that the thrust block itself, apart from the seating, was surprisingly flexible, and it was therefore decided to carry out a static deflection test on shore. This was done by Messrs. John Brown & Co. Ltd., using a thrust block belonging to a ship then under construction. A steel seating was built to carry the block and a hydraulic ram which was arranged to push on the aft end of the thrust shaft. Reference points for measurement were provided by brackets supported independently from the shop floor as shown in Fig. 11. Forward movements were recorded at the forward end of the thrust shaft, on the gland face each side at both forward and aft ends, and at the forward end of the block base. Vertical movements of the base were recorded at each end as it was realised that the steel seating would deflect and allow some tilting.

Three tests were carried out, for the first loads of zero, 30, 60, 90 and 120 tons were used. As the results showed a pronouncedly non-linear character below 30 tons the loads for the second and third tests were altered to 8·5, 38·5, 68·5, 98·5 and 128·5 tons. This change did not alter the non-linear character of the deflection and as the results of all three tests were very similar they have been averaged.

In working up the results correction has been made for the deflection caused by the measured horizontal and vertical movements of the base and the graphs of Fig. 12 therefore show the forward deflections of the shaft, the forward face and the aft gland face caused by deformation of the thrust block itself. It is evident that for vibration purposes the effective flexibility is not the total deflection but the slope of the line at the average thrust obtaining at the speed in question. Thus the total forward movement of the shaft is ·0317in. for 120 tons but the effective spring constant is represented by a deflection of ·0317in. minus ·0218, i.e. ·0099in. for an increase from 60 tons to 120 tons. This corresponds to ·0198in. for 120 tons thrust.

The plot of aft gland face deflection is peculiar in that this point apparently moves aft relative to the base. It is considered that this is probably correct and that it is caused by the deflection taking the form suggested in Fig. 13. If this deduction is correct it is evident that the thrust block could be made stiffer by thickening up the top

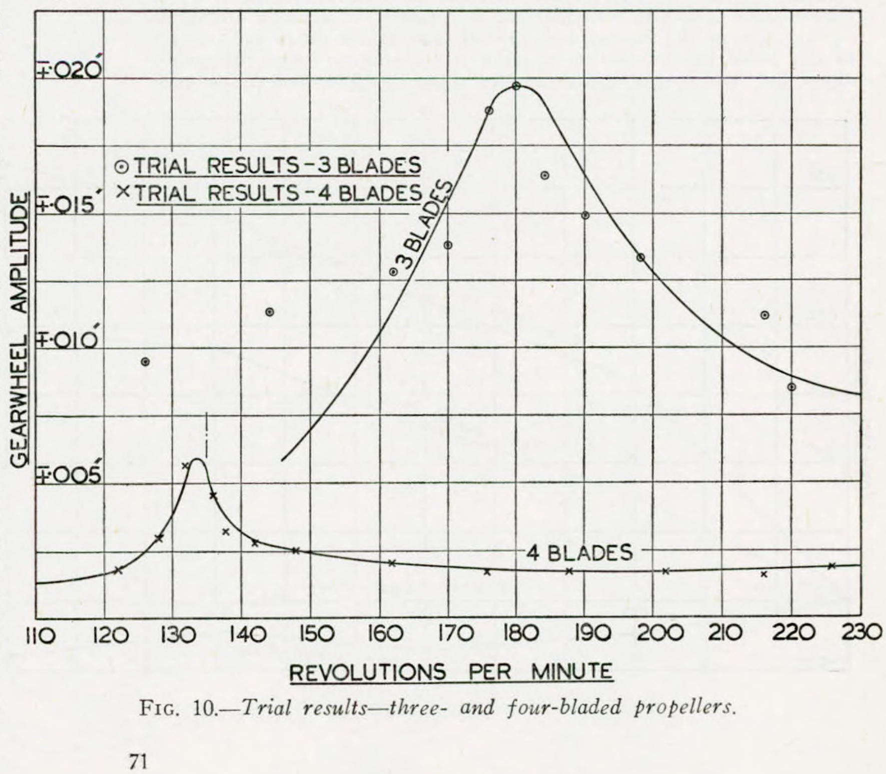

FIG. 10.—*Trial results—three- and four-bladed propellers.*

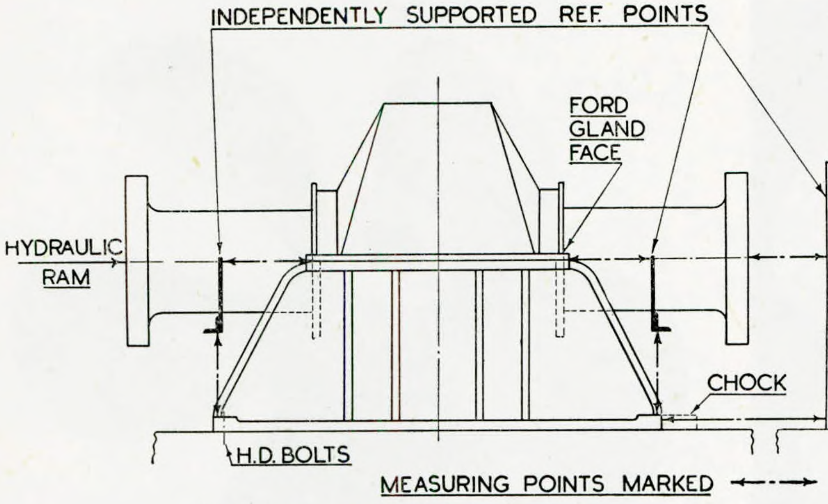

FIG. 11.—*Thrust block deflection test—arrangement.*

flange and providing horizontal ribs. Deflection due to tilting of the base could be reduced by lengthening it (there is usually room for a base twice as long) and fitting a thicker top plate to the seat welded direct to the vertical members.

Methods of Calculation

It is necessary at this stage to give an account of the standard method of calculation used in estimating critical speeds and amplitudes of vibration and in working up trial results. The method is set out in detail in the Appendices for the benefit of any prospective users and a brief explanation will suffice. The first step is to calculate or otherwise learn the weight of all the parts from the pinions to the propeller and to reduce the shafting to equivalent lengths of chosen cross section area. It is usually convenient to use two section areas of shaft by reason of the corrosion allowance on the outboard portion.

The system is then split up into (say) thirteen sections, the weight of each section is found, and the spring constant from the centre of each section to the centre of the next is calculated. This spring constant is the force in tons which would be required to increase the distance between the two points by one inch, and taking Young's modulus as 30×10^6lb./sq. in. it amounts to $106{\cdot}5 \times 10^3 \times A \div L$, where L is the length and A the cross section area of the shaft in feet and square feet respectively. It will be noted that half the weight of the thrust block casing is included, this is because the average amplitude of vibration of the casing is about half that of the thrust collar. Up to this point the calculation is quite straightforward and reasonably exact, but now two uncertain factors come in. The first is the mass of entrained water to be added to the propeller weight and the second is the spring constant between the thrust collar and the bottom of the ship or "earth".

These factors both influence the natural frequency of the system, and if one is known or assumed and the natural frequency has been observed during trials the other may be calculated. For new construction a value based on previous experience must be assumed for each. Having made the appropriate assumptions the system may be expressed diagrammatically as on Sheet 6 of *Appendix No. 1, and the natural frequency may be calculated by the well known tabulation method.(1,2,3) Starting with unit amplitude at the gearwheel end the forces in the thirteen springs and the relative amplitudes of the thirteen masses are evaluated; at any speed other than the critical there is a "remainder force" at the propeller end which is, the alternating force which would be required to produce unit amplitude at the gearwheel. At the critical speed this remainder becomes zero and if there were no damping a small force would give infinite amplitude, to find the critical speed is a matter of trial and error but it is usually possible to achieve a very small remainder at the third speed tried as the variation of remainder with speed is reasonably linear.

To estimate the amplitude of vibration at the critical two more assumptions, again based on previous experience, are required. These are the magnitude of the thrust variation, and the propeller damping factor. Dealing with thrust variation the important assumption is made throughout that the steady thrust varies as the square of the speed of revolution and that for a given propeller and hull the variation is a constant percentage of the steady thrust. This assumption appears to fit the results so far available.

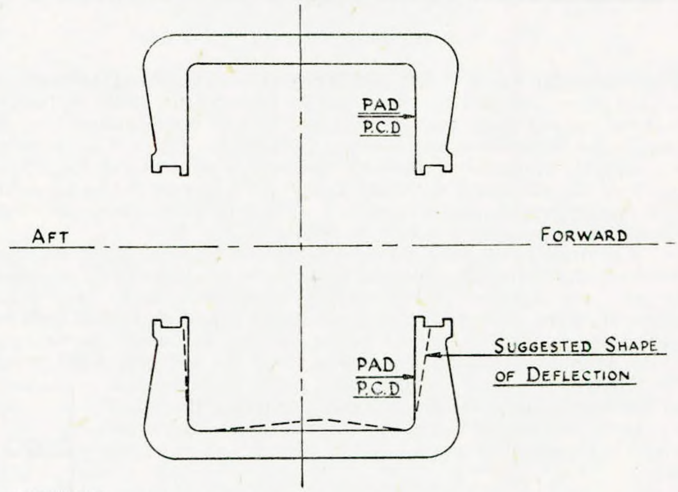

FIG. 13.—*Thrust block plan at cover joint.*

Having selected suitable values for these factors the propeller amplitude at critical may be calculated from the simple relation (see ref. 1, page 66).

Propeller amplitude = $P/\omega \times C_D$

where amplitude = inches

P = alternating force, tons

ω = (r.p.m.) × no. of blades × $2\pi/60$ radians per second.

C_D = propeller damping factor, tons per inch per second.

This applies only at the critical speed.

The gear amplitude can then be found by the relationship already established in finding the natural frequency.

Amplitudes at speeds other than the critical may be calculated by the tabulation method; as unit amplitude is assumed for the gear its actual amplitude neglecting damping is simply

gear amplitude (inches) = P/remainder force.

Damping has no appreciable effect on the

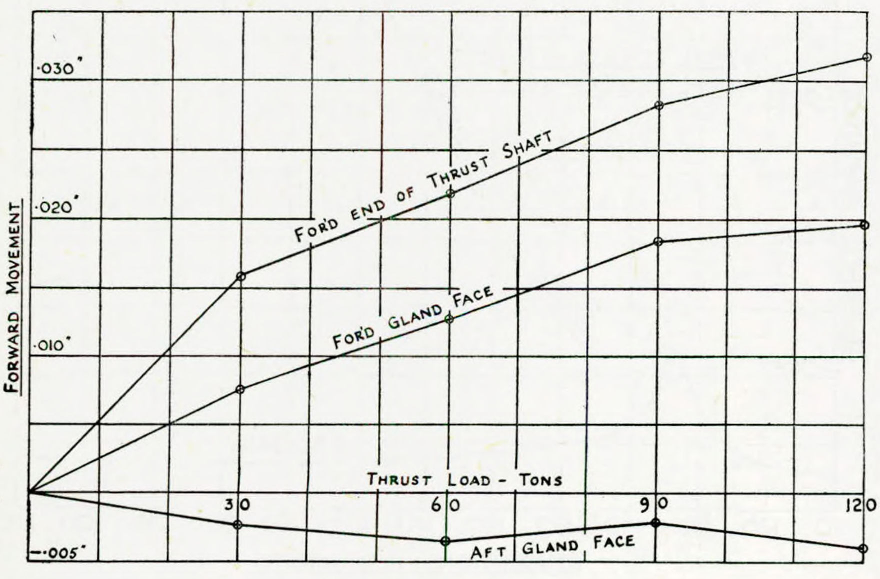

FIG. 12.—*Thrust block deflection test—results.*

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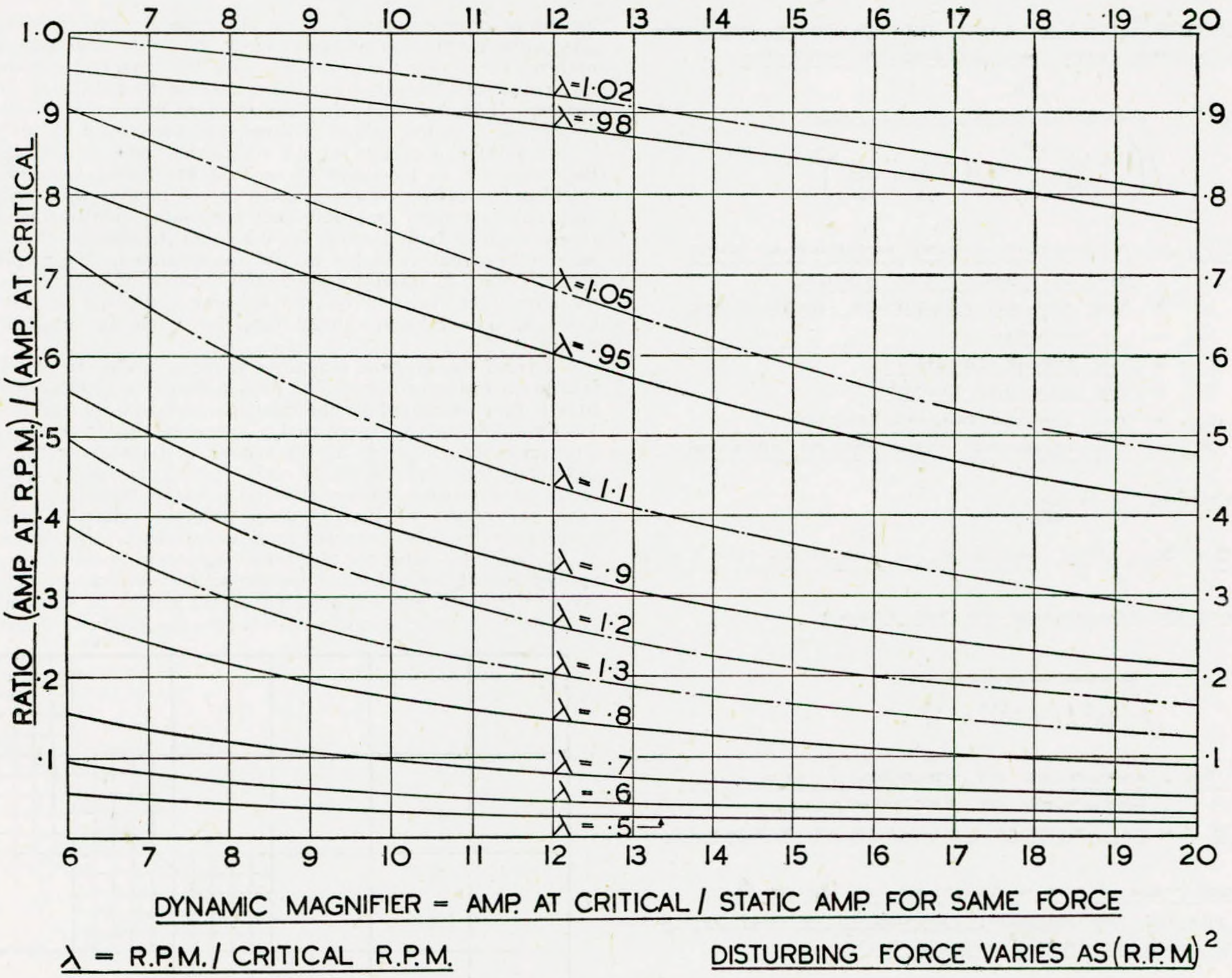


FIG. 14.—*Damped quadratic resonance curves.*

amplitude outside the range 0·8 to 1·2 times the critical speed. For a conventional line of shafting with the thrust block near the gearwheel end it is permissible to use standard resonance curves for a single degree of freedom to compute propeller amplitudes at speeds below and up to 1·3 times the first critical. This method saves a considerable amount of work and is particularly convenient in fitting a suitable damped resonance curve to trial results. The curves in Fig. 14 have been computed by the author for the purpose. The dynamic magnifier at the propeller is the calculated amplitude at the critical divided by the forward movement which would result if the same force were applied statically. It should be noted that the resonance curves cannot be used if the thrust block has been placed further aft because the system then becomes much more sharply tuned and its response is not adequately represented by a single degree of freedom. This limitation is illustrated in Appendix No. 2. The same applies to the second critical even in a conventional arrangement.

Spring Constant of Thrust Block and Seat and Propeller Entrained Water

During the first three trials efforts were made to measure the forward movement of the main gear wheel relative to the gear case, and to use the measured movement in conjunction with the calculable propeller thrust to determine the spring constant.

The most serious difficulty lay in the fact that the gearwheel was vibrating with an amplitude of some $\mp$·010 inches at full power and a further complication was introduced by the expansion of the shaft between the thrust collar and the forward end of the wheel during the appreciable time required for working up. Despite these difficulties quite reasonable results were secured in the first ship, possibly due to special skill on the part of the individual making the measurements but probably largely attributable to good fortune. Results from later ships were not reasonable and the method was therefore abandoned.

The second method of evaluation, also used in the first three ships, was the measurement of amplitude in the gland compartment by means of the vernier gauge. It was hoped that from the relationship between gland compartment, thrust block and gearwheel amplitudes and the directly calculable spring constant of the shafting it would be possible to solve for the unknown spring constant of the thrust block. This proved unsatisfactory because of the different character of the measuring instruments, the G.R. Vibration meter recording a mean value over some seconds while the vernier gauge gave the maximum peak values. Synchronisation of readings also offered difficulty, but probably this method could be made to work if sufficient attention were given to detail.

A third method which the author has not tried is the use of strain gauges to measure the alternating compressive stress in the shaft close to the thrust block, such measurements in conjunction with the vibration amplitude would give the spring constant directly. The compressive stress in a shaft is however only about 1,500lb./sq. in. under full power thrust and by the time slip ring difficulties are taken into account the accuracy of measurement would probably be low. The method on which the present assumptions are based arises from the static deflection test previously described. The considerable movement relative to each other of the forward and aft gland faces of the block under the static load has been used as a means of calculating the alternating load from vibration amplitudes recorded at the same two points during subsequent sea trials. The wing shafts of this ship are longer than the inner and so give a lower critical speed, and if the same value of entrained water is allotted to each propeller it is found that to give the observed criticals the wing thrust block must be stiffer than the inner. This is confirmed by the above calculation which gives a flexibility, expressed as forward movement under full power thrust, of ·036in. for the wing and ·042in. for the inner shaft. Using these figures the weights of entrained water required to give the observed criticals agree within 2 per cent. and the mean value of ·0481 tons per square foot of developed blade surface has been adopted as a standard.

DEN HARTOG [1] PAGE 62 GIVES THE FOLLOWING EQUATION FOR A SYSTEM WITH ONE DEGREE OF FREEDOM:-

$$x_o = \frac{\frac{P_o}{k}}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2.\frac{c}{c_c}.\frac{\omega}{\omega_n}\right)^2}}$$

WHERE P_o = A CONSTANT APPLIED ALTERNATING FORCE OF THE FORM $P = P_o \sin \omega t$

ω = THE APPLIED FREQUENCY, RADIANS/SEC.

ω_n = " NATURAL " , " "

k = THE SPRING CONSTANT.

c = THE DAMPING FACTOR.

c_c = THE CRITICAL DAMPING FACTOR.

x_o = THE RESULTING AMPLITUDE OF VIBRATION

SUBSTITUTING $\lambda = \frac{\omega}{\omega_n}$

AND $D = x_{o\lambda} \Big/ \frac{P_o}{k}$ WHERE $x_{o\lambda}$ = AMPLITUDE FOR $\lambda = 1{\cdot}0$

THIS MAY BE TRANSPOSED TO THE FORM:-

$$R_\lambda = \frac{1}{\sqrt{D^2(1-\lambda^2)^2 + \lambda^2}}$$

WHERE $R_\lambda = \frac{\text{AMPLITUDE AT FREQUENCY } \omega = \lambda\omega_n}{\text{AMPLITUDE AT FREQUENCY } \omega_n}$

AND D = THE DYNAMIC MAGNIFIER AT RESONANCE

THIS EQUATION IS FOR A CONSTANT VALUE OF P_o WHICH FOR THE THE PRESENT PURPOSE IS IN FACT TO BE PROPORTIONAL TO $(\text{R.P.M.})^2$ AND SO TO λ^2.

THE FINAL FORM USED FOR PLOTTING FIG. 14 THEN BECOMES:-

$$R_\lambda = \frac{\lambda^2}{\sqrt{D^2(1-\lambda^2)^2 + \lambda^2}}$$

FIG. 15.—*Derivation of formula for damped quadratic resonance curves.*

It should be noted that for a given first critical speed the response of the system to other frequencies does not show any marked change if entrained water is increased and thrust block flexibility reduced or vice versa until quite close to the second critical speed. The position of the latter is, however, controlled principally by the thrust block flexibility and when fitting a five-bladed propeller to a long shaft it is important to know that it will be above full power. The most satisfactory method of sorting out the true values of entrained water and flexibility would be to take the system up to the second critical with a vibration generator. It is considered that the friction in the shaft bearings would be too great if this were attempted in dry dock but consideration is being given to the possibility of attaching the vibration generator to the thrust block and carrying out the experiment while the ship is steaming at a moderate speed.

In the meantime trial results are worked up by using the above mentioned standard allowance of entrained water and calculating the thrust block flexibility from the observed critical speed. The resulting flexibilities vary in different ships from ·036 inches to ·057 inches forward deflection under full power thrust. In estimating the critical speed for a new ship with thrust block and seat of conventional proportions a value of ·045 inches deflection under full power thrust should give results not far from the truth.

Propeller Thrust Variation

The value of thrust variation deduced from trial results is dependent on the amount of entrained water assumed because that affects the calculation of thrust block flexibility and hence the force required to vibrate the gearwheel with the observed amplitude. For this reason the figures given below must not be divorced from their context: if at any future date the standard figure for entrained water is changed then the values of thrust variation must be re-calculated.

If a reliable portion of the resonance curve is available outside the range 0·8 to 1·2 times the critical the thrust variation may be calculated directly as the remainder in the tabulation method for the observed gearwheel amplitude and frequency. Unfortunately this is rarely possible both because at the small amplitudes then obtaining other vibrations are liable to add appreciably to the amplitude and because there is usually considerably scatter of the points. It has therefore been usual to find the dynamic magnifier at the propeller first and then calculate thrust variation from the amplitude at the critical.

To find the dynamic magnifier at the propeller from trial results is a curve fitting process. The ratio propeller amplitude/gear amplitude is first calculated by the tabulation method for say 0·7, 1·0 and 1·3 times the critical speed and a curve of this function is plotted. The propeller amplitude at the critical is found from the observed gear amplitude and the above ratio. Using this as the fixed point a series of resonance curves for the propeller motion are calculated from the curves of Fig. 14 using different values of the dynamic magnifier. Each is converted to gear amplitude and the best fitting is adopted. By definition the thrust variation is the force which if applied steadily would move the propeller forward (by deflection of the thrust block and compression of the shaft) an amount equal to propeller amplitude at critical divided by the dynamic magnifier.

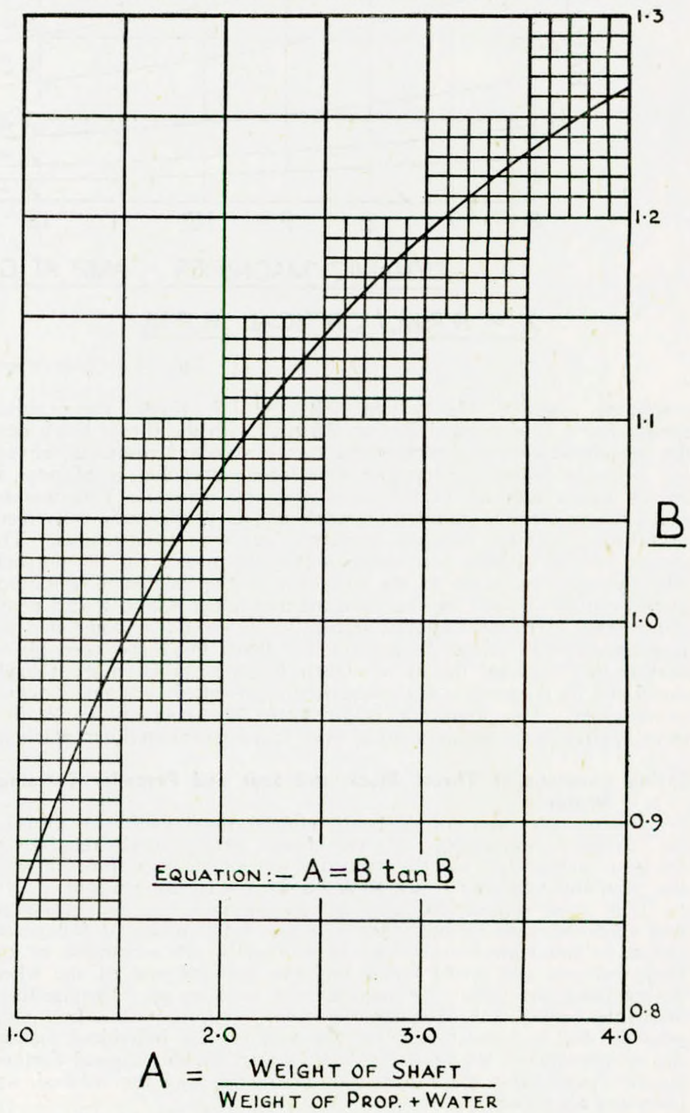


FIG. 16.—*Approximate method—graph of A and B.*

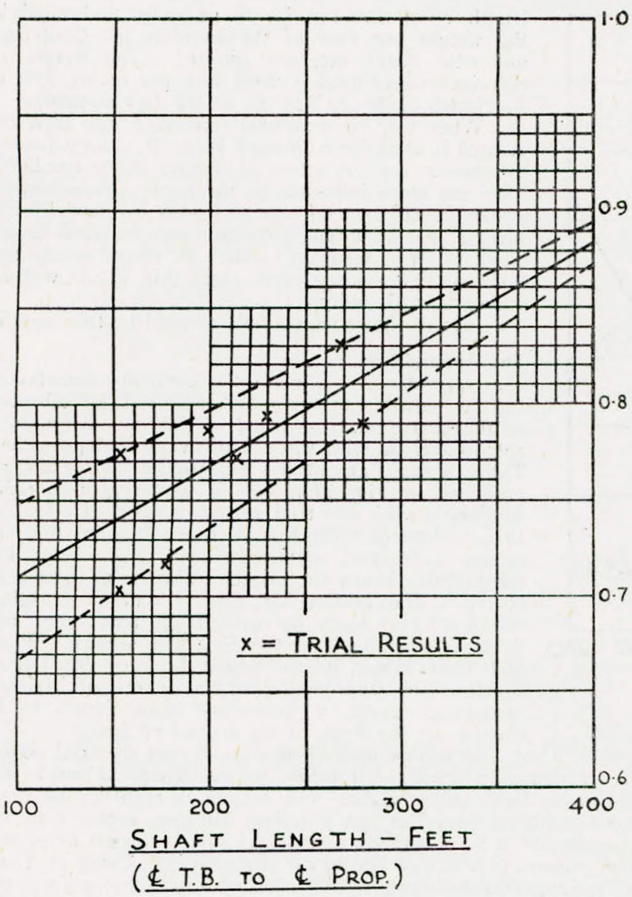


FIG. 17.—*Approximate method—flexibility factor.*

The results from the trials so far analysed show that the following figures may be taken as representative of the propeller thrust variation with the ship on a straight course:—

3 blades with "A" brackets ...	∓ 3 per cent. variation.
3 blades with bossings... ...	∓ 4 " "
3 blades centre shaft	∓ 5 " "
4 blades	⅘ of above values except for the centre shaft where the variation would be greater than with 3 blades.
5 blades	3/5 of 3-blade values.

It should be noted that H.M.S. Formidable, the basis of the figure for a centre shaft, has the structure sloped away below the shaft so that her stern is midway between a bossing and a sternpost. It is probable therefore that a single screw merchant ship would give a considerably higher thrust variation, especially with a four-bladed propeller.

The figure of 3 per cent. for three blades and "A" brackets is close to the observed value of propeller torque variation under similar conditions.[5]

Propeller Damping Factor

It has been assumed throughout that all the damping present is at the propeller. This is certainly not quite true, but so long as the shaft and gearwheel are supported on water and oil films in their bearings the only other appreciable source of damping appears to lie in friction between the poorly lubricated teeth of the flexible couplings. This is an uncertain quantity depending on the condition of the surfaces; the turbine rotors do not partake of the axial motion unless the coupling teeth have become very rough when the balanced double flow L.P. may do so. It does not appear therefore that the force is very great and when the thrust block is in the normal, forward, position it is quite satisfactory to ignore this source of damping. When however the thrust block is 200 feet from the gearwheel as in Case 2 of Appendix No. 3 it is just possible that the damping at the forward end might be sufficient to suppress the gear wheel motion and therefore invalidate the conclusions drawn regarding very long shafts. The best way to find out would be to fit an extra thrust block well aft on such a line of shafting and run trials with thrust pads fitted first in the aft block and then in the forward one.

Assuming that all the damping is at the propeller the damping factor may be found from the propeller amplitude at critical, thrust variation, and frequency, using the relation between these factors previously quoted. Up to the present the result has been expressed in terms of propeller developed surface, as for entrained water, and a representative figure is ·00739 tons per inch per second per square foot of developed blade surface. The results of the trials with four- and five-bladed propellers suggest however that the damping factor increases with the number of blades, and if this is so possibly it would be better expressed as ·0177 tons per inch per second per foot of blade edge length. This point is under investigation.

The Effect of Turns

The effect of turns has been studied under various conditions of speed and rudder angle on an inner shaft of a quadruple screw ship and on the centre shaft of H.M.S. Illustrious with the five-bladed propeller fitted. When a 4 shaft ship turns to starboard with full rudder the port inner normally suffers a violent burst of vibration soon after the swing starts, becomes quiet while the ship is doing a steady circle and then has another violent burst as she straightens up. The violent spells presumably occur when the slip stream of the wing propeller covers about half the disc of the inner, and the results from the quadruple screw ship show that there is a rudder angle less than the maximum which will cause violent vibration to be maintained throughout a circle. This does not apply to the centre shaft of the triple because there the ship's structure prevents the

LONGITUDINAL VIBRATION

DATE:- 10/47 SHAFT:- CENTRE H.M.S. ______

CRITICAL SPEED BY APPROXIMATE METHOD

LINE	ITEM	UNITS	FORMULA	FIGURE
1	PROPELLER – Nº OF BLADES			3
2	" – DEV. SURFACE	$FT.^2$		157
3	" – ENTRAINED WATER	TONS	·0481 × 2	7·55
4	WEIGHT OF PROP. WITH NUT	TONS		18·09
5	" " PROP. + NUT + WATER	TONS	3 + 4	25·64
6	IN BOARD SHAFTING – O.D.	INS.		20½
7	" " – I.D.	INS.		16¼
8	" " SECTION AREA	INS^2	$\frac{\pi}{4}\left[6^2 - 7^2\right]$	122·7
9	" " UNIT WEIGHT	TONS/FT.	·001516 × 8	·186
10	LENGTH ℄ T.B TO ℄ PROP.	FEET		212
11	WEIGHT OF SHAFT	TONS	9 × 10	39·4
12	A		11 ÷ 5	1·536
13	B		FIG. 16	·996
14	RIGID CRITICAL SPEED	R.P.M.	$\frac{161500 \times 13}{1 \times 10}$	253
15	FLEXIBILITY FACTOR		FIG. 17	·775
16	CRITICAL SPEED	R.P.M.	14 × 15	196
	NORMAL FULL SPEED	R.P.M.		230

FIG. 18.—*Approximate method—worked example.*

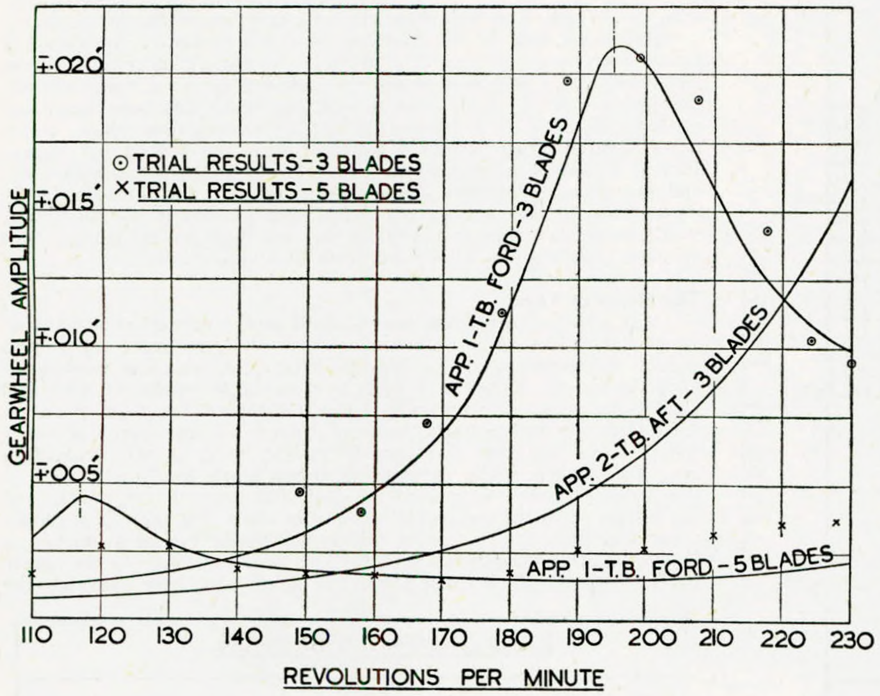


FIG. 19.—*Results from Appendices Nos. 1 and 2.*

slip stream from covering more than half the disc.

The results of the tests at different speeds show that over the range half to full revolutions there is little change in the effect. The "turn factor", amplitude on turn/amplitude on straight course, has maximum values in the quadruple screw ship of 4·96 at half speed and 5·4 at full speed while in Illustrious it is 4·1 at 102 r.p.m., 4·7 at 170, 3·7 at 210 and 3·2 at 220 r.p.m. These results are all for speeds where there is no reversal of thrust.

In estimating amplitudes for new construction a "turn factor" of 5 is used for quadruple screws in "A" brackets and a factor of 4 for the centre shaft of a triple. The latter figure does not mean that the triple suffers less on turns, merely that it is worse on straight course.

The effect of turning is less in twin screw ships, the turn factor amounting to about 2 and affecting principally the shaft on the inside of the turn.

Apart from the increase of vibration another result of turning is important where fitting a five-bladed propeller may leave the second critical close above full power and where moving the thrust block aft may place the first critical in the same position. When a quadruple screw ship turns to starboard and the slip stream of the port wing shaft covers the inner propeller the latter is relieved of its thrust and speeds up rapidly. This increase of speed is liable to take it 10 per cent. above normal full power revolutions and cannot be controlled when the tachometers fitted are of the integrating type. It is therefore necessary in the cases mentioned to consider revolutions up to 110 per cent. of normal maximum.

Approximate Method of Finding Critical Speed

When examining a proposed arrangement of shafting or drawing up a trial programme for a ship it is desirable to have a short method of estimating the critical speed. Where the thrust block is in the conventional forward position this is provided by treating the system as a weight (the propeller and entrained water) hanging on a heavy bar (the shaft between propeller and thrust collar). The flexibility of the thrust block is ignored initially and the method given by Timoshenko on p. 209 of "Vibration Problems in Engineering" is used to determine the "rigid natural frequency". His equation can be reduced to:—

$$\text{Frequency (V.P.M.)} = 161{,}500 \times \frac{B}{L}$$

where L = actual shaft length, feet, from propeller to thrust collar

and B is taken from Fig. 16 which shows it plotted against a function "A". The value of "A" is simply:—

(weight of shaft)/(weight of propeller + water)

and for this purpose the weight of the shaft is taken as the actual length from thrust collar to propeller multiplied by the weight per foot of the bare shaft. Couplings, gunmetal liners, etc. are ignored. The weight of entrained water used is ·0481 tons per square foot of developed blade surface, as in the full method.

When the "rigid natural frequency" has been calculated it must be multiplied by a "flexibility factor" to correct for the effect of thrust block flexibility. This has more influence on the natural frequency of a short shaft than on that of a long one, and Fig. 17 gives a plot of flexibility factor against shaft length based on trial results to date. If thrust blocks and their seats are made more rigid this factor will increase towards unity.

Fig. 18 shows an example worked by this method.

The Appendices

Appendix No. 1 gives the complete calculations for the natural frequency and amplitude of vibration of the centre shaft as fitted in H.M.S.'s Formidable and Illustrious with three- and five-bladed propellers. The calculated amplitudes on straight course are reproduced in Fig. 19 and it is seen that reasonable agreement with the trial results is secured. If Fig. 19 is compared with Fig. 8 it will be seen that the present calculated amplitudes with five blades fall considerably below the original estimate at the higher speeds. The reason for this is that the original estimate was made by calculating amplitudes and dynamic magnifiers at the first and second criticals and then filling in the space between by fairing together the appropriate single degree of freedom resonance curves, a procedure since found to be invalid for the flank of the second critical.

It will be noticed on Fig. 19 that the trial results for the five-bladed propeller fall below the calculated values at the left and above them at the right. The former discrepancy suggests that the five-bladed propeller has a higher damping factor than the three-bladed one, a point already discussed. The increased amplitude at higher powers is probably due to the centre shaft picking up from the hull the appreciable three per revolution vibration arising from the three-bladed wing propellers. This can be seen quite clearly on the Kelvin Vibrograph record at 200 r.p.m. where in spite of the five-bladed propeller the centre shaft breaks into free vibration at 600 v.p.m. which is apparently the natural frequency in Illustrious. From this and the position of the first critical it is concluded that Illustrious has a slightly stiffer thrust block seat than Formidable.

Regarding turns the calculations show a heavy reversal at the critical speed with a three-bladed propeller which is confirmed by hammering in the thrust block and rattling of the gears, and a slight reversal at the critical with a five-bladed propeller which does not in fact occur.

At the critical speed, on straight course, the anticipated alternating force on the thrust block seat is reduced from ± 47 tons with three blades, to ± 10 tons with five blades, and the trial results show an even greater reduction.

Appendix No. 2 deals with the same line of shafting as Appendix No. 1, but the thrust block has been moved to a new position about mid-way between gearwheel and propeller and at the same time its stiffness has been increased from $2{\cdot}27 \times 10^3$ to $3{\cdot}25 \times 10^3$ tons/inch. The result of these alterations is to raise the critical speed with a three-bladed propeller from 195 to 250 r.p.m. and as can be seen in Fig. 19 this is a distinct improvement. The amplitude at full power is however still too great and if such an alteration were contemplated it would be necessary to increase the thrust block stiffness still further, to at least $4{\cdot}0 \times 10^3$ tons/inch.

Appendix No. 3 deals with a hypothetical shaft 400ft. long having the same scantlings and running at the same speed as those in Appendices 1 and 2. The gear, thrust block and propeller weights have been rounded up and, to save labour, couplings, gunmetal liners, etc. have been disregarded.

Three different thrust block positions have been considered and in each case the stiffness has been taken as $4{\cdot}0 \times 10^3$ tons per inch. This figure is unusually high but would in the author's opinion be quite easy to achieve.

In Case 1 the thrust block is in the normal position 10 feet aft of the gearwheel and as would be expected with so long a shaft the first critical with three blades occurs at a fairly low power. The gearwheel amplitude is only ±·0058in. and on straight course the arrangement would be satisfactory except that the alternating force of ± 22·9 tons imposed on the thrust block seat might be

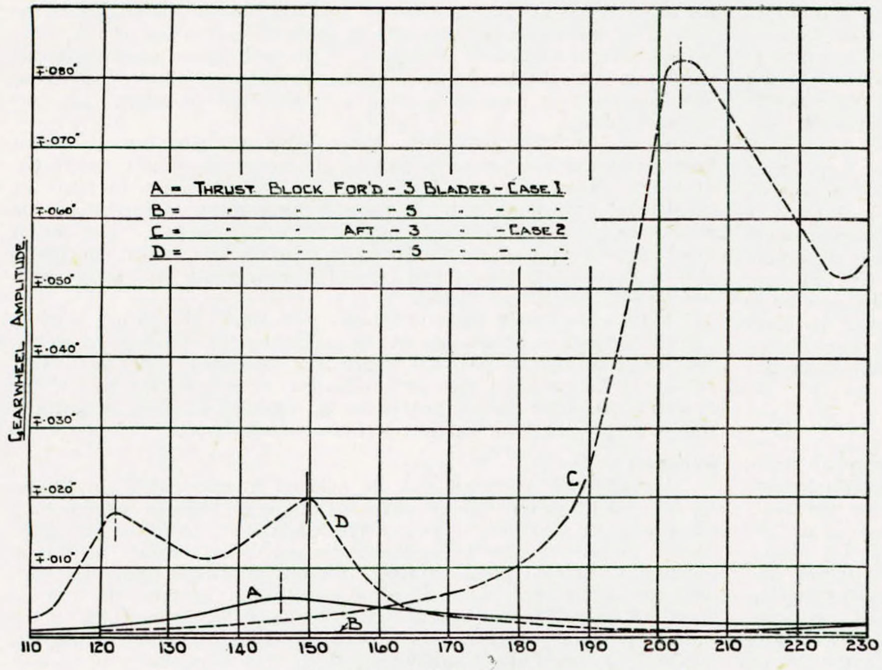


FIG. 20.—*Results from Appendix No. 3.*

In Case 3 the thrust block is placed only 80 feet from the propeller, as far aft as possible, and the force required to hold the gear stationary is again calculated. It is found that if the gear can be held the first critical will lie above full power and that the force required to hold it with three blades at 230 r.p.m. is only ±1·72 tons. It appears probable that with this arrangement the gear would actually stay still on straight course but would move when turning. The force on the thrust block seat on straight course is only ± 11·5 tons with the gear stationary.

It does seem possible that the arrangement of Case 3 might allow the use of three bladed propellers with satisfactory results, but before committing himself to such an arrangement the author would wish to secure measured values of the co-efficient of friction in flexible couplings and find a satisfactory method of predicting the resulting amplitudes.

sufficient to cause hull vibration in the event of resonance. When turning however there will certainly be a thrust reversal. If the thrust collar elects to jump the ·050in. clearance between ahead and astern pads and the pinions also shuttle the resulting amplitude will be sufficient to damage the flexible couplings. Such jumping and shuttling is probably more likely where the critical occurs at a low speed because the torque loads on coupling and gear teeth are reduced.

With five blades the first critical will be moved down to 88 r.p.m. and there may just be a thrust reversal when turning. If however the five-bladed propeller shows increased damping as in Illustrious there will certainly be no reversal. Thanks to the specially stiff thrust block the second critical is still above full power and as shown in Fig. 20 five blades will give a completely satisfactory performance throughout. It should be noted that the vertical scale of Fig. 20 is one quarter than of Fig. 19.

In Case 2 the thrust block is placed midway between gear and propeller. The amplitude of vibration at the gearwheel has been calculated neglecting any damping at that end and the results are also plotted on Fig. 20. It will be seen that even this position of the thrust block (which gives the maximum natural frequency) fails to move the first critical with three blades above full power, and that the gearwheel amplitude is very great, principally because it is now equal to that of the propeller. The results with five blades show considerably reduced but still excessive amplitudes and illustrate the adverse effect of moving the thrust block on the second critical which has been brought down quite near to the first.

The enormous amplitudes resulting when damping at the gearwheel end is neglected have an air of unreality and are evidently too great. For this reason the force required to hold the gearwheel stationary has been calculated and it is found that with three blades at 226 r.p.m. (the critical speed with the wheel held still) it would require a force of ± 10·1 tons on straight course. The l.p. turbine is double flow and therefore the only anchor available is the steam thrust of the h.p., amounting to 4·46 tons at full power. Assuming a co-efficient of friction of 0·1 for a ial sliding in the flexible couplings the force available to hold the gear is 2·6 tons and it is evident that this will not be able to keep it stationary even on straight course. To calculate the actual gear amplitude under these conditions would be a trial and error process of assuming different amplitudes, until one was found at which the energy dissipated against viscous damping at the propeller and "dry friction" at the coupling equalled the energy put in by the thrust variation. A further complication would arise because the gear motion would not be truly harmonic.

Even with the gear held still the force on the thrust block seat is ± 52·3 tons on straight course at 226 r.p.m. and for this reason the arrangement is in any case not acceptable with three blades. If five blades must be used there is no point in moving the thrust block at all.

Permissible Amplitudes

The preceding pages have given the data and the methods for predicting the amplitude of vibration with normal shaft arrangements but have not given any indication as to what is considered acceptable. From the machinery aspect the principal criterion is excessive wear in the flexible couplings. Experience in Naval vessels shows that an amplitude of ±·020in. on straight course is sufficient to cause rapid wear and that ±·050in. when turning can cause heavy scoring and even seizure of the tooth surfaces. It is concluded therefore that the amplitude should not exceed ±·010in. on straight course and ±·025in. when turning.

In twin screw ships the ±·010in. on straight course is the limiting factor, subject to the overruling consideration that a thrust reversal must not occur when turning. In triple or quadruple screw ships with a "turn factor" of four or five the amplitude on the turn is the criterion and to keep to the limit of ± ·025in. the amplitude on the straight should be kept down to ± ·005in. When the critical occurs at a low speed the avoidance of thrust reversal becomes more important than actual amplitude.

Single Screw Merchant Ships

The author has had no practical experience with any single screw vessels except the American built ships converted to aircraft carriers during the war. In these there was no longitudinal vibration problem because even with four-bladed propellers the critical was well above full power. This is probably the situation in most single screw ships, though Den Hartog[1] has apparently met with a case where the critical came at full power. This case is used for his examples 69 and 70 on page 202 of "Mechanical Vibrations".

So long as there is no critical speed in the running range there is obviously no objection to using a four-bladed propeller behind a sternpost, in fact it is quite likely that a three-bladed propeller would give a heavy six per revolution disturbance in such a situation and so bring in a second order critical. The four-bladed propeller would be expected to give a very strong four per revolution variation but not much eight per revolution disturbance.

Precise Location of Thrust Block

All the foregoing data and calculations have referred to thrust blocks which have been separate from the gearcases and supported on seatings which are virtually independent of those for the main machinery. This is a very important point, because if the thrust block is located in the gear case as for instance in the "locked train" design, the gearcase, turbines and condenser all become involved in the longitudinal vibration[6]. It is also most difficult to provide adequate fore and aft stiffness under a large gearcase because its own sump and the condenser are in the way. For this reason it is advisable where a high stiffness figure is required to sacrifice the possibility of saving space and keep the thrust block on a separate seat.

In all cases a stiff thrust block is helpful in reducing vibration amplitudes on straight course, but may increase the likelihood of thrust reversal when turning. The stiff thrust block in Appendix No. 3, Case 1, is ideal if a five-bladed propeller is to be used, but a more flexible one would perhaps reduce the severity of thrust reversal with three blades.

Increasing Shaft Flexibility

Dr. Forsyth[4] has suggested that where the critical is close to full power it would be an improvement to introduce a flexible element such as a bellows just aft of the thrust block. This would move the critical to a lower speed and also reduce the ratio of gear to propeller amplitude. The detailed design and possible flexibility of such a device require to be worked out and the resulting amplitudes could then be calculated. The only possible objection to this proposal is that the amplitude of vibration of the bulk of the shaft, which is the greatest mass in the system, would be increased relative to the propeller amplitude. The idea is under investigation and until figures are available, no final conclusion can be reached as to its merits.

Multi Bladed Propellers

It will have been noticed, with particular reference to Appendices Nos. 2 and 3, and to the preceding paragraph, that while the use of four or five blades instead of the standard naval figure of three is known to give entirely satisfactory results as regards vibration, there is a desire to explore other methods. The reason for this is that there is likely to be some loss of efficiency and increased risk of root cavitation when the number of blades is increased. The results so far do not make it possible to give any figures for efficiency but tend to suggest a slight loss, and as neither the five- nor the four-bladed propellers have yet been docked it is not known whether there has been any cavitation erosion.

GENERAL PRINCIPLES

A. Existing Ships

Once a ship is complete it would be a major operation to move the thrust block and for this reason the only practical alterations are to change the number of propeller blades or to increase the shaft flexibility. If the critical is near full power and the shaft is in "A" brackets, there is a free choice between four blades and five blades. Five blades will give the greater certainty of smooth running and the avoidance of thrust reversal on turns, but a four-bladed propeller is likely to be slightly more efficient and less prone to cavitation troubles. Where the critical is some way below full power with three blades it is possible that the use of five will bring the second critical into the running range and it is wise to calculate the probable position of this.

Where the propeller works behind a sternpost and the critical is within the running range, four blades should not be used.

It may occasionally happen that the first critical is at, or just above full power. In such a case it is worth considering the possibility of fitting a new and stiffer thrust block in the original position and at the same time stiffening the seat. In many cases it would be possible to fit a block with a base twice as long on the existing seat.

B. New Construction

Shafts may be divided, having regard to their running speed, into four classes: short, medium, long and very long.

With a short shaft, the first critical is above full power with three blades and no special precautions are required. Slightly longer shafts may be brought into this category by making a special effort to provide a stiff thrust block and seat.

With a medium length shaft the first critical comes below full power with three blades when the thrust block is near the gearwheel end, but may be raised well above full power by placing the thrust block midway between gearwheel and propeller. In this case the thrust block can be so located and a three-bladed propeller may then be retained. This arrangement requires either a self-contained, self-lubricating thrust block, or a separate lubricating oil system, because the block is too far aft to allow oil to drain back to the engine room. Allowance must also be made at the gears for expansion of the shaft between thrust block and gearwheel. This will depend on the number of plummer blocks and bulkhead glands fitted; measurements taken in one ship show a movement relative to the hull of about $\frac{5}{16}$in. per hundred feet.

A long shaft is defined as one in which the first critical cannot be moved above full power by moving the thrust block half way to the propeller. In this case it becomes necessary to change to four- or five-bladed propellers with the possible alternative of increasing the shaft flexibility, as discussed under "Existing Ships". The thrust block should be kept in the forward position and made sufficiently stiff to ensure that the second critical is well above full power with the chosen number of blades.

With a very long shaft it appears possible, if the thrust block is placed as near as practicable to the propeller, for the friction in the flexible couplings to hold the gearwheel stationary under all conditions. If the truth of this conclusion can be established, and if the retention of three-bladed propellers is regarded as very important, this arrangement will warrant serious consideration.

Summary

Longitudinal vibration can be reduced to acceptable amplitudes by fitting a propeller with an increased number of blades but this may involve some sacrifice of propulsive efficiency. The use of three-bladed propellers may be extended to shafts of medium length by locating the thrust block midway between gearwheel and propeller. In long shafts such a modification would fail to raise the critical speed above full power and it is best to keep the thrust block in the normal position and use either more propeller blades or perhaps a device to increase shaft flexibility. With very long shafts it appears possible that friction at the flexible couplings may be able to hold the gearwheel stationary if the thrust block is placed as near as possible to the propeller.

REFERENCES

(1) "Mechanical Vibrations", by J. P. Den Hartog. The McGraw-Hill Book Company, 1940.

(2) "Vibration Problems in Engineering", by S. Timoshenko. D. Van Nostrand Company, 1928.

(3) "An Investigation into the Failure of a Single Helical Geared Turbine System due to Combined Axial and Torsional Vibration", by S. F. Dorey and G. H. Forsyth. Transactions I.N.A. Vol. 84, 1942.

(4) "Some Gear Cutting Inaccuracies and Their Effect on Gear Loads and Gear Noises", by S. F. Dorey and G. H. Forsyth. Transactions N.-E. Coast Institution, Vol. 63, Part 8, 1947 (page D.205).

(5) "Torsional Vibration in Geared Turbine Propulsion Equipment", by H. Poritsky and C. S. L. Robinson. Journal of Applied Mechanics, Sept. 1940. (Trans. A.S.M.E., 1940. page A.119).

(6) "Development of Steam Turbines for Main Propulsion of High Powered Combatant Ships", by G. B. Warren. The Society of Naval Architects and Marine Engineers, Paper No. 6, 1946 (page 13).

Appendix No 1

H.M.S. "Formidable" and "Illustrious"

Calculation of Vibration Amplitudes with Three- and Five-Bladed Propellers

DATE 10/47 SHAFT CENTRE H.M.S.————

AREAS AND EQUIVALENT LENGTHS. (For Sketch see Sheet No. 5.)

INBOARD SHAFTING. O.D. $= 20\frac{1}{2}''$, I.D. $= 16\frac{1}{4}''$. Area $= 122{\cdot}7$in.^{2}

$= {\cdot}851$ft.^{2}

TAIL SHAFT. Mean O.D. $= 20\frac{7}{8}''$, I.D. $= 16\frac{1}{4}''$. Area $= 134{\cdot}8$in.^{2}

$= {\cdot}936$ft.^{2}

GEARWHEEL SHAFT.

Real Length in.	O.D. in.	I.D. in.	Area in.^{2}	Equivalent Length of Inboard Shaft
17	$27\frac{1}{2}$	20	279·8	7·4″
7	$33\frac{1}{2}$	$16\frac{1}{4}$	674·0	1·3″
34	$22\frac{1}{2}$	$16\frac{1}{4}$	190·2	21·9″
14*	$20\frac{1}{2}$	$16\frac{1}{4}$	122·7	14·0″
Total Equivalent Length				44·6″=3·72′

* To Coupling Face.

COUPLING FLANGES.
Length over pair $= 9\frac{1}{2}$in.
Diameter inside Bolt holes $= 23\frac{3}{4}$in. Bore $= 16\frac{1}{4}$in.
Section Area $= 235{\cdot}6$in.^{2}

$$\text{Equiv. Length of Inboard Shaft} = \frac{9{\cdot}5 \times 122{\cdot}7}{235{\cdot}6} = 4{\cdot}95''.$$

Amount to Deduct from Shaft Length $= 9{\cdot}5'' - 4{\cdot}95'' = 4{\cdot}55$in.
$= {\cdot}38$ft.

GUNMETAL LINER ON SHAFT IN STERN TUBE.
Liner 30·08ft. long, $21\frac{15}{16}''$ O.D., $20\frac{7}{8}''$ Mean I.D. Area $= 35{\cdot}7$in.^{2}
Equivalent Area in Steel $= 35{\cdot}7 \times 11{\cdot}5/30 = 13{\cdot}7$in.^{2}
Area of Tail Shaft $= 134{\cdot}8$in.^{2}

Total $= 148{\cdot}5$in.^{2}

$$\text{Amount to Deduct from Shaft Length} = \frac{30{\cdot}08 \times 13{\cdot}7}{148{\cdot}5} = 2{\cdot}78\text{ft.}$$

GUNMETAL LINER ON SHAFT IN "A" BRACKET.
Liner ft. long, O.D., Mean I.D. Area = in.^{2}

Equivalent Area in Steel = $\times 11{\cdot}5/30$ = in.^{2}
Area of Tail Shaft = in.^{2}

Total = in.^{2}

Amount to Deduct from Shaft Length = ——— = ft.

Correction for increased diameter in way of Plummer Blocks and Bulkhead Glands is negligible and is omitted.

AREAS, EQUIVALENT LENGTHS AND STIFFNESS.

TAIL END OF TAILSHAFT.

Real Length ft.	O.D. in.	I.D. in.	Area in.^{2}	Equivalent Length of Tailshaft
2·125	$19\frac{13}{16}$	8	258·0	1·11′
3·375	$20\frac{7}{8}$	8	292·0	1·56′
5·5	Totals			2·67′

Amount to deduct from Shaft Length = 5·5 − 2·67 = 2·83ft.

SHAFT BETWEEN GEAR WHEEL AND THRUST COLLAR.

Length from For'd Face of Collar to Gearwheel Coupling = 5·17ft.
Equivalent Length of Gearwheel Shaft = 3·72ft.

Sum = 8·89ft.
Deduction for 1 Coupling @ ·38ft. each = ·38ft.

Net Equivalent Length of Shaft = L_1 = 8·51ft.
Cross Section Area A_1 = ·851ft.^{2}

$$\text{Stiffness} = 160{\cdot}5\times10^3 \Big/ \frac{L}{A} = 160{\cdot}5\times10^3 \Big/ \frac{8{\cdot}51}{{\cdot}851} = 16{\cdot}05\times10^3 \text{ Tons/in.}$$

SHAFT BETWEEN THRUST COLLAR AND PROPELLER.

Length from Aft Face of Collar to Change of Diameter = 179·6ft.
Deduction for 6 Couplings @ ·38ft. each = 2·3ft.

Net Equivalent Length of Shaft = L_1 = Diff. = 177·3ft.

Cross Section Area A_2 = ·851ft.^{2} L_2/A_2 = 208·2

Length from Change of Diameter to C Propeller = 32·3ft.

Deduction for Couplings @ ft. each =
,, ,, Stern Tube Liner = 2·8
,, ,, "A" Bracket Liner =
,, ,, Tail End of Tailshaft = 2·8

Total Deductions = 5·6

Net Equivalent Length of Shaft = L_3 = Diff. = 26·7ft.

Cross Section Area A_3 = ·936ft.^{2} L_3/A_3 = 28·5

$$\text{Stiffness T. Collar to Prop.} = 160{\cdot}5\times10^3 \Big/ \left(\frac{L_2}{A_2}+\frac{L_3}{A_3}\right)$$
$$= 160{\cdot}5\times10^3/236{\cdot}7 = {\cdot}678\times10^3 \text{ Tons/in.}$$

THRUST BLOCK AND SEAT.

Full Power Thrust = 130 tons
Assume Elastic Forward Movement of Collar = ·0572″
Stiffness = 130/·0572 = $2{\cdot}27\times10^3$ Tons/in.

WEIGHTS.

Inboard Shafting $20\frac{1}{2}''$ O.D. $16\frac{1}{4}''$ I.D. = ·186 tons/ft
Outboard ,, $20\frac{7}{8}$ O.D. $16\frac{1}{4}''$ I.D. = ·204 tons/ft

SHAFT COUPLING FLANGES.

Pair. $32\frac{1}{2}''$ O.D. $20\frac{1}{2}''$ I.D. $9\frac{1}{2}''$ Long = 0·60 ton
Less 10 Holes $3\frac{1}{4}''$ Dia. $9\frac{1}{2}''$ Long = ·10 ton

Diff. = 0·50 ton
10 Bolts and Nuts = ·20 ton
Weight of ·38ft. of Shaft @ ·186 Tons/ft. = ·07 ton

Total = 0·77 ton

GUNMETAL LINER ON SHAFT IN STERN TUBE.

$21\frac{15}{16}''$ O.D. $20\frac{7}{8}''$ I.D. 361″ Long = 1·84 tons
2·78ft. of Shaft @ ·204 Tons/ft. = ·57 tons

Total = 2·41 tons

GUNMETAL LINER ON SHAFT IN "A" BRACKET.

O.D. I.D. Long = tons
ft. of Shaft @ Tons/ft. = tons

Total = tons

TAIL END OF PROPELLER SHAFT.

Thread $15\frac{1}{2}''$ O.D. 8″ I.D. 17″ Long = ·30 ton
Cone $18\frac{3}{4}''$ O.D. 8″ I.D. $50\frac{1}{2}''$ Long = 1·44 tons
Straight $20\frac{7}{8}''$ O.D. 8″ I.D. $40\frac{1}{2}''$ Long = 1·49 tons

Total = 3·23 tons
Allowed in Flex. Calc. 2·67ft. @ ·204 Tons/ft. = ·54 ton

Difference = 2·69 tons

MAIN GEARWHEEL AND PINIONS.

Main Gearwheel and Shaft = 26·41 tons
H.P. Pinion and $\frac{1}{2}$ Flexible Coupling = 3·16 tons
L.P. ,, ,, $\frac{1}{2}$,, ,, = 3·14 tons

Total = 32·71 tons
Less Equiv. Shaft 3·72ft. @ ·186 Tons/ft. = ·69 tons
Less one Coupling Flange = ·25 tons

Difference = 31·77 tons

MAIN THRUST BLOCK AND COLLAR.

Half Weight of Thrust Block (ex Shaft) = 4·18 tons
Thrust Collar $53\frac{3}{4}''$ O.D. $16\frac{1}{4}''$ I.D. 8″ Long = 2·08 tons

Total = 6·26 tons

PROPELLER, ETC.

Propeller (Developed Surface = 157ft.^{2}) = 17·14 tons
Cone Nut = ·95 tons
Tail End of Propeller Shaft = 2·69 tons
Entrained Water (·0481 Tons/ft.^{2}) = 7·55 tons

Total = 28·33 tons

EQUIVALENT SYSTEM (See Sketch on Sheet 5, Page 14).

WEIGHTS AND MASSES.

Section	Shaft			Concentrated Loads		Total Tons	Tons/386 Mass
	Length	Tons/ft.	Tons	Tons	Tons		
1	4·25′	·186	·79	31·77	·38	32·94	·0852
2	4·25′	,,	·79	6·26	·39	7·44	·0193
3	20·4′	,,	3·79	—	·77	4·56	·0118
4	,,	,,	,,	—	·77	4·56	·0118
5	,,	,,	,,	—	—	3·79	·0098
6	,,	,,	,,	—	·77	4·56	·0118
7	,,	,,	,,	—	·77	4·56	·0118
8	,,	,,	,,	—	—	3·79	·0098
9	,,	,,	,,	—	·77	4·56	·0118
10	,,	,,	,,	—	·77	4·56	·0118
11	14·1′	·186	2·62	—	—	3·91	·0101
	6·3′	·204	1·29	—	—		
12	20·4′	·204	4·16	—	2·41	6·57	·0170
13	—	—	—	28·33	—	28·33	·0733
						Total ...	·2953

STIFFNESS.

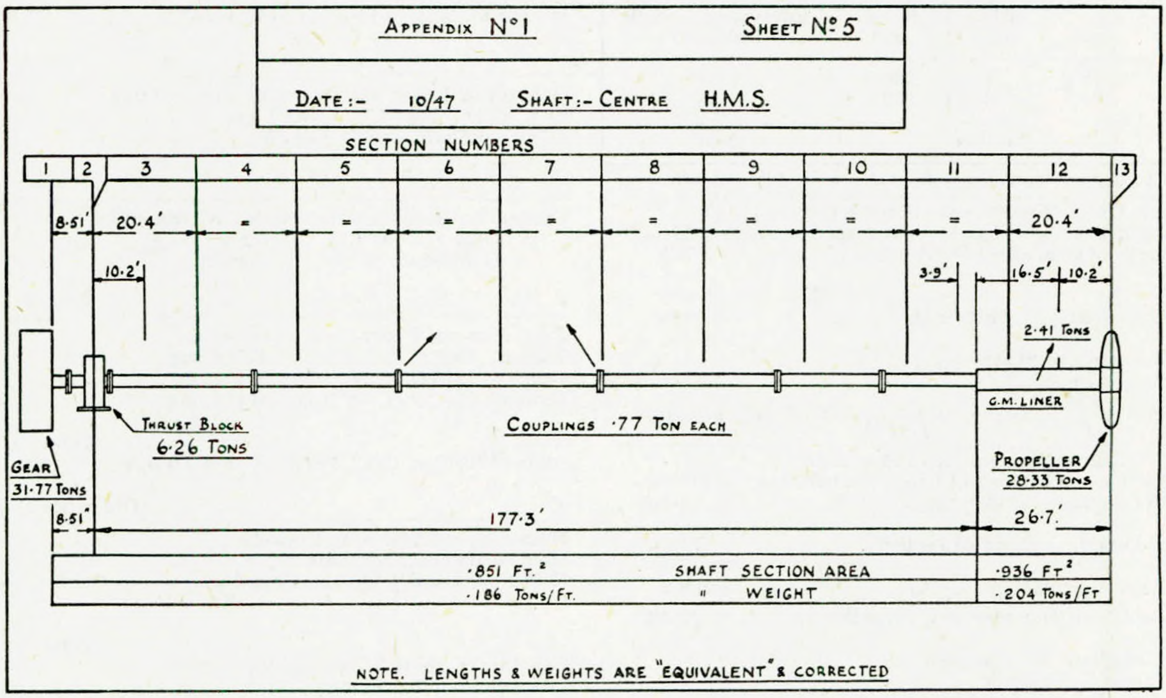
8·51′ at ·851ft.² = 160·5 × 10³ × ·851/8·51 = 16·05 × 10³ Tons/in.
10·2′ at ·851ft.² = 160·5 × 10³ × ·851/10·2 = 13·4 × 10³ Tons/in.
20·4′ at ·851ft.² = 160·5 × 10³ × ·851/20·4 = 6·7 × 10³ Tons/in.
3·9′ at ·851ft.², L/A = 3·9/·851 = 4·58
16·5′ at ·936ft.², L/A = 16·5/·936 = 17·62

Total L/A = 22·20

Stiffness = 160·5 × 10³/22·20 = 7·23 × 10³ Tons/in.

10·2′ at ·936ft.² = 160·5 × 10³ × ·936/10·2 = 14·72 × 10³ Tons/in.

Stiffness from T.B. to Earth (from page 13) = 2·27 × 10³ Tons/in.



Sketch of system.

NATURAL FREQUENCY, CRITICAL SPEED and AMPLITUDE.

NATURAL FREQUENCY.

$\lambda = 1·0$. R.P.M. = 195. Blades = 3. $\omega = 61·3$. $\omega^2 = 3758$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0852	320·0	1·000	320	320	16·05 × 10³	·020
2	·0193	72·4	·980	71	391	—	—
E	—	—			−2225	2·27 × 10³	−·980
2	—	—	·980		−1834	13·4 × 10³	−·137
3	·0118	44·3	1·117	49	−1785	6·7 × 10³	−·266
4	·0118	44·3	1·383	61	−1724	6·7 × 10³	−·258
5	·0098	36·8	1·641	60	−1664	6·7 × 10³	−·248
6	·0118	44·3	1·889	84	−1580	6·7 × 10³	−·236
7	·0118	44·3	2·125	94	−1486	6·7 × 10³	−·222
8	·0098	36·8	2·347	86	−1400	6·7 × 10³	−·209
9	·0118	44·3	2·556	113	−1287	6·7 × 10³	−·192
10	·0118	44·3	2·748	122	−1165	6·7 × 10³	−·174
11	·0101	37·9	2·922	111	−1054	7·23 × 10³	−·146
12	·0170	63·8	3·068	196	−858	14·72 × 10³	−·058
13	·0733	275·0	3·126	860	*+2		

NOTES. *This Remainder = Zero at Critical.
Remainder is Negative below First Natural Frequency.
,, ,, Positive between First and Second Natural Frequency.
,, ,, Negative above Second Natural Frequency.

AMPLITUDE AT CRITICAL ON STRAIGHT COURSE.

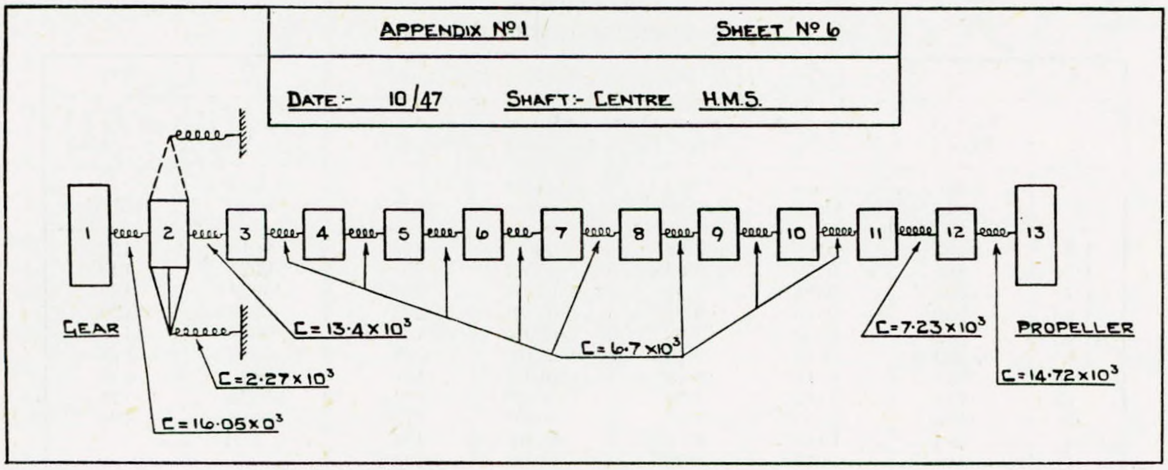
Assume Thrust Variation = ±4·57% of 130 Ton F.P. Thrust and varies as (R.P.M.)². Full Power R.P.M. = 230.
Force P at 195 R.P.M. = ·0457 × 130 × (195/230)² = ±4·28 Tons.
Assume Propeller Damping Constant C_d = 1·062.
Prop. Motion = P/C_d × ω = ±4·28/1·062 × 61·3 = ±·0656″.
Gear Motion = Prop. Motion/3·126 = ±·0656/3·126 = ±·021″.
Force at Thrust Block = 2225 × ·021 = ±46·9 Tons.

SHAPE OF RESONANCE CURVE.

Shaft T.B. to Prop. = ·678 × 10³ Tons/in. = ·001475″/Ton
T.B. and Seat = 2·27 × 10³ Tons/in. = ·000441″/Ton

Total = ·001916″/Ton

For 4·28 Ton Force Prop. would move 4·28 × ·001916 = ·0082″.
Dynamic Magnifier at Prop. = ·0656/·0082 = 8·0.



Equivalent system and stiffness.

Ratio Prop./Gear at $\lambda = \cdot 8$ (λ=R.P.M./Crit. R.P.M.)

$\lambda = \cdot 8$. R.P.M.=156. Blades=3. $\omega = 49 \cdot 0$. $\omega^2 = 2401$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0852	204·5	1·000	204	204	$16 \cdot 05 \times 10^3$	·013
2	·0193	46·3	·987	46	250	—	
E	—	—			−2240	$2 \cdot 27 \times 10^3$	−·987
2	—	—	·987		−1990	$13 \cdot 4 \times 10^3$	−·149
3	·0118	28·3	1·136	32	−1958	$6 \cdot 7 \times 10^3$	−·292
4	,,	28·3	1·428	40	−1918	,,	−·286
5	·0098	23·5	1·714	40	−1878	,,	−·280
6	·0118	28·3	1·994	56	−1822	,,	−·272
7	,,	28·3	2·266	64	−1758	,,	−·262
8	·0098	23·5	2·528	59	−1699	,,	−·254
9	·0118	28·3	2·782	79	−1620	,,	−·242
10	,,	28·3	3·024	86	−1534	,,	−·229
11	·0101	24·2	3·253	79	−1455	$7 \cdot 23 \times 10^3$	−·201
12	·0170	40·8	3·454	141	−1314	$14 \cdot 72 \times 10^3$	−·089
13	·0733	176·0	3·543	624	−690		

Thrust Variation $= 130 \times \cdot 0457 \times (156/230)^2 = \pm 2 \cdot 74$ tons.
Gear Amp. undamped $= 2 \cdot 74/690 = \pm$ **·0040″**.

Ratio Prop./Gear at $\lambda = 1 \cdot 2$

$\lambda = 1 \cdot 2$. R.P.M.=234. $\omega = 73 \cdot 5$. $\omega^2 = 5402$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0852	460	1·000	460	460	$16 \cdot 05 \times 10^3$	·029
2	·0193	104·1	·971	101	561	—	
E	—	—			−2204	$2 \cdot 27 \times 10^3$	−·971
2	—	—	·971		−1643	$13 \cdot 4 \times 10^3$	−·123
3	·0118	63·7	1·094	70	−1573	$6 \cdot 7 \times 10^3$	−·235
4	,,	63·7	1·329	85	−1488	,,	−·222
5	·0098	52·9	1·551	82	−1406	,,	−·210
6	·0118	63·7	1·761	112	−1294	,,	−·193
7	,,	63·7	1·954	125	−1169	,,	−·174
8	·0098	52·9	2·128	113	−1056	,,	−·158
9	·0118	63·7	2·286	146	−910	,,	−·136
10	,,	63·7	2·422	154	−756	,,	−·113
11	·0101	54·5	2·535	138	−618	$7 \cdot 23 \times 10^3$	−·085
12	·0170	91·8	2·620	240	−378	$14 \cdot 72 \times 10^3$	−·026
13	·0733	396	2·646	1049	+671		

Thrust Variation $= 130 \times \cdot 0457 \times (234/230)^2 = \pm 6 \cdot 16$ tons.
Gear Amp. undamped $= 6 \cdot 16/671 = \pm$ **·0092″**.

AMPLITUDES.
DYNAMIC MAGNIFIER=8.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Fig. 14 Page 7	Amp.λ=1 × (2)	Sheet 11 Page 17	(3)/(4)	Crit. × (1)	$(5) \times \left[\frac{3}{5}\right]^3$	$(6) \times \left[\frac{3}{5}\right]$
λ	Amp. Ratio	±Prop. Amp.	Prop./ Gear	±Gear Amp.	R.P.M.	±Gear Amp.	R.P.M.
1·3	·297	·0195″	2·39	·0082″	254	·0018	152
1·2	·387	·0254″	2·646	·0096″	234	·0021	140
1·1	·600	·0394″	2·91	·0135″	214·5	·0029	129
1·05	·828	·0543″	3·02	·0180″	205	·0039	123
1·02	·972	·0638″	3·09	·0206″	199	·0044	119
1·0	1·000	·0656″	3·126	·0210″	195	·0045	117
·98	·931	·0611″	3·18	·0192″	191	·0041	115
·95	·735	·0481″	3·23	·0149″	185	·0032	111
·9	·458	·0300″	3·36	·0089″	175·5	·0019	105
·8	·213	·0140″	3·543	·0040″	156		
·7	·118	·0077″	3·70	·0021″	136·5		
·6	·070	·0046″	3·80	·0012″	106		
		3 Blades				5 Blades	

Forces on Thrust Block Seat—Assume Turn Factor=4.

3 *Blades at Critical—Turning.*
Gear Amp.=±·021″×4=±·084″.
Force on Seat (from Page 14)=±·084×2225=±187 Tons.
Steady Thrust=130×(195/230)^2=93 Tons.
∴ There will be a heavy reversal of Thrust.

5 *Blades at Critical—Turning.*
Force on Seat=±187×(3/5)^3=±40·4 Tons.
Steady Thrust=130×(117/230)^2=33·6 Tons.
∴ There may still be a slight reversal.

5 *Blades at Full Power—Turning.*
From Table below.
Gear Amp.=±·0022×4=±·0088″.
Force on Seat=±·0088×2095=±18·4 Tons.
Steady Thrust=130 Tons.

AMPLITUDES WITH 5 BLADES.
THRUST VARIATION AT 230 R.P.M.=130×·0457×3/5=±**3·57 tons.**

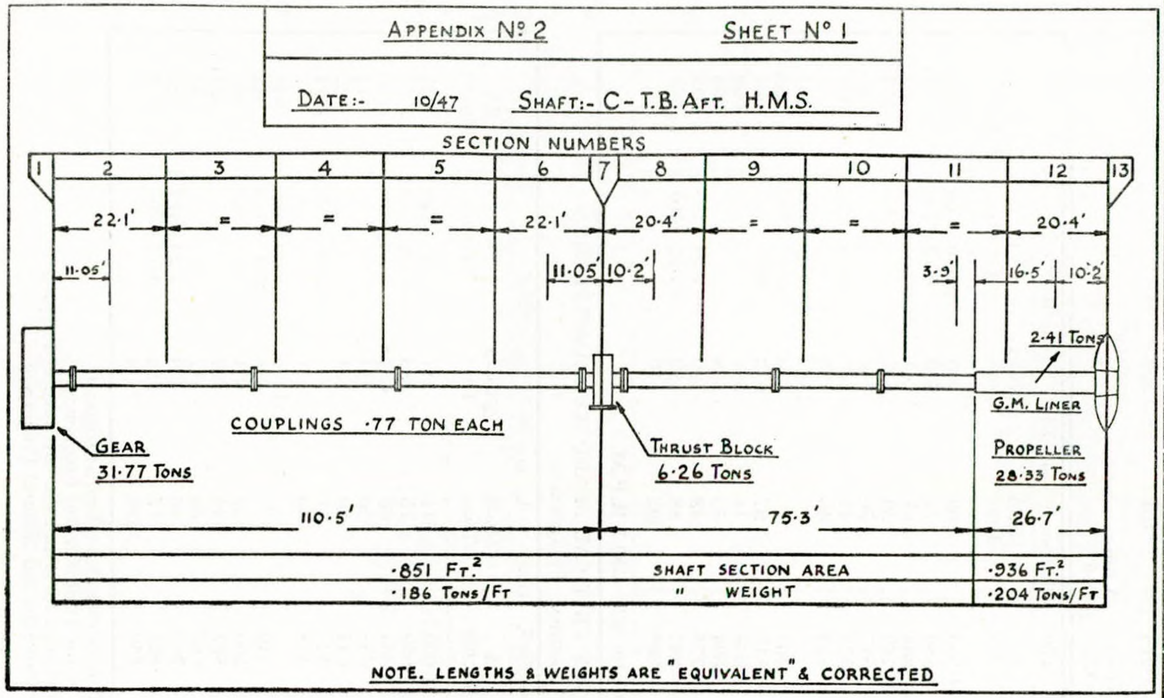
λ=1·965. R.P.M.=230. Blades=5. ω=120·4. ω²=14500.							
Section	Mass Tons/386	Mω²	Δ Inches	Mω²Δ Tons	ΣMω²Δ Tons	C Tons/in.	Σ/C Inches
1	·0852	1237	1·000	1237	1237	16·05×10³	·077
2	·0193	280	·923	258	1495	—	
E	—	—			−2095	2·27×10³	−·923
2	—	—	·923		−600	13·4 ×10³	−·045
3	·0118	171	·968	165	−435	6·7 ×10³	−·065
4	,,	171	1·033	177	−258	,,	−·038
5	·0098	142	1·071	152	−106	,,	−·016
6	·0118	171	1·087	186	+80	,,	+·012
7	,,	171	1·075	184	264	,,	·039
8	·0098	142	1·036	147	411	,,	·061
9	·0118	171	·975	167	578	,,	·086
10	,,	171	·889	152	730	,,	·109
11	·0101	146	·780	114	844	7·23×10³	·117
12	·0170	246	·663	163	1007	14·72×10³	·068
13	·0733	1062	·595	632	1639		
Gear Amp.=±3·57/1639=±**·0022″** at 230 R.P.M.							

λ=1·622. R.P.M.=190. Blades=5. ω=99·5. ω²=9900.							
Section	Mass Tons/386	Mω²	Δ Inches	Mω²Δ Tons	ΣMω²Δ Tons	C Tons/in.	Σ/C Inches
1	·0852	844	1·000	844	844	16·05×10³	·053
2	·0193	191	·947	181	1025	—	
E	—	—			−2150	2·27×10³	−·947
2	—	—	·947		−1125	13·4 ×10³	−·084
3	·0118	117	1·031	121	−1004	6·7 ×10³	−·150
4	,,	117	1·181	138	−866	,,	−·129
5	·0098	97	1·310	127	−739	,,	−·110
6	·0118	117	1·420	166	−573	,,	−·086
7	,,	117	1·506	176	−397	,,	−·059
8	·0098	97	1·565	152	−245	,,	−·037
9	·0118	117	1·602	188	−57	,,	−·009
10	,,	117	1·611	189	+132	,,	+·020
11	·0101	100	1·591	159	291	7·23×10³	·040
12	·0170	168	1·551	261	552	14·72×10³	·037
13	·0733	726	1·514	1100	1652		
Thrust Variation=±3·57×(190/230)²=±2·44 tons. Gear Amp.=±2·44/1652=±**·00143″** at 190 R.P.M.							

Appendix No. 2

H.M.S. "Formidable"

Calculation of Vibration Amplitudes with Thrust Block Moved Aft and Stiffened

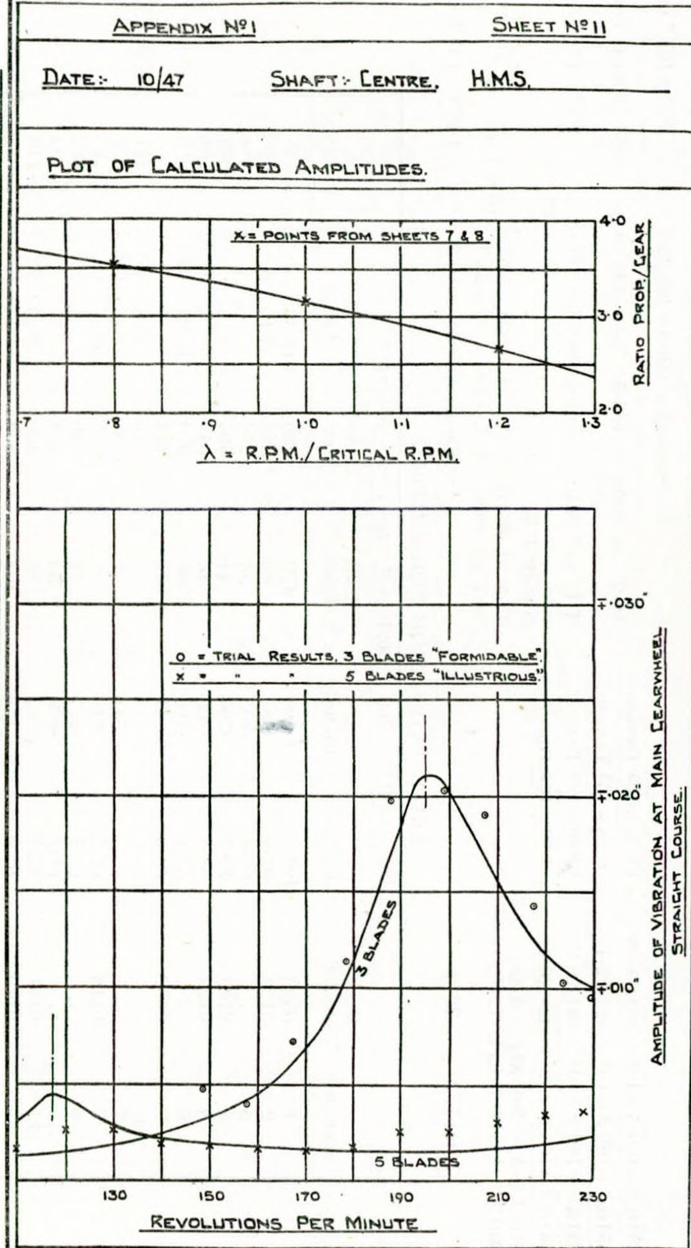


Sketch of system.

EQUIVALENT SYSTEM.

Weights and Masses.

Section	Shaft			Concentrated Loads		Total Tons	Tons/386 Mass
	Length	Tons/ft.	Tons	Tons	Tons		
1	—	—	—	31·77	—	31·77	·0823
2	22·1′	·186	4·11	—	·77	4·88	·0126
3	22·1′	,,	4·11	—	·77	4·88	·0126
4	22·1′	,,	4·11	—	—	4·11	·0106
5	22·1′	,,	4·11	—	·77	4·88	·0126
6	22·1′	,,	4·11	—	·77	4·88	·0126
7	—	—	—	6·26	—	6·26	·0162
8	20·4′	·186	3·79	—	·77	4·56	·0118
9	20·4′	,,	3·79	—	·77	4·56	·0118
10	20·4′	,,	3·79	—	·77	4·56	·0118
11	14·1′ 6·3′	·186 ·204	2·62 1·29	— —	— —	3·91	·0101
12	20·4′	·204	4·16	—	2·41	6·57	·0170
13	—	—	—	28·33	—	28·33	·0733
						Total ...	·2953



For "Sheets 7 & 8" read "Pages 14 & 15".

Stiffness.

11·05′ at ·851ft.^{2}=160·5×10^3×·851/11·05 =12·36×10^3 Tons/in.

22·1′ at ·851ft.^{2}=160·5×10^3×·851/22·1 = 6·18×10^3 Tons/in.

10·2′ at ·851ft.^{2}=160·5×10^3×·851/10·2 =13·40×10^3 Tons/in.

20·4′ at ·851ft.^{2}=160·5×10^3×·851/20·4 = 6·70×10^3 Tons/in.

3·9′ at ·851ft.^{2} L/A= 3·9/·851= 4·58
16·5′ at ·936ft.^{2} L/A=16·5/·936=17·62

Total L/A =22·20

Stiffness=160·5×10^3/22·2 = 7·23×10^3 Tons/in.

10·2′ at ·936ft.^{2}=160·5×10^3×·936/10·2 =14·72×10^3 Tons/in.

T.B. and Seat. Assume ·040″ for 130 tons = 3·25×10^3 Tons/in.

Shaft T.B. to Prop.

75·3′ at ·851ft.^{2} L/A=75·3/·851=88·4 }
26·7′ at ·936ft.^{2} L/A=26·7/·936=28·5 } 116·9 Total.

Stiffness=160·5×10^3/116·9 =1·375×10^3 Tons/in.

TRIAL AND ERROR FOR CRITICAL.

λ=·919. R.P.M.=230. Blades=3. ω=72·2. ω^2=5213.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	429	1·000	429	429	12·36×10^3	·035
2	·0126	65·8	·965	63	492	6·18×10^3	·080
3	,,	65·8	·885	58	550	,,	·089
4	·0106	55·3	·796	44	594	,,	·096
5	·0126	65·8	·700	46	640	,,	·104
6	,,	65·8	·596	39	679	12·36×10^3	·055
7	·0162	84·0	·541	45	724	—	
E	—	—			−1759	3·25×10^3	−·541
7	—	—	·541		−1035	13·4 ×10^3	−·077
8	·0118	61·5	·618	38	−997	6·7 ×10^3	−·149
9	,,	61·5	·767	47	−950	,,	−·142
10	,,	61·5	·909	56	−894	,,	−·133
11	·0101	52·7	1·042	55	−839	7·23×10^3	−·116
12	·0170	88·6	1·158	103	−836	14·72×10^3	−·057
13	·0733	382	1·215	464	−372		

Thrust Variation=130×·0457=±5·95 tons.
Gear Amp. undamped=5·95/372=±·016″.

λ=·999. R.P.M.=250. Blades=3. ω=78·5. ω^2=6160.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	506	1·000	506	506	12·36×10^3	·041
2	·0126	77·6	·959	74	580	6·18×10^3	·094
3	,,	77·6	·865	67	647	,,	·105
4	·0106	65·4	·760	50	697	,,	·113
5	·0126	77·6	·647	50	747	,,	·121
6	,,	77·6	·526	41	788	12·36×10^3	·064
7	·0162	99·8	·462	46	834	—	—
E	—	—			−1503	3·25×10^3	−·462
7	—	—	·462		−669	13·4 ×10^3	−·050
8	·0118	72·7	·512	37	−632	6·7 ×10^3	−·094
9	,,	72·7	·606	44	−588	,,	−·088
10	,,	72·7	·694	50	−538	,,	−·080
11	·0101	62·2	·774	48	−490	7·23×10^3	−·068
12	·0170	104·8	·842	88	−402	14·72×10^3	−·027
13	·0733	451	·869	392	−10		

From above Crit.=250+(20×10/362)=**250·5 R.P.M.**

NATURAL FREQUENCY CRITICAL SPEED AND AMPLITUDE.

Natural Frequency.

λ=1·0. R.P.M.=250·5. Blades=3. ω=78·7. ω^2=6200.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	511	1·000	511	511	12·36×10^3	·041
2	·0126	78·1	·959	75	586	6·18×10^3	·095
3	,,	78·1	·864	68	654	,,	·106
4	·0106	65·7	·758	50	704	,,	·114
5	·0126	78·1	·644	50	754	,,	·122
6	,,	78·1	·522	41	795	12·36×10^3	·064
7	·0162	100·4	·458	46	841	—	
E	—	—			−1489	3·25×10^3	−·458
7	—	—	·458		−648	13·4 ×10^3	−·048
8	·0118	73·1	·506	37	−611	6·7 ×10^3	−·091
9	,,	73·1	·597	44	−567	,,	−·085
10	,,	73·1	·682	50	−517	,,	−·077
11	·0101	62·6	·759	48	−469	7·23×10^3	−·065
12	·0170	105·3	·824	87	−382	14·72×10^3	−·026
13	·0733	455	·850	386	*+4		

Notes. *This Remainder=Zero at Critical.
Remainder is Negative below First Natural Frequency.
,, ,, Positive between First and Second Natural Frequency.
,, ,, Negative above Second Natural Frequency.

AMPLITUDE AT CRITICAL ON STRAIGHT COURSE.

Assume Thrust Variation $= \pm 4{\cdot}57\%$ of 130 ton F.P. Thrust and varies as $(\text{R.P.M.})^2$. Full Power R.P.M. = 230.

Force P at 250·5 R.P.M. $= {\cdot}0457 \times 130 \times (250{\cdot}5/230)^2 = \pm 7{\cdot}05$ Tons.

Assume Propeller Damping Constant $C_d = 1{\cdot}062$.

Prop. Motion $= P/C_d \times \omega = \pm 7{\cdot}05/1{\cdot}062 \times 78{\cdot}7 = \pm {\cdot}0841''$.

Gear Motion = Prop. Motion$/{\cdot}850 = \pm {\cdot}0841/{\cdot}850 = \pm {\cdot}099''$.

SHAPE OF RESONANCE CURVE.

Shaft T.B. to Prop.	$= 1{\cdot}375 \times 10^3$ Tons/in.	$= {\cdot}000727$ Inch/ton
T.B. and Seat	$= 3{\cdot}25 \times 10^3$ Tons/in.	$= {\cdot}000308$ Inch/ton
	Total	$= {\cdot}001035$ Inch/ton

For 7·05 Ton Force Prop. would move $7{\cdot}05 \times {\cdot}001035 = {\cdot}0073''$.

Dynamic Magnifier at Prop. $= {\cdot}0841/{\cdot}0073 = 11{\cdot}5$.

AMPLITUDES UNDAMPED.

$\lambda = {\cdot}7$. R.P.M. = 175. Blades = 3. $\omega = 55{\cdot}0$. $\omega^2 = 3025$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	249	1·000	249	249	$12{\cdot}36 \times 10^3$	·020
2	·0126	38·1	·980	37	286	$6{\cdot}18 \times 10^3$	·046
3	,,	38·1	·934	36	322	,,	·052
4	·0106	32	·882	28	350	,,	·057
5	·0126	38·1	·825	31	381	,,	·062
6	,,	38·1	·763	29	410	$12{\cdot}36 \times 10^3$	·033
7	·0162	49	·730	36	446	—	
E	—	—			−2372	$3{\cdot}25 \times 10^3$	−·730
7	—	—	·730		−1926	$13{\cdot}4 \times 10^3$	−·144
8	·0118	35·6	1·874	31	−1895	$6{\cdot}7 \times 10^3$	−·283
9	,,	35·6	1·157	41	−1854	,,	−·277
10	,,	35·6	1·434	51	−1803	,,	−·269
11	·0101	30·5	1·703	52	−1751	$7{\cdot}23 \times 10^3$	−·242
12	·0170	51·4	1·945	100	−1651	$14{\cdot}72 \times 10^3$	−·112
13	·0733	221·5	2·057	455	−1196		

Thrust Variation $= 5{\cdot}95 \times (175/230)^2 = \pm 3{\cdot}45$ tons.
Gear Amp. undamped $= 3{\cdot}45/1196 = \pm {\cdot}0029''$.

$\lambda = {\cdot}839$. R.P.M. = 210. Blades = 3. $\omega = 66$. $\omega^2 = 4356$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	358	1·000	358	358	$12{\cdot}36 \times 10^3$	·029
2	·0126	54·9	·971	53	411	$6{\cdot}18 \times 10^3$	·066
3	,,	54·9	·905	50	461	,,	·075
4	·0106	46·2	·830	38	499	,,	·081
5	·0126	54·9	·749	41	540	,,	·087
6	,,	54·9	·662	36	576	$12{\cdot}36 \times 10^3$	·047
7	·0162	70·5	·615	43	619	—	
E	—	—			−1999	$3{\cdot}25 \times 10^3$	−·615
7	—	—	·615		−1380	$13{\cdot}4 \times 10^3$	−·103
8	·0118	51·5	·718	37	−1343	$6{\cdot}7 \times 10^3$	−·200
9	,,	51·5	·918	47	−1296	,,	−·193
10	,,	51·5	1·111	57	−1239	,,	−·185
11	·0101	44	1·296	57	−1182	$7{\cdot}23 \times 10^3$	−·164
12	·0170	74·1	1·460	108	−1074	$14{\cdot}72 \times 10^3$	−·073
13	·0733	319·5	1·533	490	−584		

Thrust Variation $= 5{\cdot}95 \times 210/230)^2 = \pm 4{\cdot}96$ tons.
Gear Amp. Undamped $= 4{\cdot}96/584 = \pm$ **·0085″**.

$\lambda = {\cdot}5$. R.P.M. = 125. Blades = 3. $\omega = 39{\cdot}3$. $\omega^2 = 1543$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0823	127	1·000	127	127	$12{\cdot}36 \times 10^3$	·010
2	·0126	19·5	·990	19	146	$6{\cdot}18 \times 10^3$	·024
3	,,	19·5	·966	19	165	,,	·027
4	·0106	16·4	·939	15	180	,,	·029
5	·0126	19·5	·910	18	198	,,	·032
6	,,	19·5	·878	17	215	$12{\cdot}36 \times 10^3$	·017
7	·0162	25	·861	22	237	—	
E	—	—			−2798	$3{\cdot}25 \times 10^3$	−·861
7	—	—	·861		−2561	$13{\cdot}4 \times 10^3$	−·191
8	·0118	18·2	1·052	19	−2542	$6{\cdot}7 \times 10^3$	−·380
9	,,	18·2	1·432	26	−2516	,,	−·375
10	,,	18·2	1·807	33	−2483	,,	−·371
11	·0101	15·6	2·178	34	−2449	$7{\cdot}23 \times 10^3$	−·339
12	·0170	26·2	2·517	66	−2383	$14{\cdot}72 \times 10^3$	−·162
13	·0733	113·1	2·679	303	−2080		

Thrust Variation $= 5{\cdot}95 \times (125/230)^2 = \pm 1{\cdot}76$ tons.
Gear Amp. Undamped $= 1{\cdot}76/2080 = \pm$ **·00085″**.

AMPLITUDES.

DYNAMIC MAGNIFIER=11·5 at Prop.						No Damping	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Fig. 14 Page 7	Amp.λ=1 ×(2)	Sheet 8 Page 21	(3)/(4)	Crit. ×(1)	Pages 18 & 19	
λ	Amp. Ratio	±Prop. Amp.	Prop./ Gear	±Gear Amp.	R.P.M.	±Gear Amp.	R.P.M.
1·3							
1·2							
1·1							
1·05							
1·02							
1·0	1·000	·0841″	·850	·099″	250·5		
·98	·888	·0746″	·95	·0786″	245·5		
·95	·617	·0519″	1·09	·0475″	238	·016	230
·9	·341	·0286″	1·30	·0220″	225·5		
·8	·151	·0127″	1·70	·0075″	200	·0085	210
·7	·081	·0068″	2·057	·0033″	175	·0029	175
·6	·047	·0040″	2·4	·0017″	150		
·5	·029	·0024″	2·679	·0009″	125	·00085	125

It is evident that the amplitudes given by the single degree of freedom resonance curve are too great.

The undamped figures may be slightly high but have been used.

Force on T.B. Seat—Assume Turn Factor=4.

3 *Blades at Full Power—Turning.*

Gear Amp.=±·016″×4=±·064″.

Force on Seat (from Page 18)=·064×1759=±112·5 Tons.

Steady Thrust=130 Tons, so there is no reversal.

CONCLUSION.

The gear amplitude at full power is unacceptable.

If T.B. is to be moved aft the T.B. and Seat must be made still more stiff, probably greater than $4{\cdot}0\times10^3$ Tons/in.

Unless this can be achieved it will be preferable to retain the forward position and fit a 5 bladed propeller.

Appendix No. 3

Shaft 400 feet Long

Case 1 Thrust Block 10 feet from Gearwheel. Calculation of Vibration Amplitudes with Three and Five Bladed Propellers

Case 2 Thrust Block 200 feet from Gearwheel. Calculation of Vibration Amplitudes and Force Required to Hold Gearwheel Stationary

Case 3 Thrust Block 80 feet from Propeller. Calculation of Force Required to Hold Gearwheel Stationary

CASE 1.—SYSTEM AND 5 BLADES AT FULL POWER.

SECTION 1.

Gear+5ft.=30+·9=30·9 Tons =·0801 Tons Sec.^{2}/in.

SECTION 2.

T.B.+5ft.= 7+·9= 7·9 Tons =·0205 Tons Sec.^{2}/in.

SECTIONS 3 TO 12 INCLUSIVE.

39ft. of Shaft=7·26 Tons=·0188 Tons Sec.^{2}/in.

Propeller, etc.=30 Tons =·0778 Tons Sec.^{2}/in.

STIFFNESS.

10′ @·851ft.^{2} =$160{\cdot}5\times10^3\times{\cdot}851/10$ =$13{\cdot}67\times10^3$ Tons/in.

19·5′@·851ft.^{2} =$160{\cdot}5\times10^3\times{\cdot}851/19{\cdot}5$ = $7{\cdot}0\times10^3$ Tons/in.

39′ @·851ft.^{2} =$160{\cdot}5\times10^3\times{\cdot}851/39$ = $3{\cdot}5\times10^3$ Tons/in.

T.B. and Seat = = $4{\cdot}0\times10^3$ Tons/in.

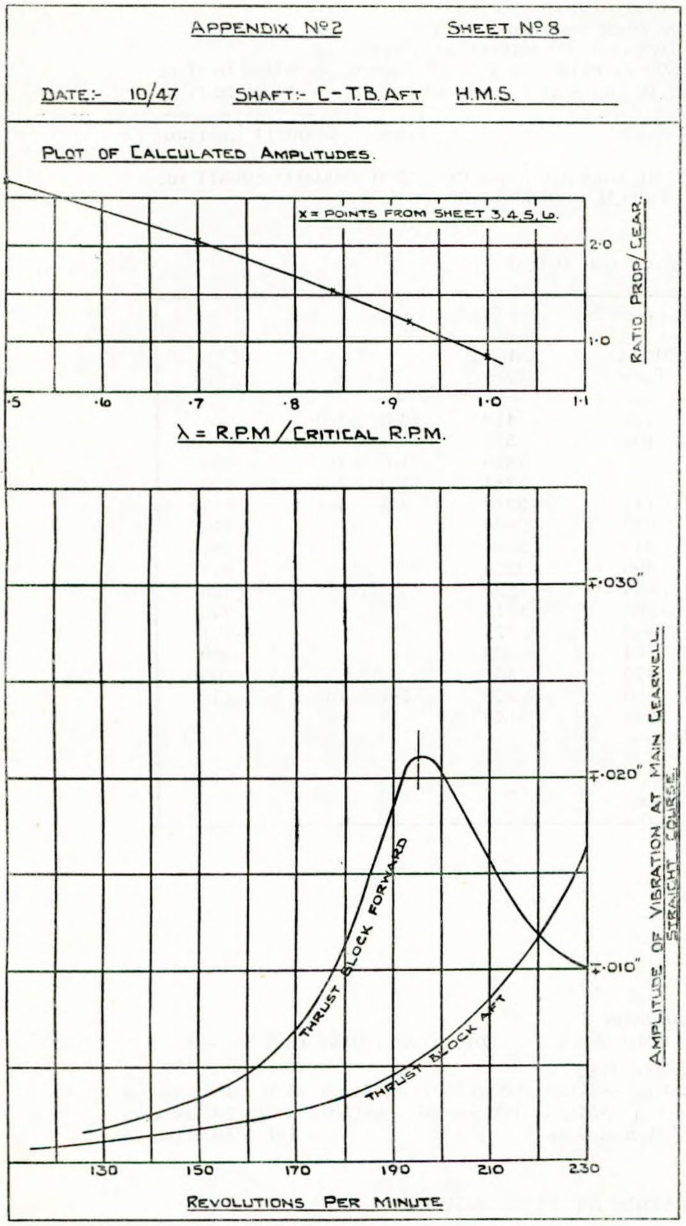
R.P.M.=230. Blades=5. ω=120·4. ω^2=14500.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0801	1161	1·000	1161	1161	$13{\cdot}67\times10^3$	·085
2	·0205	297	·915	272	1433	—	
E	—	—			−3660	$4{\cdot}0\times10^3$	−·915
2	—	—	·915		−2227	$7{\cdot}0\times10^3$	−·318
3	·0188	272·5	1·233	336	−1891	$3{\cdot}5\times10^3$	−·541
4	,,	,,	1·774	484	−1407	,,	−·401
5	,,	,,	2·175	593	−814	,,	−·232
6	,,	,,	2·407	656	−158	,,	−·045
7	,,	,,	2·452	668	+510	,,	+·146
8	,,	,,	2·306	629	1139	,,	·325
9	,,	,,	1·981	540	1679	,,	·479
10	,,	,,	1·502	410	2089	,,	·597
11	,,	,,	·905	247	2336	,,	·667
12	,,	,,	·238	65	2401	$7{\cdot}0\times10^3$	·343
13	·0778	1128	−·105	−118	+2283		

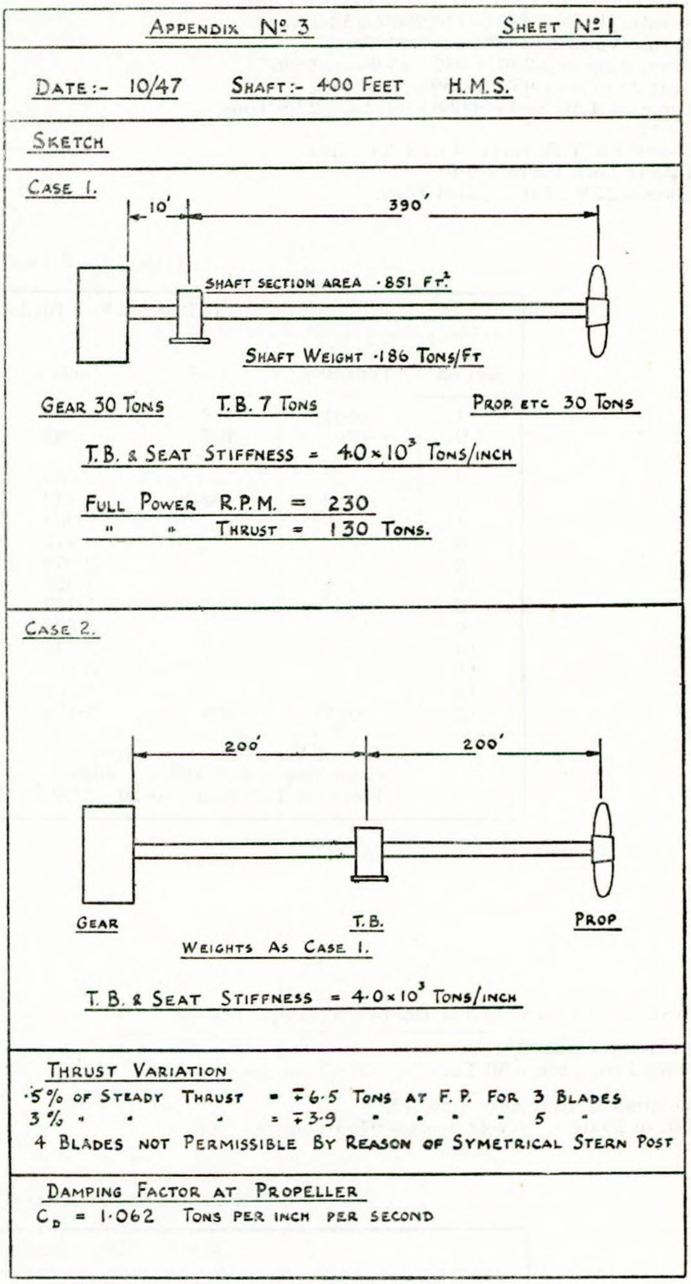
Thrust Variation=±3·9 tons.

Gear Amp.=3·9/2283=**±·0017″.**

Force on T.B. Seat=·0017×3660=**±4·65 tons on straight.**



For "Sheets 3, 4, 5, 6" read "Pages 18 & 19".



CASE 1.—3 BLADES CRITICAL.

R.P.M. = 146. Blades = 3. $\omega = 45{\cdot}9$. $\omega^2 = 2100$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0801	168	1·000	168	168	$13{\cdot}67 \times 10^3$	·012
2	·0205	43·1	·988	43	211	—	
E	—	—			−3952	$4{\cdot}0 \times 10^3$	−·988
2	—	—	·988		−3741	$7{\cdot}0 \times 10^3$	−·535
3	·0188	39·5	1·523	60	−3681	$3{\cdot}5 \times 10^3$	−1·051
4	,,	,,	2·574	102	−3579	,,	−1·022
5	,,	,,	3·596	142	−3437	,,	−·982
6	,,	,,	4·578	181	−3256	,,	−·930
7	,,	,,	5·508	218	−3038	,,	−·868
8	,,	,,	6·376	252	−2786	,,	−·796
9	,,	,,	7·172	284	−2502	,,	−·715
10	,,	,,	7·887	312	−2190	,,	−·625
11	,,	,,	8·512	336	−1854	,,	−·530
12	,,	,,	9·042	357	−1497	$7{\cdot}0 \times 10^3$	−·214
13	·0778	163·3	9·256	1512	+15		

Amplitude at Critical on Straight Course.
Steady Thrust = 130 × (146/230)^2 = 52·3 Tons.
Thrust Variation = 5% = ±2·61 Tons.
Prop. Amp. = ±2·61/1·062 × 45·9 = ±·0536″.
Gear Amp. = ±·0536/9·256 = ±·0058″.
Force on T.B. Seat = ·0058 × 3952 = ±22·9 Tons.

Force on T.B. Seat when Turning.
Taking Turn Factor = 4·0.
Force = 22·9 × 4·0 = ±91·6 Tons.

Steady Thrust = 52·3 Tons.
So there is a thrust reversal.
Dynamic Magnifier at Propeller.
390′ @ ·851ft.^{2} = ·35 × 10^3 Tons/in. = ·00286 In./Ton.
T.B. and Seat = 4·0 × 10^3 Tons/in. = ·00025 In./Ton.

Total = ·00311 In./Ton.

2·61 Tons will move Prop. 2·61 × ·00311 = ·00811 In.
∴ D.M. = ·0536/·00811 = 6·6.

CASE 1.—3 BLADES AT 230 R.P.M.

R.P.M. = 230. Blades = 3. ω = 72·3. ω^2 = 5220.

Section	Mass Tons/386	Mω^2	Δ Inches	Mω^{2}Δ Tons	ΣMω^{2}Δ Tons	C Tons/in.	Σ/C Inches
1	·0801	418	1·000	418	418	13·67 × 10^3	·031
2	·0205	107	·969	104	522	—	
E	—	—			−3876	4·0 × 10^3	−·969
2	—	—	·969		−3354	7·0 × 10^3	−·479
3	·0188	98·1	1·448	142	−3212	3·5 × 10^3	−·917
4	,,	,,	2·365	232	−2980	,,	−·852
5	,,	,,	3·217	316	−2664	,,	−·761
6	,,	,,	3·978	390	−2274	,,	−·649
7	,,	,,	4·627	454	−1820	,,	−·520
8	,,	,,	5·147	505	−1315	,,	−·376
9	,,	,,	5·523	542	−773	,,	−·221
10	,,	,,	5·744	564	−209	,,	−·060
11	,,	,,	5·804	570	+361	,,	+·103
12	,,	,,	5·701	559	+920	7·0 × 10^3	+·131
13	·0778	406	5·570	2263	+3183		

Thrust Variation = ±6·5 tons.
Gear Amp. = 6·5/3183 = ±**·0020″.**
Force on T.B. Seat = ·0020 × 3876 = **±7·9 tons.**

CASE 2.—**System and 3 Blades at Full Power.**

Sections 1 and 13.
Gear = Prop., etc. = 30 Tons = ·0778 Tons Sec.^{2}/in.

Sections 2 to 6 and 8 to 12.
40ft. of Shaft = 7·44 Tons = ·0193 Tons Sec.^{2}/in.

Section 7.
Thrust Block = 7·0 Tons = ·0181 Tons Sec.^{2}/in.

Stiffness.
20′ @ ·851ft.^{2} = 160·5 × 10^3 × ·851/20 = 6·84 × 10^3 Tons/in.
40′ @ ·851ft.^{2} = 160·5 × 10^3 × ·851/40 = 3·42 × 10^3 Tons/in.
T.B. and Seat = 4·0 × 10^3 Tons/in.

CASE 2.—SYSTEM AND 3 BLADES AT FULL POWER.

R.P.M. = 230. Blades = 3. ω = 72·3. ω^2 = 5220.

Section	Mass Tons/386	Mω^2	Δ Inches	Mω^{2}Δ Tons	ΣMω^{2}Δ Tons	C Tons/in.	Σ/C Inches
1	·0778	406	1·000	406	406	6·84 × 10^3	·059
2	·0193	100·9	·941	95	501	3·42 × 10^3	·147
3	,,	,,	·794	80	581	,,	·170
4	,,	,,	·624	63	644	,,	·188
5	,,	,,	·436	44	688	,,	·201
6	,,	,,	·235	24	712	6·84 × 10^3	·104
7	·0181	94·5	·131	12	724	—	
E	—	—			−524	4·0 × 10^3	−·131
7	—	—	·131		+200	6·84 × 10^3	+·029
8	·0193	100·9	·102	10	210	3·42 × 10^3	·061
9	,,	,,	·041	4	214	,,	·063
10	,,	,,	−·022	−2	212	,,	·062
11	,,	,,	−·084	−8	204	,,	·060
12	,,	,,	−·144	−15	189	6·84 × 10^3	·028
13	·0778	406	−·172	−70	+119		

Thrust Variation = ±6·5 tons.
Gear Amp. = 6·5/119 = ±**·0546″.**
5 Blades at 138 R.P.M.
Gear Amp. = ·0546 × (·6)^3 = ±**·0118″.**

CASE 2.—3 BLADES CRITICAL.

R.P.M.=203·7. Blades=3. $\omega=64{\cdot}0$. $\omega^2=4096$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0778	319	1·000	319	319	$6{\cdot}84\times10^3$	·047
2	·0193	79·1	·953	75	394	$3{\cdot}42\times10^3$	·115
3	,,	,,	·838	66	460	,,	·135
4	,,	,,	·703	56	516	,,	·151
5	,,	,,	·552	44	560	,,	·164
6	,,	,,	·388	31	591	$6{\cdot}84\times10^3$	·087
7	·0181	74·2	·301	22	613	—	—
E	—	—			−1204	$4{\cdot}0\times10^3$	−·301
7	—	—	·301		−591	$6{\cdot}84\times10^3$	−·087
8	·0193	79·1	·388	31	−560	$3{\cdot}42\times10^3$	−·164
9	,,	,,	·552	44	−516	,,	−·151
10	,,	,,	·703	56	−460	,,	−·135
11	,,	,,	·838	66	−394	,,	−·115
12	,,	,,	·953	75	−319	$6{\cdot}84\times10^3$	−·047
13	·0778	319	1·000	319	0		

AMPLITUDE AT CRITICAL ON STRAIGHT COURSE.
Steady Thrust$=130\times(203{\cdot}7/230)^2=102$ Tons.
Thrust Variation$=\pm5\%=\pm5{\cdot}1$ Tons.
Prop. Amp.$=5{\cdot}61/1{\cdot}062\times64{\cdot}0=\pm{\cdot}0826''$.
Gear Amp.$={\cdot}0826/1{\cdot}000=\pm{\cdot}0826$.
Force on T.B. Seat$={\cdot}0826\times1204=\pm99{\cdot}5$ Tons.
FORCE ON T.B. SEAT WHEN TURNING.
Taking Turn Factor=4·0.
Force$=99{\cdot}5\times4=\pm398$ Tons.
Steady Thrust=102 Tons.
So there is a heavy Thrust Reversal.

DYNAMIC MAGNIFIER AT PROPELLER.
200ft.@·851ft.^{2} $={\cdot}684\times10^3$ Tons/in. $={\cdot}001462$ In./ton.
T.B. and Seat $=4{\cdot}0\times10^3$ Tons/in. $={\cdot}00025$ In./ton.
Total $={\cdot}001712$ In./ton.

5·1 Tons will move Prop. $5{\cdot}1\times{\cdot}001712={\cdot}00873''$.
∴ D.M.$={\cdot}0826/{\cdot}00873=9{\cdot}5$.
5 BLADES CRITICAL.
Gear Amp.$={\cdot}0826\times(3/5)^3=\pm{\cdot}0178''$.
At 122 R.P.M.

CASE 2.—5 BLADES AT 230 AND 2ND CRITICAL.

R.P.M.=230. Blades=5. $\omega=120{\cdot}4$. $\omega^2=14500$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0778	1128	1·000	1128	1128	$6{\cdot}84\times10^3$	·165
2	·0193	280	·835	234	1362	$3{\cdot}42\times10^3$	·398
3	,,	,,	·437	122	1484	,,	·434
4	,,	,,	·003	1	1485	,,	·434
5	,,	,,	−·431	−121	1364	,,	·399
6	,,	,,	−·830	−232	1132	$6{\cdot}84\times10^3$	·166
7	·0181	262	−·996	−261	871	—	
E	—	—			3984	$4{\cdot}0\times10^3$	+·996
7	—	—	−·996		4855	$6{\cdot}84\times10^3$	·711
8	·0193	280	−1·707	−478	4377	$3{\cdot}42\times10^3$	1·280
9	,,	,,	−2·987	−836	3541	,,	1·036
10	,,	,,	−4·023	−1128	2413	,,	·705
11	,,	,,	−4·728	−1322	1091	,,	·319
12	,,	,,	−5·047	−1413	−322	$6{\cdot}84\times10^3$	−·047
13	·0778	1128	−5·000	−5640	−5962		

Thrust Variation$=\pm3{\cdot}9$ tons.
Gear Amp.$=3{\cdot}9/5962=$ **·00065″.**

R.P.M.=149·5. Blades=5. $\omega=78{\cdot}3$. $\omega^2=6125$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0778	476	1·000	476	476	$6{\cdot}84\times10^3$	·070
2	·0193	118	·930	110	586	$3{\cdot}42\times10^3$	·172
3	,,	,,	·758	89	675	,,	·197
4	,,	,,	·561	66	741	,,	·217
5	,,	,,	·344	41	782	,,	·228
6	,,	,,	·116	14	796	$6{\cdot}84\times10^3$	·116
7	·0181	111	0	0	796	—	
E	—	—			0	$4{\cdot}0\times10^3$	0
7	—	—	0		796	$6{\cdot}84\times10^3$	·116
8	·0193	118	−·116	−14	782	$3{\cdot}42\times10^3$	·228
9	,,	,,	−·344	−41	741	,,	·217
10	,,	,,	−·561	−66	675	,,	·197
11	,,	,,	−·758	−89	586	,,	·172
12	,,	,,	−·930	−110	476	$6{\cdot}84\times10^3$	·070
13	·0778	476	−1·000	−476	0		

Thrust Variation$=3{\cdot}9\times(149{\cdot}5/230)^2=\pm1{\cdot}65$ tons.
Gear Amp.=Prop. Amp.$=1{\cdot}65/1{\cdot}062\times78{\cdot}3=\pm{\cdot}0199''$.

CASE 2.—CRITICAL SPEED WITH GEAR HELD STILL.

R.P.M. = 226. Blades = 3. $\omega = 71 \cdot 0$. $\omega^2 = 5041$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0778	392	0	0	−1000	$6{\cdot}84 \times 10^3$	—·146
2	·0193	97·2	·146	14	−986	$3{\cdot}42 \times 10^3$	—·288
3	,,	,,	·434	42	−944	,,	—·276
4	,,	,,	·710	69	−875	,,	—·256
5	,,	,,	·966	94	−781	,,	—·228
6	,,	,,	1·194	116	−665	$6{\cdot}84 \times 10^3$	—·097
7	·0181	91·1	1·291	118	−547	—	
E	—	—			−5164	$4{\cdot}0 \times 10^3$	−1·291
7	—	—	1·291		−5711	$6{\cdot}84 \times 10^3$	—·836
8	·0193	97·2	2·127	207	−5504	$3{\cdot}42 \times 10^3$	−1·610
9	,,	,,	3·737	363	−5141	,,	−1·502
10	,,	,,	5·239	509	−4632	,,	−1·353
11	,,	,,	6·592	640	−3992	,,	−1·167
12	,,	,,	7·759	755	−3237	$6{\cdot}84 \times 10^3$	—·474
13	·0778	392	8·233	3225	−12		

Thrust Variation = $6{\cdot}5 \times (226/230)^2 = \pm 6{\cdot}28$ Tons.
Prop. Amp. = $6{\cdot}28/1{\cdot}062 \times 71{\cdot}0 = \pm{\cdot}0833$.
Force to keep gear still = $1000 \times {\cdot}0833/8{\cdot}233 = \pm 10{\cdot}1$ Tons.
Force on T.B. Seat = $5164 \times {\cdot}0833/8{\cdot}233 = \pm 52{\cdot}3$ Tons.

POSSIBLE HOLDING FORCE AT 230 R.P.M.
L.P. Turbine—Balanced—Nil.
H.P. Turbine—4·46 Tons net steam thrust.
Torque Load on Flex. Coupling Teeth = 26·45 Tons.
Taking Coeff. of Friction = 0·1 Axial Force = 2·645 Tons.

CONCLUSIONS.
Gear will not be held still even on straight course. Even if it were the force on T.B. is excessive.

CASE 3.—T.B. 80′ FROM PROP.—GEAR HELD STILL.

R.P.M. = 230. Blades = 3. $\omega = 72 \cdot 3$. $\omega^2 = 5220$.

Section	Mass Tons/386	$M\omega^2$	Δ Inches	$M\omega^2\Delta$ Tons	$\Sigma M\omega^2\Delta$ Tons	C Tons/in.	Σ/C Inches
1	·0778	406	0	0	−1000	$6{\cdot}84 \times 10^3$	—·146
2	·0193	100·9	·146	15	−985	$3{\cdot}42 \times 10^3$	—·288
3	,,	,,	·434	44	−941	,,	—·275
4	,,	,,	·709	72	−869	,,	—·254
5	,,	,,	·963	97	−772	,,	—·226
6	,,	,,	1·189	120	−652	,,	—·191
7	,,	,,	1·380	139	−513	,,	—·150
8	,,	,,	1·530	154	−359	,,	—·105
9	,,	,,	1·635	165	−194	$6{\cdot}84 \times 10^3$	—·028
10	·0181	94·5	1·663	157	—37	—	
E	—	—			−6652	$4{\cdot}0 \times 10^3$	−1·663
10	—	—	1·663		−6689	$6{\cdot}84 \times 10^3$	—·979
11	·0193	100·9	2·642	267	−6422	$3{\cdot}42 \times 10^3$	−1·879
12	,,	100·9	4·521	456	−5966	$6{\cdot}84 \times 10^3$	—·873
13	·0778	406	5·394	2190	−3776		

Thrust Variation = $\pm 6{\cdot}5$ Tons.
Force to keep gear still = $1000 \times 6{\cdot}5/3776 = \pm 1{\cdot}72$ Tons.
Prop. Amp. = $5{\cdot}394 \times 6{\cdot}5/3776 = \pm{\cdot}0093''$.
Force on T.B. Seat = $6652 \times 6{\cdot}5/3776 = \pm 11{\cdot}5$ Tons.
POSSIBLE HOLDING FORCE.
From above—Friction in H.P. Turb. Coupling = 2·645 Tons.
Turbine Thrust = 4·46 Tons.

CONCLUSIONS.
Gear might be held still on straight course.
On turns (Factor = 4) it would not be held still.

Discussion

Captain (E) J. G. C. Given, C.B.E., R.N. (Member) said that this was probably the first time, as the author had suggested, that anything had been published dealing with the particular problem to which the paper was devoted. Personally, he had some knowledge of the problem, since the author worked with him when, as mentioned in the paper, he first joined the Engineer-in-Chief's department. It might be of interest, therefore, to amplify a little the historical side of the paper.

The author mentioned how the troubles in H.M.S. Warspite first brought the matter to a head. In the first place lubrication difficulties were looked for as a cause. It was really due to the personal interest taken in the matter by the late Sir George Preece, then Engineer-in-Chief, and to the suggestions which Sir George made, that attention was directed to the fact that the source of the trouble was not so much in the lubrication or in the engine room but was coming from further aft and was in the propellers.

From 1937-38 to 1943 seemed rather a long time. It was, of course, appreciated that the easing method described in the paper was only a palliative, and that the problem had to be investigated thoroughly at some time; but it would be realized that a great deal was going on in the period prior to 1939, with the heavy pressure of work in connexion with rearmament, and there then followed the early part of the war. It might be thought that the matter had been neglected, but at that time it was possible to deal only with problems which were so urgent that lack of attention to them would cause definite failures, and in the case of the problem in question a palliative had been found which could be used at any rate temporarily.

There were two questions which he would like to put to the author. The first concerned the actual cyclic thrust variation. Apart from the effect of dynamic magnification on resonance, of what order could that be in certain circumstances even on a straight course, as opposed to turning conditions? Was it liable to give trouble in possible designs of new prime movers or new conceptions of transmission systems, especially where some of the thrust was taken either by the design of the prime mover itself or by the gearing, and the thrust block itself was designed to carry only the balance of thrust?

It would be of interest to have some actual figures of the cyclic thrust variation at the propeller and its order.

In the final conclusions and recommendations, he did not feel very happy about the idea of holding the gearwheel stationary by the friction at the flexible couplings, especially with the more complex mechanical linkages involved with double reduction gears. That was rather a feeling on his part than a definite factual criticism.

With regard to the actual shaft movements which had undoubtedly been experienced, and not only when a fairly large amplitude due to resonance occurred, he would like to know whether any evidence was available of the effect of this movement on the rate of wear which occurred in the linings of the A-brackets or stern bushes.

With regard to the author's criteria of amplitude, he thought that every case must be considered on its merits, having regard to the type of ship, type of machinery and all the other factors involved. To extrapolate from one type of ship to another without some previous experience, would, he suggested, be liable to cause trouble.

Commander (E) L. Baker, D.S.C., R.N. (Member), said that the problem outlined by the author had forced itself on his notice whilst serving in an aircraft carrier of the Illustrious class. Although the flexible couplings between the turbine and pinions had just been renewed, they were again seized and so badly damaged as to require further renewal after three months' service. External examination of the shaft and seatings showed clearly that there was an axial movement of the centre shaft resulting in a loose thrust block seating. A similar but less severe vibration was observed in the wing shafts.

At about this time the ship left for the Far East, and it was decided to undertake an investigation with such facilities as were available to the ship. Interest was primarily centred on the movement of the pinion, because of the harmful effect on the flexible couplings, and an apparatus was made up from a recording pressure gauge, the Bourdon tube of which was disconnected, and the balance wheel removed from the clock, so that the axial vibration of the pinions could be recorded on paper. With this apparatus, which was firmly mounted on the gear-case, it was clearly shown that the main vibration had a frequency of three times the propeller revolutions, and that a critical speed existed at about the figure quoted by the author. Furthermore, the increased amplitude on turning was observed, but one slight divergence from the author's results was obtained, namely that there was a very large increase in the amplitude whilst turning, when the rudder was taken off at about 20 deg. position. It was perhaps significant that the maximum load on the steering gear occurred in the same conditions. Speaking from memory, the orders were: running at the critical speed, straight, $\pm 0{\cdot}02$; turning, $\pm 0{\cdot}040$; taking off the helm, $\pm 0{\cdot}128$, under which conditions the instrument broke.

In order to obtain more satisfactory records than could be achieved with this apparatus, even if it had been rebuilt, an apparatus was designed and manufactured to utilize a cathode ray tube with a variable time and amplitude base. The principle adopted was to transfer the pinion movement to the compression of a spring against a carbon pile, so that the resistance of the pile changed approximately linearly with the movement of the pinion. That apparatus had been set up and its satisfactory functioning had been demonstrated when the war ended, but, owing to the dispersal of staff, no further work was done. It might, however, be worth recording that the picture obtained on the cathode ray tube suggested that the simple critical was by no means the only one, and that there appeared to be lost motion due to the pinion clearances on one side only of the axis of vibration. The explanation was not clear, and the problem had not been solved when the apparatus was dismantled.

Further serious wear of couplings was avoided by operating off the critical speeds and accepting the resultant loss in endurance. The staggered shaft speeds, however, set up hull vibrations which for any given ship speed could be moved along the ship between the stern and the bridge superstructure by changing the relative speeds of the wing and centre shafts.

Mr. R. K. Craig (Vice-Chairman of Council) remarked that in merchant ships they had always looked to a flexible coupling to accommodate the expansion of the turbine, and they had not associated it with longitudinal vibration. The author had not said anything about the materials of which the flexible coupling was made, and probably with different grades of steel, it might be possible to overcome the seizing of the claws of the coupling.

BY CORRESPONDENCE

Mr. S. Archer, B.Sc. (Member), wrote that on p. 67 it was stated that by "easing" the wing propeller, the amplitude of vibration was reduced from $\pm 0{\cdot}050$in. to $\pm 0{\cdot}025$in. Presumably this would refer to the amplitude at the propeller, not at the gears.

On p. 68 as regards the mechanism by which excessive vibration was built up on the inner shafts during turning, it would appear that the effects of interaction between blades and dead water close to the ship on one side of the inner screw and between blades and outer screw slip stream on the opposite side would at first tend to counteract each other as regards axial excitation. As, however, more and more of the outer half of the innner screw disk became affected by the outer screw slip stream, so the axial forces on opposite sides of the inner screw would become more and more unbalanced, inducing bending moments with resulting unbalanced reactions at the A-brackets and thus giving rise to the excessive hull vibration reported aft.

On the subject of the amount of the propeller thrust variation with twin screws and liner type bossings, as mentioned on pp. 70 and 75, the following results of torsiograph tests carried out by Lloyd's Register during the war on a well-known twin screw liner might be of interest, since the torque variation well clear of the torsional critical speed should be of the same order as the thrust variation also under non-resonant conditions.

The propellers were three-bladed, bronze, 18ft. 6in. in diameter, 17ft. 6in. pitch with 106 sq. ft. developed surface and 88 sq. ft. projected surface. They absorbed 15,000 s.h.p. each at 140 r.p.m. and had liner type plated bossings.

Records were taken at 120 r.p.m. and the results were given below. Torsional natural frequency was calculated at 148 r.p.m., i.e. third order critical speed at 49·3 r.p.m.

Third order torque variation $= \pm\ 275{,}000$lb. in.

Mean torque at 120 r.p.m. $= 4{\cdot}96 \times 10^6$lb. in.

$$\text{Torque variation} \quad = \frac{0{\cdot}275 \times 100}{4{\cdot}96} = \pm\ 5{\cdot}5 \text{ per cent.}$$

From this it would seem that with the narrower mechant marine type of propeller blades, the thrust variation might be somewhat greater than the $\pm$ 4 per cent estimated by the author for naval screws.

With reference to the methods of calculation described on p. 72 it could be stated that the replacement of a heavy shaft by a number of concentrated masses connected by springs had been in use at Lloyd's Register for many years and had proved both accurate and simple. Some idea of its accuracy might be shown as follows:—

For a uniform, cylindrical, heavy "free-free" shaft of length L in., modulus of elasticity E and mass density ρ, the natural frequencies of axial vibration were given exactly by

$$F_r = 9{\cdot}55\ \frac{r\pi}{L} \times \sqrt{\frac{E}{\rho}}$$

where $\quad r$ = number of nodes

Let the shaft length L be divided into n equal parts and imagine each part connected with its neighbours by imaginary massless springs of axial stiffness C lb. per in. and sectional area A sq. in.

Also let M = mass of each section in lb.-in.$^{-1}$ sec.^{2} and

$$a^2 = C/M$$

$$\text{Now } C = \frac{EA}{L/n} \text{ and } M = \rho AL/n$$

$$a^2 = \frac{EA}{L/n} \times \frac{1}{\rho AL/n}$$

$$= \frac{E}{\rho L^2/n^2}$$

$$a = n/L\ \sqrt{(E/\rho)} = \sqrt{(C/M)}$$

Then the natural frequencies could be expressed in the form

$$F = 9{\cdot}55ka$$

$$= 9{\cdot}55kn/L\ \sqrt{(E/\rho)}$$

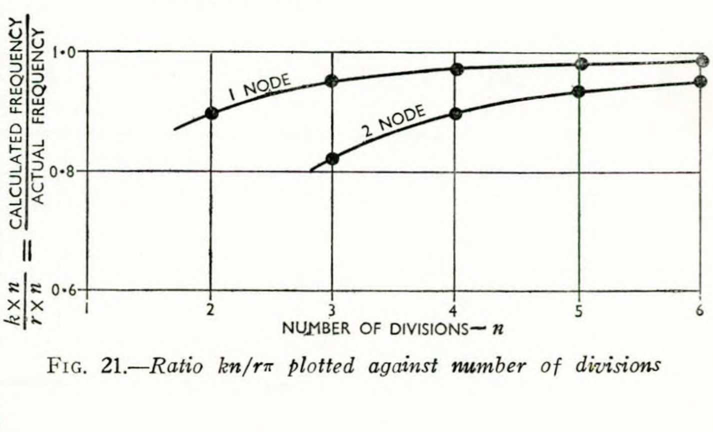


FIG. 21.—*Ratio kn/rπ plotted against number of divisions*

Where k was a factor depending on the number of nodes r and the number of divisions n.

The values of the products k × n and the ratio kn/rπ were given in Table I below for one-node and two-nodes and the ratio kn/rπ was plotted against the number of divisions in the graph Fig. 21.

This ratio showed the accuracy of the approximation in relation to the exact natural frequencies.

$$= \frac{\partial T}{\partial S}\left(\frac{-101\ 1/3}{PN}\right)$$

$$\text{or}\quad \frac{\partial T}{\partial V_A} = \frac{-101\ 1/3}{PN}\frac{\partial T}{\partial S}$$

Using this method, an analysis had been made of a merchant marine type of four-bladed bronze propeller for which the following performance data were available:—3,650 b.h.p.; 126·5 r.p.m.; speed on trial, 14·827 knots; diameter, 15ft.; pitch (mean) 11ft. 5in.; developed area, 86 sq. ft.; number of blades, 4; pitch ratio, 0·763; blade thickness fraction, 4·7 per cent; mean width ratio, 0·238; disk area ratio (developed), 0·486; analysis wake (Taylor), 0·39.

Table 1

	One-node, r=1					Two-node, r=2			
No. of divisions, n	2	3	4	5	6	3	4	5	6
k	$\sqrt{2}$	1	0·7654	0·61806	0·5177	$\sqrt{3}$	$\sqrt{2}$	1·176	1·00
kn	2·828	3·0	3·0616	3·0903	3·1062	5·196	5·656	5·88	6
kn/rπ	0·90	0·955	0·975	0·984	0·988	0·827	0·90	0·936	0·955

These results showed that for a plain, heavy "free-free" shaft without end masses it would probably be sufficiently accurate to take no more than five to six divisions for the one-node mode, whereas for the two-node mode for the same degree of accuracy it was necessary to take double the number of divisions, i.e. say ten to twelve divisions, the error then being of the order of 1½ to 1 per cent.

Where, however, as in marine shafting systems, the end masses were comparable in magnitude with the total shaft mass, it was clear that for the same degree of accuracy still fewer subdivisions could be used, say, three and six for one and two-nodes respectively.

In any case there seemed little point in striving for great accuracy with the shaft when the governing factors were probably the thrust stiffness and mass of entrained water at the propeller, the latter being adjusted to give the observed frequencies when taken in conjunction with measured values of the thrust stiffness.

It was noted that the thrust block deflexion test indicated in Fig. 11 was carried out in the shop, and it would be of interest to know whether the results obtained had since been confirmed by actual measurements in the ship, having regard to possible difference in the stiffness of the thrust block seatings.

It might be noted that the standard figure given for entrained water in Fig. 18, i.e., 0·0481 tons per sq. ft. of developed blade surface corresponded to an addition of 42 per cent to the mass of the propeller. The practice at Lloyd's Register hitherto had been to add 50 per cent to the propeller mass and this had given good results in conjunction with measured thrust stiffness.

As regards the propeller damping factor assumed by the author, it was clear that provided performance curves for the propeller were available, it was possible to estimate the damping coefficient as indicated below, the procedure being comparable with that adopted for determining torsional vibration damping coefficients.

Let T = thrust in pounds.
V_A = speed of advance of propeller relative to water in which it works, knots.
S = true slip ratio.
D = propeller diameter, feet.
P = pitch, feet.
N = rotational speed, r.p.m.

$$\lambda = \frac{101\ 1/3 \times V_A}{ND}$$

For constant ship speed and constant revolutions, N

$$S = 1 - \frac{101\ 1/3/\times V_A}{PN}$$

$$\frac{\partial T}{\partial V_A} = \frac{\partial T}{\partial S}\frac{S}{\partial V_A}$$

$$\text{From this data}\quad \frac{\partial T}{\partial V_A} = \frac{-101\ 1/3}{11{\cdot}42 \times 126{\cdot}5}\frac{\partial T}{\partial S}$$

$$= -0{\cdot}07\ \frac{\partial T}{\partial S}$$

$$V_A = 14{\cdot}827\ (I - w)$$

$$= 14{\cdot}827 \times 0{\cdot}61 = 9{\cdot}04 \text{ knots.}$$

The performance curves used were Troosts' Type B.4.40 (Trans. North-East Coast Inst., 1938).

$$\lambda = \frac{101\ 1/3}{126{\cdot}5 \times 15} \times V_a$$

$$= 0{\cdot}0534 V_A$$

$$C_1 = \frac{T}{\rho D^4 (N/60)^2}$$

$$T = \rho D^4 \left(\frac{N}{60}\right)^2 C_1$$

$$= 1{\cdot}99 \times (15)^4 \times \frac{(126{\cdot}5)^2}{3{,}600} \times C_1$$

$$T = 448{,}000\ C_1 \text{ lb.}$$

$$S = 1 - \frac{V_A \times 101\ 1/3}{PN}$$

$$= 1 - \frac{101\ 1/3\ V_A}{ND} \times \frac{D}{P}$$

$$= 1 - \frac{\lambda}{P/D} \qquad P/D = 0{\cdot}763$$

$$S = 1 - 1{\cdot}31\lambda$$

The values of thrust, T, were plotted against slip ratio, S, in Fig. 22. It would be noted that with varying propeller speed of advance, over range taken, the torque slip character of this propeller was approximately linear at constant revolutions and constant ship speed.

$$\text{From Fig. 22}\quad \frac{\partial T}{\partial S} \text{ at } V_A = 9{\cdot}04 \text{ knots} = +118{,}600$$

$$\frac{\partial T}{\partial V_A} = -0{\cdot}07 \times 118{,}600$$

$$= -8{,}300\text{lb. per knot}$$

$$\frac{\partial T}{\partial V_A} = -410\text{lb. per in. per sec.}$$

Total developed surface = 86 sq. ft.

$$\text{Therefore}\quad \frac{\partial T}{\partial V_A} = -\frac{410}{2{,}240 \times 86} \text{ tons per in. per sec. per sq. ft. developed area.}$$

$$= -0{\cdot}00212 \text{ tons per in. per sec. per sq. ft. developed area.}$$

Table 2

		C_1				
V_A (kn)	$\lambda=0{\cdot}0534V_A$	P/D=0·6	P/D=0·763	P/D=0·80	$T=448\times10^3C_1$	$S=1-1{\cdot}31\lambda$
8·00	0·427	0·1125	0·1814	0·197	81,400	0·44
8·50	0·454	0·1016	0·1716	0·1875	77,000	0·405
8·75	0·467	0·097	0·1682	0·1844	75,500	0·387
9·04	0·483	0·0905	0·1630	0·1796	73,150	0·367
9·25	0·494	0·086	0·1585	0·1750	71,100	0·353
9·50	0·507	0·0796	0·1541	0·1710	69,100	0·335
10·000	0·534	0·068	0·1444	0·1617	64,800	0·300

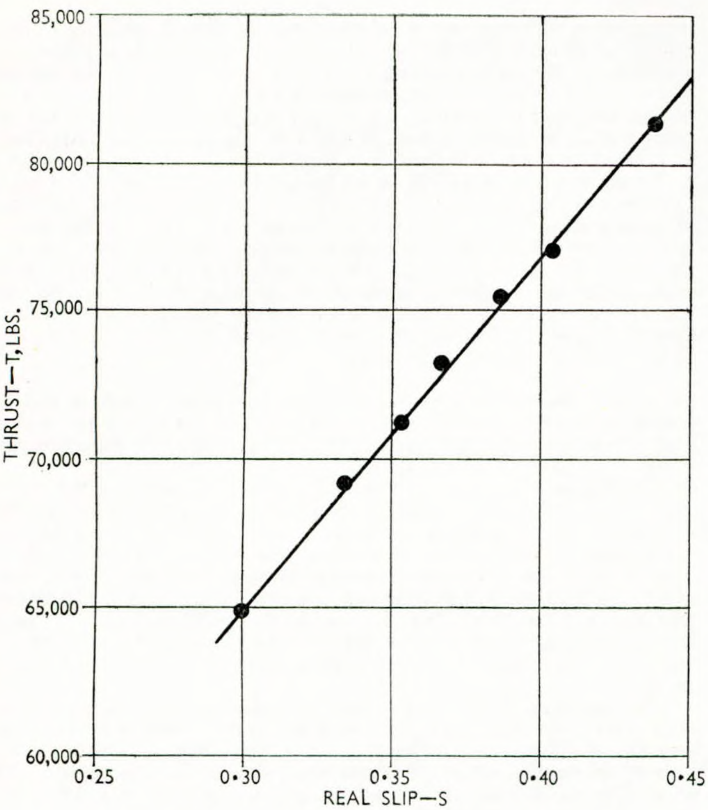


FIG. 22.—*Values of T plotted against slip ratio, S*

If total blade length was criterion,

$$L = 4 \times (7{\cdot}5 - 1{\cdot}5) = 24\text{ft.}$$

$$\text{and } \frac{\partial T}{\partial V_A} = \frac{410}{2{,}240 \times 24}$$

$$= -0{\cdot}00762 \quad \text{tons per in. per sec. per ft. of blade length.}$$

These figures compared with the corresponding representative values given by the author on p. 75 for three-bladed, naval type screws, viz.: 0·00739 and 0·0177, respectively. The latter carried very much heavier thrust loading and were probably considerably more sensitive to small changes of slip, which doubtless accounted for the major part of the difference indicated. He considered the author's assumption that substantially all the damping was contributed by the propeller might still be open to question, and it would be interesting if the author would check his value for the damping coefficient on the basis of the actual performance curves for the propellers in question. However, it was probable that for purposes of practical calculation the assumption made was close enough.

Figs. 16 and 17 should be found very useful for rapid approximate calculation of frequency.

In conclusion, it would seem that in cargo vessels the axial vibration problem was not likely to prove troublesome, even with the four-bladed propellers commonly fitted. This was doubtless due to the relatively stiff shafting and low revolutions.

With increasing revolutions and longer shaft lines, however, such as might apply in the liner class of vessels in which three-bladed propellers were often fitted, it would appear a wise precaution for designers to investigate the axial vibration characteristics of the shafting in the design stage, particularly with multi-screw installations.

Dr. T. W. F. Brown (Member) wrote that he was in general agreement with the author's findings except for the important statement in trying to link the measured results with improved designs.

All the data and calculations in the paper referred to thrust blocks separate from the gear cases and supported on independent seatings. The author considered that it was difficult to provide adequate fore and aft stiffness under a large gearcase because its own sump and condenser was in the way. This point was made after referring to the locked train design of gearcase. It should be recognized that with this type of gearcase a main wheel small in diameter could be used, as double reduction allowed large ratios to be obtained without a correspondingly large main wheel. The axial length or face of the gear was also reduced since in a compound turbine arrangement four secondary pinions transmit the load.

If first of all the stiffness of the independent thrust block was compared with the thrust block integral with the main gearcase it would be seen that the thrust block without considering the additional stiffness arising from the main gearcase was already stiffer than the independent thrust block, and as it butted on, and was welded to, the main cross girder carrying the forward main bearing, he thought that it was considerably stiffer. Similarly if the position of the thrust block was examined in relation to the seating it would be found that even under the thrust block alone the seating was deeper as the thrust block occurred at the forward part of the gearcase and had a longer base than corresponding drawings for seatings for independent thrusts. This seating was, in fact, part of the whole seating for the gearcase and, as already explained, the cut-away in the seating to accommodate the sump was smaller than in any of the earlier designs.

It was agreed that the amount of flexibility in the thrust housing itself would not be greatly affected as to whether it was integral or separate to the main gearcase, but there was no doubt that the overall stiffness of the gearcase plus seating for the integral thrust block was certainly greater than that where the thrust block was an independent unit. In addition, the weight of the gearcase itself bolted round the seating surrounding the thrust block contributed to the stiffness and this stiffness of seating was distributed over a very much greater area of ship. In view of this it would indeed be surprising if the integral thrust did not prove in practice to be an improved arrangement.

The information given in Fig. 14 provided a useful and rapid method of plotting damped resonance curves for systems which approximated fairly closely to the single degree of freedom system. For systems vibrating in a more complex manner, however, the method could give misleading results as the author showed in Appendix 2, since the shape of the resonance curve was then no longer determined solely by the dynamic magnification.

In such cases it was possible to plot resonance curves which took account of propeller damping by using a slight extension of the usual tabular method. For any given value of frequency the table was worked through in the usual manner to obtain the value of the remainder force at the propeller, and then use was made of the fact that the remainder force, the thrust variation and the damping force on the propeller must at every instant be in equilibrium.

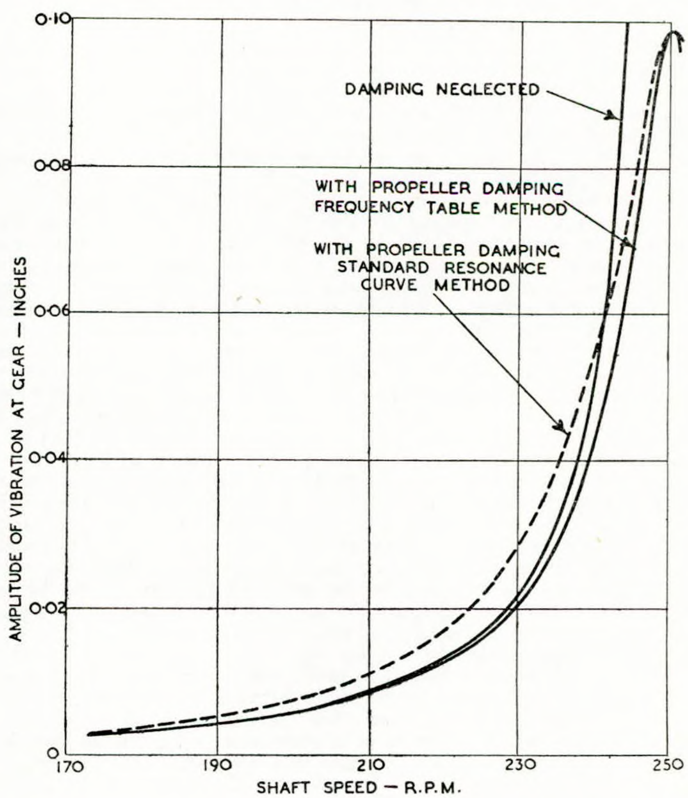


FIG. 23.—*The damped resonance curve for the system of Appendix 2 together with the undamped curve*

Thus assuming sinusoidal motion of the propeller, at any instant

$$P_t + xF \sin \omega t - xC_D\omega\Delta \cos \omega t = 0$$

where P_t = instantaneous value of thrust variation,

F = amplitude of remainder force (corresponding to unit amplitude at the gear) read from the foot of the $M\omega^2\Delta$ column in the frequency table,

Δ = amplitude of the propeller motion (corresponding to unit amplitude at the gear) read from the foot of the Δ column, and

x = amplitude of the gear motion.

It followed that the amplitude of the thrust variation P was given by—

$$P = \sqrt{(xF)^2 + (xC_D\omega\Delta)^2}$$

i.e. $$= x \frac{P}{\sqrt{F^2 + (C_D\omega\Delta)^2}}$$

For the special case of resonance, in which F was zero, the expression reduced to a similar formula to that given on p. 72, the only difference being that the gear amplitude was now given directly.

The above method had been used to calculate the damped resonance curve for the system of Appendix 2 and the results were plotted in Fig. 23, along with the undamped curve. The figures given in column (5) of the last table in Appendix 2, which related to the single degree of freedom system having the same dynamic magnification, were also plotted for purposes of comparison. Although the curves were continued beyond the practically useful range of speed, they illustrated the author's point about the system being more sharply tuned than the corresponding single degree of freedom system.

The damped resonance curve calculated as above was probably still in error due to the neglect of various additional sources of damping such as shaft hysteresis, oil films, couplings, joints etc. which might have, in sum, an appreciable effect when the amplitude at the gear was comparable with that at the propeller. Some published test data for a torsional system indicated that this type of damping accounted for some 30 per cent energy loss per cycle, and one would expect the corresponding figure for an axial system to be of the same order. The calculation of a resonance curve taking account of such additional damping would probably require the damping forces to be introduced into the frequency calculating table, which would then include the phase as well as the amplitude of the displacement and forces. However, it was realized that such considerations were of academic interest only until further test data became available.

Mr. Alexander Kari, M.Sc., wrote that the history of the problem was presented in an exhaustive manner and the author's method of mathematical approach left nothing to be desired, yet the solution put forward lowered the efficiency of the propeller, for a five-bladed screw could not be made as efficient as a three-bladed screw and a reduction in blade width with an increase in the number of blades predisposed the screw towards cavitation.

In the circumstances one might perhaps be excused for seeking alternative ways of remedying the trouble. In this connexion it should be noted that vibration was magnified by resonance between the natural frequency of the system and the impulses arising from the screw. The number of blades of the offending screw was important only in as much as it represented one of a number of propeller characteristics capable of variation, and the change from three to five blades was important only in so far as the author succeeded in altering the resonance by a change in the mass of the screw, at the same time altering the frequency and magnitude of propeller impulses. A change such as was aimed at by an increase in the number of blades could, in his opinion, also be produced by a change in the radial distribution of blade area combined with a modification of pitch distribution such that the outer portion of the blade contributel in a smaller degree to the propeller effort, while the portion of the blade which traversed the belt of least wake vibration was made to contribute most of the effort. In passing the hull the outer portion of the blade traversed a belt of relatively high wake intensity. With the wide tipped blades usual in naval practice and maximum effort being allocated to the tip area, a succession of high and low wake intensities encountered by the referred portion of the blade during a revolution would cause the emission of impulses, the magnitude and frequency of which could be altered by:—

(1) The narrowing down of tip width.

(2) Simultaneous reduction in pitch; the combination of (1) and (2) being carried to the limit dictated by considerations of face and back cavitation at say 0·9 propeller radius.

(3) Compensating increase in blade width and pitch further in from the tip and modification in blade section characteristics.

A reduction in propeller diameter or an increase in propeller rake would obviously be helpful.

One other point called for comment. In Figs. 2 and 4 the author indicated a partial overlap of the slipstream, and as a solution an arrangement of propellers all abreast was suggested, which was dismissed as unacceptable for operational reasons. He was somewhat mystified by the author failing to draw conclusions from the nature of the overlap. It was not the overlap that conduced to the trouble but the fact that the overlap was partial and not complete. A partial overlap such as shown in Figs. 2 and 4 would accentuate inequalities of the wake pattern while with a complete overlap the supply of feed to the inner screws would be more uniformly accelerated. To obtain a more complete overlap of slipstream the shafts of the inner set of screws should be made slightly divergent and those of the shafts of the outer set of screws slightly convergent, the degrees of departure from the fore and aft plane being such as to provide for a non-interference of slipstreams when steaming on a straight course. A slight divergence of shafts of inner screws would also contribute towards better tip clearance thereby improving the feed to the screws with consequent benefits all round.

Mr. F. McAlister (Member) wrote that the paper was a factual record of an important investigation into one aspect of the propulsion problem, an aspect which fortunately did not make its presence felt in the vast majority of ships.

In making these few comments he sought to elaborate the background of the problem rather than question the practical or mathematical investigation which the author had propounded, and with this end in view he would refer to the source of the inner shafting vibration of quadruple screw ships during turning. At other times, and in other places, he had pointed out one important distinction between warships and merchant ships in that it was more desirable in warships to have speed in manœuvring and small tactical diameter whilst more stress was laid on merchant ships to have good course keeping qualities.

It was therefore, more important during the many turning manœuvres of warships at high speed that they should be free of any mechanical difficulties interfering with such manœuvres than in the case of quadruple screw merchant ships which were, more or less, on a constant course, always excepting the zig-zagging imposed by war conditions on such ships.

Vibration would always be induced during turning by impingement of the outer screw race on the inner propeller, as described by the author, but this should be avoided, if at all possible, by other means than having all propellers abreast as in Fig. 3. Apart from the practical difficulties imposed, a most unpleasant beating might occur with all four propellers abreast when the revolutions synchronized.

Vibration could also be induced by inadequate tip clearance, and the propeller tips must always be clear of the frictional belt surrounding the hull form. In vessels of the type treated by the author, the tips should be at least 4 feet clear of the hull to avoid this source of trouble.

In general it was true to say that the smaller the blade number the greater the propeller efficiency, but this statement needed some qualification. The vast majority of propellers worked without cavitation, and in this region the best blade number depended to a great extent on the power, r.p.m., speed relationship. The higher the speed and the less the power and revolutions in proportion thereto, the less the number of blades for maximum efficiency. At some extraordinary combination of the three variables there was probably a sound case, from the efficiency point of view, in having a one-bladed propeller, with, naturally, an offset boss to give proper balance. For fast vessels, such as torpedoes and motor boats, again from the efficiency point of view, and not from the vibration angle, there was a sound case for two-bladed propellers. For all normal fast warships three blades were preferable and for all slower type merchant vessels four blades were desirable.

Further, if the combination of power, speed and rotational speed was such that cavitation intervened then for comparable conditions the lower the blade number the smaller the loss in cavitation. Therefore, it was necessary to be clear sighted with the author in surveying the implications of his investigation. Every ship was a conglomeration of compromises, and every propeller certainly contained one or two compromises. Here, therefore, was the author's problem presented for the compromise. In large quadruple-screw ships freedom from vibration during turning by the adoption of five-bladed propellers must be balanced against whatever loss in efficiency and damage through erosion might be associated with the five blades.

In his own professional work he had not yet been faced with this particular vibration trouble in such an acute form. It might be owing to his preference for four blades with quadruple screw vessels, but he would say that the paper had made him ponder deeply.

Mr. D. Laugharne Thornton, M.A., M.I.Mech.E. (Member) wrote that he would like to refer to an aspect of the matter which did not

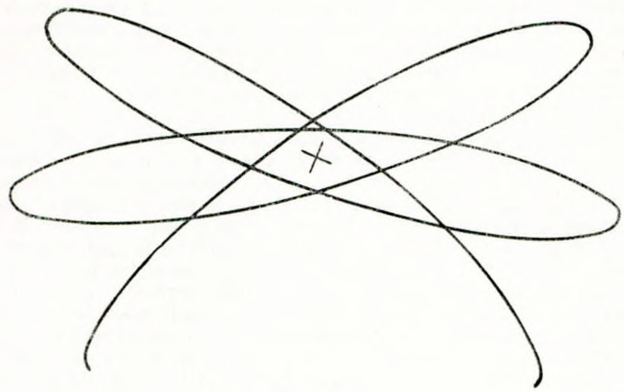


FIG. 24.

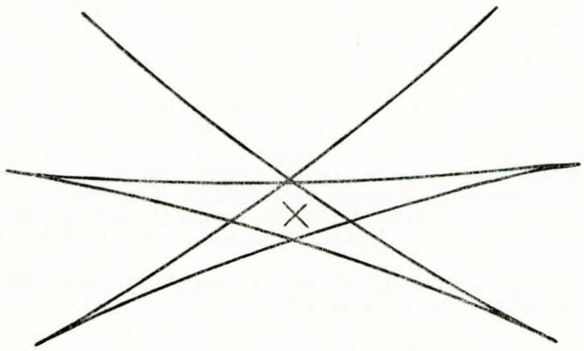


FIG. 25.

appear to have received due attention in the paper.

In essentials the system consisted of a large structure and sources of disturbance that arose from a combination of the effects of rotating masses of considerable size, and forces and couples of hydrodynamic origin. It was hardly possible to investigate the disturbed motion of such a system in terms of less than two degrees of freedom, and the author's attempt to do so probably accounted for his remark that "it was necessary to make an unexpectedly large allowance for flexibility in the thrust block and seat in order to match the observed speed". As regards the thrust block, its position also was of first importance in this connexion. In general the vibratory motion would ultimately depend on the relative values of quantities which presented themselves as ratios between the impressed frequency on the one hand, and on the other coefficients representing the masses, the stiffnesses and the damping. Hence the formulation of the contributory factors would be a matter of some difficulty and uncertainty unless each case was examined in the light of information concerning the proportions and class of ship, and the distribution of loading, static as well as dynamic.

With a ship having four shafts, the initial effects of a turn were most conveniently considered as consisting of two phases. These phases were, in terms of time, divided by the instant at which the inner propeller on the outside of the turn first entered the wake of the outer propeller on the same side of the ship. Therefore the difference between the vibration in the first phase and that in the second was to be attributed mainly to the effect of the wake on the inner propeller. Thus, in the case of a turn to starboard, for instance, the path of a point on the system of the inner port propeller might be represented diagrammatically by Fig. 24 during the first phase, and by Fig. 25 during the second. In these figures the amplitudes were not necessarily equal, and the co-ordinates were generalized co-ordinates. The causes of the difference in shape of these paths had, in his opinion, much to do with the author's observation that in such circumstances "the port inner normally suffers a violent burst of vibration soon after the swing starts, then becomes quiet while the ship is doing a steady circle and then has another violent burst as she straightens out". It was unnecessary to go into details here as discussion of them was to be found elsewhere*. He would like to ask the author whether, in the case of a ship having four shafts, any tests were conducted with a combination of four-bladed and five-bladed propellers on the outer and inner shafts respectively.

* Thornton, D. Laugharne. 1939. "Mechanics Applied to Vibration and Balancing". (Chapman and Hall, London).

Lt.-Com'r G. F. A. Trewby (Visitor) wrote that in the calculations of Appendices Nos. 1, 2 and 3 the author had apparently assumed that the total mass of the shafting was concentrated at various points and that these concentrated masses were connected by shafts having stiffness but no mass. It would be interesting to know why this method had been used as it would appear that the labour of calculation could be reduced considerably if the mass of the shafting were assumed to be uniformly distributed along its length as, in fact, it was. Under these conditions the necessity for dividing the shafting into a large number of sections for calculation purposes disappeared and greater accuracy should be obtained with less labour.

If the mass of the shaft be uniformly distributed along its length, and the vibration was assumed to be simple harmonic, the force and amplitude at any section vary as simple sine and cosine functions respectively of the distance along the shaft. Thus if the force and amplitude at one end of the shaft were known the corresponding force and amplitude at the other end could at once be determined for any frequency of vibration.

As an example the natural frequency calculation of the system shown in Appendix No. 3, Sheet No. 1, Case 1 (p. 87) had been carried out treating the mass of the 390ft. shaft as uniformly distributed along its length, and the results were given below to compare with the first six columns of the authors table at the foot of p. 87.

Section	Concentrated masses, tons	$\frac{M^2}{386}$	Amplitude, in.	$-M^2$ tons	Force, tons
1	30·9	168	1·00	−168	−168
2	7·9	43·1	0·988	−43	−211
E	—	—	—	—	3,952
Inboard end of shafting	Mass uniformly distributed	—	0·988	—	3,741
Outboard end of shafting			9·240	—	1,497
Propeller	30	163·3	9·240	−1,508	−11

Even in a practical case such as that given in Appendix No. 1, p. 80, the whole of the shafting from the thrust block to the propeller could be dealt with in one section without loss of accuracy by the distributed mass method.

The shafting was first reduced to an equivalent length at some standard area. In this case a convenient standard area is 0·851 sq. ft. and, if that portion of the couplings proud of the shaft was assumed to make no contribution to the stiffness of the shaft, then the total equivalent shaft length between thrust block and propeller could be taken as 205ft.

The additional mass of the couplings was allowed for by using an equivalent density which in this case worked out to be 543lb. per cu. ft.

The extra mass of the gunmetal liner (1·2 tons)) which had not been accounted for in the uniform mass distribution, due both to the higher density and lower stiffness of gunmetal compared with steel, was added to the mass of the propeller etc. which thus became 30·07 tons.

The natural frequency calculation for this example was given on p. 96 to compare with the first six columns of the author's table on p. 80.

It would be seen that in both cases the agreement between the two sets of figures was close and it would appear that all the calculations could have been considerably shortened by the use of the distributed mass method.

The author had given a very comprehensive survey of the attempts which had been made to reduce the amplitude of vibration by increasing the frequency of the disturbing force and by increasing the natural frequency of the shafting system. It would be interesting to know if the author had made any calculations or carried out any trials to find out what reduction in amplitude could be expected if a tuned damper were fitted to the system. The mass of the damping

Section	Concentrated masses, tons	M2 386	Amplitude, in.	—M2 tons	Force, tons
1	32·94	320·0	1·000	—320	—320
2	7·44	72·4	0·980	—71	—391
E	—	—	—	—	2,225
Inboard end of shafting	4·2 (6 in No. couplings)	Mass of shafting uniformly distri-buted	0·980	—	1,834
Outboard end of shafting			3·162	—	924
Propeller (+allowance for gunmetal liner, etc.)	30·07	293	3·162	—925	—1

element could be relatively small provided it was tuned to the same frequency as the main system and, theoretically, if the damping were suitablv chosen the resonant peak should be practically eliminated.

Such a damper, consisting of a mass attached to the main system by springs and provided with viscous damping, could be conveniently fitted in a length of shafting and for maximum efficiency should be placed as close to the propeller as possible.

Dr. W. Ker Wilson wrote that in recent years a number of cases had appeared where longitudinal vibration of a shaft system seemed to have been at least a contributory cause of the trouble encountered, but in many of these cases the problem was complicated by the possibility of rather close coupling between longitudinal and other modes. Examples were, coupled torsional and longitudinal vibration of oil engine crankshaft systems, and of installations employing helical gearing.

In the paper, however, the trouble appeared to have been mainly due to resonant longitudinal vibration strongly excited by propeller action during an essential manœuvre, and though the author dealt exclusively with investigations carried out on a number of large naval vessels, the findings might well have considerable significance in other applications.

Many would recall, for instance, reports of vibration trouble on some of the world's largest passenger liners where the symptoms were not unlike those described in the paper and where in some cases the trouble was reduced to tolerate proportions by methods similar to those advocated by the author, such as changing from a three-bladed to a four-bladed propeller.

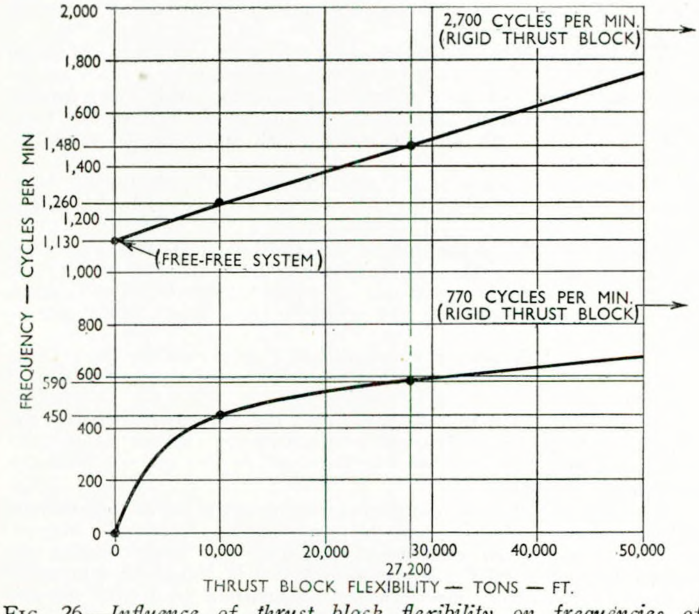


FIG. 26.—*Influence of thrust block flexibility on frequencies of longitudinal vibration*

This suggested the need for an inquiry into the general problem of vibration from propeller action in multi-screw ships, covering such items as propeller location both fore-and-aft and athwartships, with parallel and splayed shafts, the influence of rudder form and location, and the effect of afterbody shape, as well as the question of resonance in the shaft system itself.

Broadly speaking, avoidance of resonance was the chief preoccupation of the vibration specialist whose aim was to produce a system so tuned that no significant critical condition occurred throughout the range of service speeds. Naturally, it was far better—and in most cases easier—to build the correct tuning into a project in the design stage since there were usually several principal masses or elasticities which could be used as adjustable variables for this purpose. In certain industries—notable in the aero-engine industry—examination of the tuning characteristics of each prototype installation was now regarded as a routine design requirement and there was no doubt that this procedure—coupled with a vigorous experimental policy—had resulted in improved reliability and better utilization of structural weight.

In recent years this work had been greatly facilitated by the use of "tuning curves" based on the concepts of effective inertia, dynamic stiffness or mechanical impedance, and dynamic flexibility or mechanical admittance, according to the requirements of the particular problem. These methods lent themselves readily to the solution of problems involving longitudinal vibration and a description of the underlying principles was available.*

Fig. 26 showed the results of an estimate of the influence of thrust block flexibility on the frequencies of the system described in Appendix I of the paper. This diagram was prepared from roughly sketched tuning curves based on the effective inertia parameter, and showed the change of frequency for both the fundamental and first higher modes of longitudinal vibration as the thrust block flexibility was varied from zero (free-free system) to infinity (rigid thrust block).

The corresponding critical speeds with three-, four- and five-bladed propellers were given in Table 3.

Table 3. Critical Speeds, r.p.m.

Number of propeller blades	Thrust Block Flexibility—tons/ft.							
	0 (Free-free)		10,000		27,200 (Appendix I)		Infinity (Rigid)	
	1st	2nd	1st	2nd	1st	2nd	1st	2nd
3	0	380	150	420	200	490	260	900
4	0	280	110	320	150	370	190	670
5	0	230	90	250	120	300	150	540

Assuming that for satisfactory operation the critical speed should be 30 per cent above full power r.p.m. or below one-half of full power r.p.m. the criticals in this example should occur above 300 or below 115 r.p.m. since, according to Appendix 1, full power was obtained at 230 r.p.m.

On the foregoing basis the following conclusions appeared reasonable.

(a) A "free-free" system with a three-bladed propeller placed all criticals well above full power r.p.m., but such a system was not acceptable with a four- or five-bladed propeller because the critical speed was too near full power r.p.m. This arrangement was, of course, of academic interest only.

(b) A system having a thrust block flexibility of 10,000 tons-ft. was satisfactorv with a four-bladed propeller, and might also be satisfactory with a three-bladed propeller depending on whether or not a first critical at 150 r.p.m. was acceptable.

Such a system was not acceptable with a five-bladed propeller because the second critical was too near full power r.p.m.

Assuming the full power thrust of 130 tons quoted in Appendix I, the movement of the thrust block would be of of the order of 0·16in. at full speed and this would have to be accommodated between the thrust block and the gearing.

Incidentally, an increase of thrust bearing flexibility had been used successfully in radial aero-engines to overcome

* Manley, R. G. 1942. "Fundamentals of Vibration Study". (Chapman and Hall); Morris, J. 1947. "The Escalator Method in Engineering Vibration Problems". (Chapman and Hall); Ker Wilson, W. 1940. "Practical Solution of Torsional Vibration Problems". Second Edition, Vol. I. (Chapman and Hall).

trouble due to axial vibration of the large single throw crankshaft used in this type of engine. In that case the additional flexibility was obtained by reducing the thickness of the crankcase web which contained the bearing assembly, thus providing a more flexible diaphragm.

(c) A system having the thrust block flexibility quoted in Appendix I, i.e. 27,200 tons-ft., was satisfactory with a five-bladed propeller, and it might also be acceptable with a four-bladed propeller, depending on whether or not a fundamental critical at 150 r.p.m. could be tolerated.

It was not acceptable with a three-bladed propeller because the fundamental critical occurred too near full power r.p.m.

(d) A system having a rigid thrust block might be acceptable with either a three- or a five-bladed propeller. The acceptability of a three-bladed propeller depended on whether the estimated position of the fundamental critical, i.e., 260 r.p.m., could be sufficiently closely realized in practice, bearing in mind that some flexilibity would inevitably be present and this would, of course, bring the critical nearer to full power r.p.m. In the case of a five-bladed propeller much depended on whether a fundamental critical at 150 r.p.m. was permissable or not.

In that case, however, the effect of unpredictable flexibility would be to lower the fundamental critical speed and for this reason the use of a five-bladed propeller with a "rigid" thrust block probably represented the safest choice.

A four-bladed propeller was not acceptable with a "rigid" thrust block because the fundamental critical was too near full power r.p.m.

The foregoing "slide-rule" analysis should be regarded as purely tentative, the main purpose in presenting it was to demonstrate the type of useful information which could be gleaned from a few roughly sketched tuning curves.

The author's remarks on the vital question of instrumentation indicated the importance of care in selecting and operating vibration measuring equipment to avoid spurious results leading to false diagnosis. For this reason some of the leading manufacturing firms, whose research programmes included an appreciable amount of dynamic test work, had found it essential to establish instrument maintenance sections whose duties included the modification of instruments and the manufacture of special equipment to meet the needs of particular investigations.

The frequency range below about 600 cycles per min. was not very well covered by standard commercially available equipment, while it was probably true to say that for frequencies below about 300 cycles per min. mechanical rather than electrical instruments were to be preferred. For example, the Geiger mechanical vibrograph was probably one of the most versatile instruments available for measurements in these lower frequency ranges.

The standard instrument was suitable for the range 500 to 4,000 cycles per min., and the range could be extended both upwards and downwards by the use of easily fitted adaptors. In addition, it could be quickly concerted for use as a torsiograph or a strain recorder. It was to be hoped that this instrument—which was of German origin—or an equivalent would continue to be available.

With regard to the author's remarks on the unsatisfactory low-frequency response of the linear pick-up units used with Sperry-M.I.T. equipment it would be interesting to know which type of pick-up was in question, because according to the maker's lists the larger unit should have a range from 180 to 18,000 cycles per min. Incidentally the author referred to these pick-ups as of the inductive type whereas the usual M.I.T. linear units were of the moving-coil, self-excited, electro-magnetic type which require dynamic calibration. Inductive pick-ups, as usually understood, required independent excitation and could therefore be calibrated statically, though this very fact rendered them more susceptible to drift under static strain.

With regard to resistance strain gauges, there should be no great difficulty in recording alternating stresses of the order of $\pm 1{,}500$lb. per sq. in.

A single resistance strain gauge was capable of measuring a stress of ± 600lb. per sq. in. in steel with an accuracy of ± 5 per cent, and the sensitivity could be considerably increased by using a multiplicity of gauges.

Turning now to the questions of entrained water allowance, and propeller damping torque it was interesting to note that the entrained water allowance was evaluated on a basis of developed blade area. This was perhaps rather more rational than the quite arbitrary practice of basing this quantity on the mass polar moment of inertia of the propeller in the case of torsional vibration studies. The propeller damping torque, however, was evaluated on a basis of propeller diameter whereas in torsional vibration studies it was deduced from model propeller experiments using the torque-speed relationship.

It would therefore appear to be more rational to evaluate the damping of longitudinal vibrations on the basis of the thrust-speed relationship.

It would be useful to have the author's comments on these points.

Finally, it was noted that in the summary on p. 78 there was a suggestion that for very long shafts it might be possible that friction at the sliding couplings might hold the gearwheel stationary in cases where the thrust block was located right aft. It would, however, appear to be preferable to tune the system so that the gearwheel amplitudes were negligible, thus enabling the gearwheel to take up a normal running position free from external restraint.

Mr. H. G. Yates wrote that the author remarked that although the use of multi-bladed propellers would probably eliminate the troubles experienced in the installations described, there was a desire to explore other methods since the multi-bladed propeller was not ideal from other considerations. The author mentioned a suggestion due to Dr. Forsyth that it might be advantageous to fit a flexible bellows element just aft of the thrust block in order to reduce the natural frequency of the system and bring it to a value so low that propeller excitation wold be unable to induce serious vibration. To obtain a rough estimate of the possibilities of this method it was not necessary to work out an example in complete detail, and a rough indication might be arrived at by considering a simplified equivalent system having the same natural frequency as the actual case.

In a system comprising a propeller at the end of a uniform heavy shaft whose other end was fixed, the fundamental natural frequency was the same as that of a simple system comprising a spring of negligible mass and having a stiffness equal to that of the shaft, loaded at its free end by a mass equal to that of the propeller plus a fraction of the shaft mass, which varied from $\frac{1}{3}$ for a light shaft to $4/\pi^2$ for a heavy shaft without end loading. In the example of Appendix I the propeller mass including entrained water was approximately 28 tons and that of the shaft about 46 tons. The total flexibility was 0·00196in. per ton and a single mass of about 54 tons would give the correct natural frequency. The shaft was thus equivalent to about 26 tons, i.e. considerably more than $\frac{1}{3}$ of its actual weight but this was not unexpected since some of the flexibility was between thust block and "earth" so that there was no point of the shaft which had zero velocity in the vibrating condition, and furthermore the thrust block and main gear wheel took part in the motion.

If now, additional flexibility was introduced to reduce the resonant frequency to half its present value, this would, in the simple system, require the total flexibility to be multiplied by four. As the author pointed out, a still greater proportion of the shaft weight would take part in the vibration, so that the flexibility required would be of the order of 4·7 times its present value. This meant that the flexibility of the added bellows element required to be in the region of 0·007in. per ton. Under the full power thrust of 130 tons this would deflect 0·91in. and must therefore be capable of containing a stored energy of about 59in.-tons without experiencing unduly high stresses.

Assuming that the material of the bellows element was stressed in double cantilever fashion, and that the maximum tensile stress was not to exceed say 15 tons per sq. in. then the weight of the flexible parts of the bellows would require to be about 8 tons. This might be reduced somewhat by very careful shaping of the section to render the stress more equally distributed, but the additional weight of flanges, bolts etc. would act in the reverse direction and the bellows would be both heavy and bulky.

An alternative method would be the addition of virtual mass by fitting a second thrust block as far aft as possible and encircling it with a floating thrust housing. This would be attached by links to one or more masses carried on stout levers pivotally attached to the ship's structure, the point of attachment of the links being much closer to the pivot than was the centre of gravity of the added masses. The system could then float by the amount necessary to take up compression of the main shaft and also thermal expansions, but if the shaft vibrated longitudinally the masses would be forced to oscillate about their neutral position. If the lever ratio was say 5/1 the effective added mass was 25 times the actual mass of the weight and the natural frequency could be appreciably lowered by this means. The forces to be carried by this additional thrust block would of course be very large and he did not suggest that the scheme was any more practical than the fitting of a bellows piece.

It was worth examining for a moment the relative performance of these two alternative methods. For this purpose he found the following method more convenient than the one due to Den Hartog. The velocity of a system with one degree of freedom might be calculated simply for the resonant condition by dividing the exciting force by the damping constant. At other frequencies the velocity induced would be shifted in phase by an angle ϕ and reduced in amount by the factor cos ϕ where tan ϕ was given by $Q\ (\lambda - \frac{1}{\lambda})$, Q being the dynamic magnifier at resonance and λ the frequency ratio as used in the paper. The factor Q was identical with D in the Den Hartog notation and was more commonly used nowadays. If the amplitude of response was required at reasonance or any other frequency, this might be obtained by dividing the velocity by ω.

If the natural frequency of the system was now reduced to half the existing value, the velocity of vibration at the new resonant condition would be the same whether the frequency change was brought about by the addition of flexibility or of virtual mass, and consequently the amplitudes would also be the same. The load on the main thrust block would however be much less in the system with added flexibility, than that in the system with increased virtual mass.

As a more practical alternative to either of these proposals he would suggest a vibration damper. This could take the form of one or more dashpots coupled in the same way as described for the addition of virtual mass, but a more convenient arrangement was shown more or less diagrammatically in Fig. 27. It comprised a length of main shaft incorporating a number of projecting collars (four were shown in the figure) and surrounded by a housing which was split on the horizontal centre line and provided with a flanged joint. The housing contained a number of compartments encircling the shaft collars and was arranged with fine radial clerances of the order of 0·015in. The two ends of the housing carried bearings which supported it on the shaft, and axial motion was prevented by suitable links (not shown), which at the same time permitted slight lateral movements if required. A torsion bar would also be fitted to restrain the housing from rotation. Sufficient axial clearance was left in each compartment to accommodate the necessary shaft movement due to load or temperature. The damper was fitted as far aft as convenient. In operation the system was maintained full of oil so that axial movement of the shaft axis caused pressure differences to be built up across the collars due to viscous flow of oil in the narrow annular spaces between adjacent peripheral surfaces. The first and last compartments were provided with a large recirculating connexion to prevent the build up of pressure near the bearings in order to avoid unnecessary loss of oil. This of course rendered inoperative the outer faces of first and last collars. In addition the forward end of each inner compartment was provided with a small vent hole at the highest point to prevent air locks. Calculations showed that such a system would provide a damping constant more than 10 times that derived from the propeller.

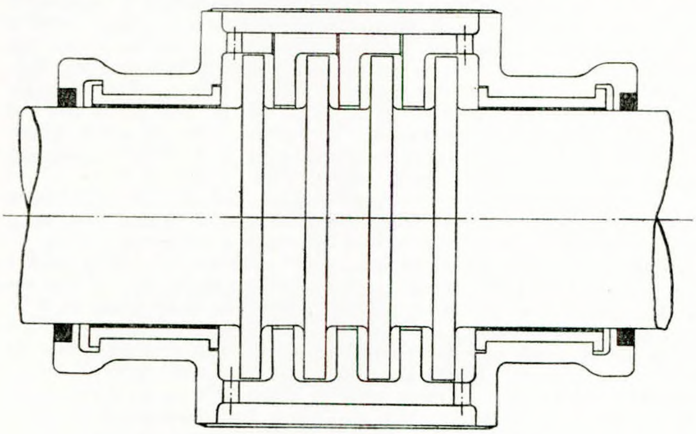


FIG. 27.—*One form of vibration damper*

There was no risk of applying unduly heavy loads to the housing of the damper if full power was applied rapidly, since a damping constant of the order of 10 tons per in. per sec. would permit working up to full power in about half a second without overloading the axial ties and the turbine operation could not be so rapid as this. The loads to be handled by these ties under vibratory conditions could not exceed the propeller exciting forces. As indicated in the first figure the diameter over the shaft collars was less than that of the coupling flanges, so that the arrangement was quite compact. It was suitable for fitting in an existing ship, and in the case of new designs it would leave the designer free to determine the shaft characteristics from other considerations and ignore longitudinal resonances.

The author did not state specifically how the propeller damping constant of 1·062 in Appendix I was arrived at but it appeared to be an empirical deduction from the measured vibration figures. He would suggest however that at least the greater part of it was deducible from theoretical considerations. To at least a first approximation the thrust produced by a propeller was proportional to the product of two velocities, one being the mean velocity of water through the propeller disk and proportional to the mass flow taking part in the propeller action, and the other being a measure of the increase of momentum given to each affected particle of water and corresponding to the true slip velocity. Therefore it was possible to write $P = kV_AVs$.

Under steady conditions and assuming that the ship resistance was proportional to the square of the speed, this result was compatible with the cube law if a fixed ratio was maintained between Vs and V_A. If now the ship was steaming steadily but the propeller was vibrating in a fore and aft direction, the effective speed of slip would vary above and below the mean by the amount of the propeller vibration velocity. The rate of change of thrust with respect to Vs was therefore the damping constant applicable to the vibration calculation, and it would be seen that this was given by kV_A or by P/Vs. If the velocity was expressed in inches per second and the thrust in tons, then the units of the damping constant were tons per in. per sec. as used in the paper.

In the absence of exact data of propeller and hull form he had made assumptions of speed of advance and percentage true slip which appeared to be reasonable for the centre shaft of a three-shaft installation similar to that analysed in Appendix I, at the critical speed. These gave a damping constant 2 per cent below the figure of 1·062 used in the paper. This very close agreement was no doubt largely fortuitous and it would be unwise to draw precise conclusions from it, but it did suggest that this method of predicting damping for axial propeller vibration gave results which were at least of the correct order of magnitude.

For ships of the same hull form, operating at about the same speed with the same percentage of true slip, the damping constant would be proportional to the thrust and hence approximately proportional to the blade area of the propeller if the proportions of the latter remain the same. This agreed with the author's practice of expressing the propeller damping constant as a percentage of the developed blade area. The adoption of this practice however suggested that the author visualized the damping as arising from some form of fluid friction, but it was difficult to see how this could give anything like the amount of damping actually present.

If the theory outlined above was approximately correct, it followed that the damping constant at reduced speed and revolutions must fall more or less proportionally to the revolutions, since P varied as the square of the revolutions. If so, the fitting of a propeller having a greater number of blades but with the same effective mass and operating at the same slip should excite the same critical frequency (at lower revolutions) with a more sharply tuned resonance. This followed from the fact that the dynamic magnifier was $m\omega/C_D$ and both m and ω remained constant. The trial results in Fig. 10 confirmed this, with in fact an even greater increase of Q than would be anticipated from an increase in number of blades from 3 to 4. Fig. 8 re-plotted to a logarithmic scale, showed about the same sharpness of resonance with five blades as with three blades but a sharp peak might have been present at about 125 r.p.m.

The variation of damping directly as the speed was of course only an approximate deduction from the suggested theory. In fast vessels there would be a tendency for the percentage slip to fall with reduction in revolutions in the upper ranges of power, possibly associated also with reduction in the wake factor. Both of these would tend to reduce the fall in propeller damping as speed was reduced.

The Author's Reply to the Discussion

Mr. Rigby, in reply, said that Captain Given's remarks had led him to realize that he might have been more remiss in what he had said in introducing the paper than in the paper itself, and might have made it sound as though the department had been negligent from 1937 to 1943. He would like to make it clear that he had no such thought in his mind. In fact, other ships of the same class —including, he believed, the Queen Elizabeth—when re-engined were fitted with over-size stiffer thrust blocks, which certainly reduced the trouble, though the easing procedure was still applied. Also in later new construction every effort was made to get the largest possible fore-and-aft separation between the inner and wing propellers. It was hoped that that increased separation would reduce the strength of the slipstream impinging on the inner screw and reduce the interaction effects when turning. Unfortunately, those hopes were not realized. The slipstream seemed to maintain its strength for a very considerable distance. The separation in Warspite was some 24ft., while in certain later ships it was more of the order of 50ft.; but even so the slipstream was still very strong. He thought, however, that the steps taken were quite reasonable. The fact that the trouble occurred at full power in Warspite was unfortunate, because it did not give a real lead to the effect of resonance in building up amplitudes, a fact which he was fortunately led to consider by an entirely different department coming to ask what caused the critical speed in Furious.

With regard to the cyclic thrust variation, before quoting any figures he ought to make it clear that in carrying out the calculations from trial results the firm information available consisted of the critical speed, the amplitude and the shape of the resonance curve. That put one in the position of having only three equations to solve, with four unknowns. From the known weights and stiffnesses it was possible to work out the theoretical critical with no entrained water and no flexibility in the thrust block, but to get to the actual critical speed, which was much lower than that, it was necessary to allow for both entrained water and flexibility. The difficulty lay in assessing the relative importance of each. The value which he had obtained for entrained water, and on which he had standardized, came from using the static deflexion test on the thrust block to provide the fourth equation, and it influenced all the other values which followed; everything was based on the figure of 0·0481 tons per sq. ft. of developed surface for the entrained water. With that consideration, from which nothing else must be divorced, the thrust variations deduced were about 3 per cent with steady thrust for a shaft in A-brackets, 4 per cent with bossings, and 5 per cent with the centre shaft, but this centre shaft was cut away below the centre line to some extent so that it was midway between a stern post and a bossing. Probably a true stern post would give appreciably more than this. Before there was any magnification, therefore, there was this variation of some 3 per cent.

He could not say that he was really any happier than Captain Given about using the flexible couplings, and therefore the turbine, as a means of holding the gearwheel still if the thrust block was placed right aft. The ideal, of course, would be to have a flexible coupling providing axial freedom immediately forward of the thrust block in such a situation; but whether there was such a thing as a flexible coupling which would really provide axial freedom he was doubtful; at any rate he had not yet discovered one.

He himself had no statistics on the question of wear in stern bushes. He felt sure however that excessive longitudinal amplitudes did contribute to wear. The movement at the propeller when turning was of the order of $\frac{3}{8}$in. total—i.e. $\pm\frac{3}{16}$in.

He agreed very heartily with Captain Given that every case must be treated on its merits, and that it was not possible to make generalizations. He did not want to suggest that the trouble was one which was necessarily of very great importance in every ship; it depended entirely on the individual case. If there was a longitudinal critical at dead-on full power in a passenger ship he thought that it would be just as important as in a warship, because the increase of alternating forces would undoubtedly cause vibration somewhere; but probably in many cases in a passenger ship it did not matter if it came at a low power. It did not always matter in a warship, nor did flexible couplings always seem to object to it; they seemed to get used to it (if he might use such an expression) after a while. Commander Lane, who had Warspite for a considerable period, told him that when they were bombed they sometimes forgot the easing procedure, but the couplings did not weld themselves together as they had on the trials. Partly, presumably, it was a question of getting them run in. They had, of course, to build them up $\frac{1}{4}$in. by welding every now and again, so that apparently they were still wearing.

He had no clear explanation thought out for the phenomenon observed by Commander Baker, where the amplitude increased violently on taking the rudder off. He did not personally attend the trials of Formidable, but in quadruple-screw ships there was the same tendency that was recorded for Formidable; on turning there was a violent peak on entering the turn, followed by a comparatively quiet period, and then another violent peak as the ship came out of the turn. His own explanation had been that the worst vibration occurred at the condition when the slipstream half covered the disk of the inner propeller. With triple-screw ships the conditions were rather different, because it could not completely swing across, but in a quadruple-screw ship the slipstream of the wing shaft could completely cover the disk of the inner propeller. The inner shaft on the outside of the turn speeded up to about 110 per cent of normal full power revolutions when in the middle of a circle, and for vibration purposes it was necessary to allow for that, because with the usual form of integrating tachometer it was not possible to control the increase of revolutions, which was faster than the integrating action.

A jerky motion about a datum was quite evident in the old records from Warspite, and he concluded that it was when the thrust collar actually jumped off the pad that one had that unidirectional rather than sinusoidal motion. He would like to ask whether it was only when turning and with large amplitudes that that was observed.

Commander Baker replied in the negative; it was always observed.

Mr. Rigby mentioned that all his records were taken off the gearwheel and thrust block rather than from the pinions.

Commander Baker remarked that it was not discernible unless the cathode ray tube was used, when a definite flat could be seen.

Mr. Rigby said he had seen it on photographic records from Warspite, and it was commented on by the A.R.L. in their reports.

He would like to ask Mr. Richmond to reply to Mr. Craig's question about the material of couplings.

Mr. G. W. Richmond said that, before doing so, he would like to take the opportunity of saying something about the effect on the ship's personnel of these severe shaft vibrations. He personally would describe the conditions on board such ships when running near the critical speed as nothing less than diabolical.

He had had an opportunity during the war of spending a few weeks in one of the triple-screw aircraft carriers, referred to in the paper, in connexion with the work of replacement of a set of main gearing and subsequent acceptance trials at sea. On completion of the work a programme of runs at speeds of 150, 170, 190 and 210 r..p.m. arranged to bed in the gears prior to the acceptance trial and to enable records of noise and vibration to be obtained over this speed range.

The change from 170 to 190 r.p.m. was timed to be made at 11 p.m. but when he experienced the severity of the general hull vibration at the higher speed he made an urgent request for the revolutions to be reduced to 150 r.p.m. for the night. It was impossible to sleep, to think or to carry out any work efficiently in such a turmoil, and the impression obtained in the engine room at the higher speed was that the gears were being subjected to very severe dynamic loading.

With regard to the question on the flexible couplings he would suggest that if the coupling teeth were subject to severe wear and scuffing or seizing of the driving faces it would be reasonable to expect that bettter service would be obtained if the tooth surfaces were hardened. There was in fact some evidence to that effect.

Interrogation of German technicians led to the conclusion that the German warships had experienced little or no trouble with their claw tooth type couplings which were of normal design and materials but with the tooth surfaces flame hardened. He had no information, however, as to whether the German vessels had suffered from severe axial vibration of their propeller shafting.

In general he would recommend that where trouble with these couplings was experienced hardening of the tooth surfaces would provide a palliative. Lubrication was also a matter of some importance in determining whether wear or seizure would occur. Alteration to the design of coupling to provide a generous flow of oil for cooling and lubrication of the coupling teeth was certainly worth considering and the gear tooth type possessed advantages over the claw type in this respect.

Mr. Rigby wrote in reply that the amplitudes quoted by Mr. Archer, $\mp 0{\cdot}050$ and $\mp 0{\cdot}025$in., occurred at the main gearwheel, the corresponding motions of the propeller would have been approximately three times as great.

He was interested to see that the torque variation in a twin screw ship with bossings was of the same order as the thrust variation he had quoted, it did seem reasonable that narrow blades should give a greater variation.

He agreed that thirteen sections were an unnecessarily large number when the thrust block was in the forward position, but had found it convenient to standardize on that in order to have a reasonable number of sections between propeller and thrust block when the latter was moved aft.

The thrust block deflexion test was carried out in such a way as to eliminate the effect of the quite different seating and the results plotted represented the forward deflexions relative to the base flange of the block. The block was then used, during the sea trials, as a meter for alternating thrust, by measuring vibration amplitudes at the same points and relating the differences in amplitude of forward and aft gland faces to the differences between their static deflexions as described on p. 73.

It appeared from Mr. Archer's remarks that Lloyd's Register had actually measured the stiffness of thrust blocks and seats in a ship and it would be interesting to have details of the methods employed. He had himself been unable to see anything solid enough to push against or to use as a datam and had concluded that the use of a vibration exciter in conjunction with a known added weight would be the only possible method.

The question of propeller damping had been dealt with separately.

Replying to Dr. Brown, he thought that he had perhaps been rather too dogmatic in the conclusions expressed in the section "Precise Location of Thrust Block" and that in a destroyer for instance there was certainly no objection to locating the thrust block in a locked train gearcase. In big ships, however, it became more difficult to provide adequate stiffness and the passage in the paper by Warren,

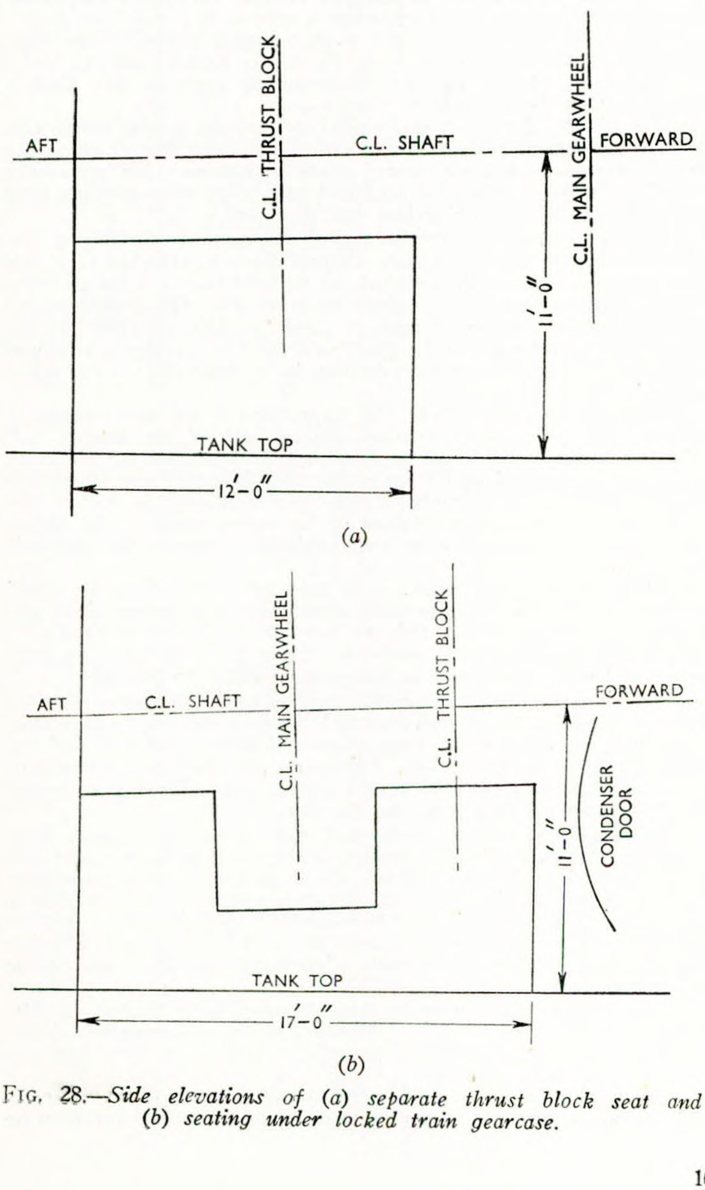


FIG. 28.—*Side elevations of (a) separate thrust block seat and (b) seating under locked train gearcase.*

to which reference was made, indicated that actual experience had not been altogether happy.

He found that Dr. Brown's statement about the reduced diameter of main gearwheels was not valid for naval designs because advantage had been taken of the double reduction to reduce propeller and increase turbine revolutions and also because the diameter of the main wheel was governed to a large extent by the centre distance required between high pressure and low pressure turbines. The result was that the reduction in diameter only amounted to two or three inches at the most and was not significant.

As to the relative stiffness of the seatings Fig. 28 showed side elevations of (a) a typical separate thrust block seat and (b) the girders under the centreline of a big locked train gearcase. Admittedly the girders at the sides of the gearcase would not be cut out for the sump but he felt some doubt as to the ability of the front wall of the gearcase to transfer thrust to the sides.

He was indebted to Dr. Brown for the method of calculating a damped resonance curve. This cleared up an unsatisfactory piece of work and would be incorporated in future calculations.

He agreed that there must be a considerable amount of damping in the system apart from that due to the propeller. This became much more important when the thrust block was moved aft and it was probable that the gearwheel ampltitudes calculated in Appendix 2 were for this reason appreciably too great. If such damping consisted mainly of "dry friction" in the flexible couplings the gearwheel would not in such an arrangement follow a true resonance curve but would remain stationary until the forces exceeded the friction and then move in a jerky manner. Experimental work was required to clear up this point.

Replying to Mr. Kari, he was not certain that in the absence of cavitation a five-bladed propeller need necessarily be less efficient than one with three blades. A part of the efficiency deduction for high developed surface ratio might well be due to the great width of the individual blades and if this were so the five narrower blades might show an advantage.

While it might be possible to reduce the thrust variation by a re-distribution of work and blade surface it seemed doubtful whether very much could be done in this way with Naval propellers of high developed surface ratio because there was little space in the disk which was not already occupied. A reduction in thrust variation on straight course could certainly be obtained by increasing the tip clearance, but none of these methods would alter the frequency of the impulses which would remain fundamentally the product of number of blades and r.p.m. and he doubted their adequacy in dealing with the more serious interaction effect when turning.

With reference to this he agreed that it was the partial nature of the overlap that caused the trouble, but pointed out that present arrangements only gave a small clearance between slipstreams on straight course—in some vessels they did actually overlap. He did not see how it would be possible to arrange for (a) non-interference on straight course and (b) complete overlap for all angles of turning in the same shaft arrangement.

He agreed with Mr. McAlister that the ability to carry out turning manœuvres at high speed without mechanical difficulties was of much greater importance in warships than in merchant ships. At the same time experience in the twin screw aircraft carriers had shown that unacceptable vibration of the structure could be caused, even on straight course, by the increased alternating thrust arising from longitudinal vibration and he felt that in any passenger ship it would be worth while trying to avoid having a critical in the usual operating range.

He fully concurred with Mr. McAlister's desire to have a tip clearance of at least four feet in ships of the type considered and thought that this would materially reduce lateral and vertical vibration in the after part of the ship as well as longitudinal vibration of the shaft.

In reply to Mr. Thornton he wrote that he would like to make it clear that the use of the single degree of freedom resonance curves was confined to the limited cases and regions where they could be proved to fit without undue error. In general by using thirteen masses and thirteen springs he was, in fact, providing sufficient degrees of freedom.

The "unexpectedly large allowance for flexibility" was rather physically than mathematically unexpected; it was surprising to find that a structure which looked so rigid was deflecting as much as 0·050in. under full power thrust.

He agreed that there were two distinct phases in a turn divided by the instant at which the inner propeller entered the slipstream from the wing shaft.

He had not had occasion to try a combination of four- and five-bladed propellers on a four-shafted ship because the wing shafts had not suffered from longitudinal vibration. The possibility that the

DATE 3/48. SHAFT CENTRE. H.M.S. FORMIDABLE.

NATURAL FREQUENCY CRITICAL SPEED AND AMPLITUDE.

NATURAL FREQUENCY WITH BELLOWS FITTED AFT OF T.B.

R.P.M.$=97{\cdot}5$. Blades$=3$. $\omega=30{\cdot}65$. $\omega^2=939{\cdot}5$.

Section	Mass, tons/386	$M\omega^2$	Δ, in.	$M\omega^2\Delta$, tons	$\Sigma M\omega^2\Delta$, tons	C tons per in.	Σ/C in.
1	0·0852	80·0	1·000	80	80	$16{\cdot}05\times10^3$	0·005
2	0·0193	18·1	·995	18	98	—	
E	—	—	—	—	−2,259	$2{\cdot}27\times10^3$	−0·995
2	—	—	·995	—	−2,161	$0{\cdot}22775\times10^3$	−9·489
3	0·0118	11·1	10·484	116	−2,045	$6{\cdot}7\times10^3$	−0·305
4	0·0118	11·1	10·789	120	−1,925	,,	−0·287
5	0·0098	9·2	11·076	102	−1,823	,,	−0·272
6	0·0118	11·1	11·348	126	−1,697	,,	−0·253
7	0·0118	11·1	11·601	129	−1,568	,,	−0·234
8	0·0098	9·2	11·835	109	−1,459	,,	−0·218
9	0·0118	11·1	12·053	134	−1,325	,,	−0·198
10	0·0118	11·1	12·251	136	−1,189	$6{\cdot}7\times10^3$	−0·177
11	0·0101	9·5	12·428	118	−1,071	$7{\cdot}23\times10^3$	−0·148
12	0·0170	16·0	12·576	201	−870	$14{\cdot}72\times10^3$	−0·059
13	0·0733	68·8	12·635	869	*−1		

NOTES. *This remainder = zero at critical.
Remainder is negative below first natural frequency.
,, ,, positive between first and second natural frequency.
,, ,, negative above second natural frequency.

AMPLITUDE AT CRITICAL ON STRAIGHT COURSE.
Assume thrust variation$=4{\cdot}57$ per cent of 130 ton F.P. thrust and varies as (r.p.m.)^2. Full power r.p.m.$=230$.
Force P at 97·5 r.p.m.$=0{\cdot}0457\times130\times(97{\cdot}5/230)^2=\mp1{\cdot}07$ tons.
Assume propeller damping constant $C_D=1{\cdot}062$ tons per in. per sec.
Propeller motion$=P/C_D\times\omega=1{\cdot}07/1{\cdot}062\times30{\cdot}65=\mp0{\cdot}0328$ in.
Gear motion$=$Propeller motion$/12{\cdot}635=0{\cdot}0328/12{\cdot}635=\mp0{\cdot}0026$ in.
Force at thrust block$=2259\times0{\cdot}0026=\mp5{\cdot}87$ ton.

FIG. 29.—*A check of calculation showing a bellows of flexibility of 0·004316lb. per ton was required to move the critical from 195 to 97·5 r.p.m.*

number of blades on the wing propeller might influence the vibration of the inner had been considered but early trials had suggested that the transfer of vibration energy was only small. This conclusion had since been confirmed by results in Illustrious. He thought, however, that the combination of four-bladed wing and five-bladed inner propellers would in most cases result in an exceptionally smooth running ship.

He wished to thank Lt.-Com'r Trewby for his suggestion that it would save labour to treat the shaft as a distributed mass, a method he had not previously considered. The necessary equations could be derived from Den Hartog* and were:—

$$\Delta = a\cos\left[L\sqrt{\frac{\mu\omega^2}{AE}}\right]-\frac{t}{\sqrt{AE\mu\omega^2}}\sin\left[L\sqrt{\frac{\mu\omega^2}{AE}}\right]$$

$$T = t\cos\left[L\sqrt{\frac{\mu\omega^2}{AE}}\right]+a\sqrt{AE\mu\omega^2}\,.\,\sin\left[L\sqrt{\frac{\mu\omega^2}{AE}}\right]$$

where Δ = value of Δ at propeller, i.e. at section 13, in.
T = value of $\Sigma M\omega^2\Delta$ in line 14 of standard table (i.e. value opposite section 12), tons.
a = value of Δ at thrust collar (i.e. value opposite section 2), in.
t = value of $\Sigma M\omega^2\Delta$ in line 4 of standard table, tons. (See standard table on p. 014).
and L = length from thrust collar to propeller, in.
μ = shaft mass per unit length, tons/386, per in.
ω = applied frequency, radians/sec.
A = cross-section area of shaft, sq. in.
E = Youngs modulus, tons per sq. in.

In a practical application L, μ, A might be "equivalent" to cover for couplings, change of section, etc.

The signs in the equations had been adjusted to suit the sign conventions used in the tabulation method, e.g. if t was negative (as it usually was) and the sine and cosine were both positive, then the two terms of the first equation added together. He noted that Lt.-Com'r Trewby had changed the sign of $M\omega^2$ in the table instead of changing the equation.

* Den Hartog, J. P. 1947. "Mechanical Vibrations". (McGraw-Hill Book Company, New York and London), p. 173, equation 93a.

He had checked the equations against the three tabular calculations given for Case 1 of Appendix No. 3 and found them correct in each case. The saving of time was however not so great as might have been expected; although he had evaluated in advance the parts which remained constant for the particular shaft arrangement, i.e. $L\sqrt{\frac{\mu}{AE}}$ and $\sqrt{AE\mu}$ it had still taken him some fifteen minutes to calculate Δ and T for one frequency, without checking.

Checking of arithmetic was certainly necessary when using this method, whereas in the tabular method a visual inspection for discontinuities in the rate of change of Δ and Σ/C was adequate, and taking this into account it would evidently take at least twenty to twenty-five minutes to do the complete table, against the normal time of thirty minutes for the thirteen sections.

Even so, any saving was welcome and he was setting out to produce a standard calculation sheet by which to proceed from line 4 to line 14 via the equations without the need for any more thought than was required in the standard table. To do it without thought was essential if time was to be saved, and that meant, amongst other things, choosing units and operations so that only three, or at most four, significant figures lay to right or left of the decimal point.

Modified calculation sheets would, if desired, be available for publication at a later date.

With reference to tuned dampers, he had at one time made some calculations regarding the possibility of fitting one inside the aftermost section of intermediate shafting but unfortunately could not now put his hand on them. The results had not, if he remembered correctly, been altogether encouraging. It had appeared that a suitable weight for the piston would have required it to be some 16ft. long and 16in. diameter, and the stresses in the spring giving it longitudinal attachment to the shaft would necessarily have been high.

Damping would have been provided by filling the section of shaft with oil, and the adjustment of the leakage space to a rather critical optimum offered some difficulty.

The project was not, however, pursued very far because changing the number of propeller blades provided a simpler solution.

Replying to Dr. Ker Wilson, he wrote that coupling between longitudinal and torsional vibration had not been observed in this case, at any rate it did not show up on torsional vibration records taken from the pinions. Some coupling might be expected to arise through the torque-thrust relationship of the propeller, on the other hand the natural longitudinal frequency of the system was nearly four times as great as the one node torsional frequency.

He was not familiar with the methods of calculation described and was therefore unable to comment as to whether they were appreciably simpler than those given in the appendices but was grateful for the additional references given. The curves showed close agreement with his own results, but he thought that in drawing conclusions from them Dr. Ker Wilson had perhaps laid too much emphasis on the position of the critical speed and too little upon the amplitude of vibration. For instance, while fitting a more flexible thrust block would move the critical to a lower power it would at the same time increase the amplitude of vibration at the gearwheel. Also, where the critical lay at a low power the question of whether a thrust reversal would occur when turning was of more importance than its actual position.

With reference to instruments, the Sperry M.I.T. pick-up in question was of the small type; no doubt the larger unit would be satisfactory. He regretted the rather loose description of these pick-ups as "inductive", they were in fact "moving coil, self excited, electromagnetic".

The difficulty about using strain gauges was that to secure reasonably steady conditions it was necessary to take the readings on straight course and this meant that even with the severe condition at the critical speed with three blades on a centre shaft the alternating stress was only some $\mp$ 700lb. per sq. in. With A-brackets, or off the critical, it would be considerably less, and the accuracy of the strain gauges would be further reduced by the unavoidable use of slip rings of rather large diameter. However he felt that it was worth trying at the next opportunity.

The question of propeller damping had been dealt with separately.

He had checked Mr. Yates's calculation by the tabulation method and found that to move the critical from 195 to 97·5 r.p.m. would require the insertion of a bellows with a flexibility of only 0·004316in. per ton.

Fig. 29 showed a check of this calculation; it would be noted, if it were compared with the table on p. 014 that the stiffness between sections 2 and 3, where the bellows was fitted, had been reduced by the latter from $13{\cdot}4\times10^3$ to $0{\cdot}22775\times10^2$ tons per in. As a result the ratio of propeller amplitude to gear amplitude had increased from 3·126 to 12·635 and the gear amplitude at the critical had been reduced from $\mp$0·021in. to $\mp$0·0026in.

He agreed however that to provide even 0·004in. per ton flexibility might be difficult, but suggested that it was perhaps over ambitious to seek so great a change. He thought that it would be adequate if the critical were moved to 120 r.p.m., which would require a bellows flexibility of only 0·0024in. per ton and would give a gear amplitude on straight course of $\mp$0·0049in. although it was true that there would then be a risk of a slight thrust reversal when turning.

The use of various types of detuners and dampers had received consideration from time to time but had not been followed up because fitting a four- or five-bladed propeller had provided a simpler solution. If, however, the increased number of blades was found to cause any serious reduction in efficiency the straightforward hydrualic damper suggested by Mr. Yates would be a very convenient alternative.

A rough calculation suggested that radiation from the casing would provide adequate cooling and it should therefore be possible to make the damper self-contained with its own oil sump on the lines of the usual air cooled self-lubricating plummer block. He would be interested to have Mr. Yates's views on this point.

He must plead guilty to some negligence in his treatment of the propeller damping constant; the figures given had been derived purely from the vibration trial results and he had given no thought to the precise origin of the damping forces. He had assumed that the factor was a constant for a given propeller at all speeds.

Dr. Ker Wilson had suggested that the damping factor could be calculated from the performance of the propeller as was done for torsional vibration; he knew of three such methods, differing in detail*.

The methods now suggested for longitudinal vibration by Mr. Yates and Mr. Archer similarily differed from each other. With regard to the former, he was not happy about the apparent assumption that V_A was constant. Surely it varied by an amount equal and opposite to the variations of V_S?

Mr. Archer's method appeared to cover the equal and opposite variation of V_A and V_S in a satisfactory manner, he regretted that time had not permitted him to apply this method to H.M.S. Formidable, but he would certainly do so and would let Mr. Archer know the result.

In figures, applying Mr. Yates's $\frac{T}{V_S}$ to Mr. Archer's data gave a damping factor at $V_A = 9{\cdot}04$ knots of 0·309 tons per in. per sec. compared with the latter's $\frac{410}{2240} = 0{\cdot}183$.

Applying $\frac{T}{V_S}$ (both rather elusive figures!) to H.M.S. Formidable at 195 r.p.m. gave only about 0·5 tons per in. per sec. compared with 1·062 found from the vibration trial results. The disparity between these figures was alarming in that it rendered distinctly questionable the practice of assuming all the damping to be at the propeller and a further contribution from Dr. Ker Wilson as to the validity of the two methods of calculating the damping factor would be very welcome.

An aspect of the matter which was perhaps more important than the absolute value of the factor was the implication that for a given hull and propeller it would vary approximately in direct proportion to rotational speed. The calculations given in the appendices were based on the assumption that both damping factor and percentage thrust variation remained constant; the trial results could however be matched equally well if both quantities were treated as proportional to rotational speed.

* Ker Wilson, W. 1941 "Practical Solution of Torsional Vibration Problems". (Chapman and Hall, London). Vol. 2, p. 44; Den Hartog, J. P. 1947 "Mechanical Vibration" (McGraw-Hill Book Company, New York and London). p. 260; Donovan, W. J. 1941 Trans, A.S.M.E. Vol. 63, p. A94.

CORRESPONDENCE

Sea Water Contamination of Boiler Fuel Oil and its Effects

By Engr. Rear Admiral C. J. Gray and Wycliffe Killner

Mr. H. Mackegg (Member) wrote that the question of dealing with emulsions which developed as a result of sea-water contamination of boiler fuel had had his firm's attention for a long time, and they found that it was a relatively straightforward problem which could be dealt with by effective centrifugal separation.

If the fuel oil was passed through a centrifugal separator of the correct design at a low throughout capacity at the correct temperature it was possible to split the emulsion formed by sea-water contamination. The temperature must be in the region of 180 deg. F., at which temperature a certain amount of the asphaltic matter, amongst other things, went into solution with the oil, and it was that finely divided matter which tended to stabilize the emulsion.

It was appreciated, of course, that it was possible to obtain some stable emulsion which even at high temperature would not readily split, but in any case, if the procedure suggested was adopted, it was possible to recover the bulk of the oil, leaving the stable emulsion, which would discharge with the water from the centrifuge, and this emulsion could then be treated with reagents.

From the point of view of marine practice, the problem of reducing the volume of the contaminated oil on board a given vessel was therefore considerably reduced.

In the experiments which Mr. Lamb had carried out in the "Auricula" (with which he had been associated), the oil was treated in the manner which he suggested, using boiler oil of 1,500 secs. Redwood No. 1 viscosity, and emulsion troubles were not experienced, even though hot wash water was used to ensure continuous discharge of the separated solids. That could be accounted for by the fact that the oil under treatment was passed through centrifuges of correct design at a temperature of 180 deg. F. at approximately 2 tons per hr.

He did not think that the authors could have considered this angle of the problem, and that view was supported by the fact that the authors stated that their remarks referred to experience in H.M. ships, because, as far as he was aware, the Admiralty never purchased high capacity centrifuges of the type which he had in mind, and he thought that the largest centrifugal separators used in the Fleet were those installed in the "T" class submarines, which only had a throughout capacity of 1 ton per hr. on Diesel oil, which machines would be inadequate to deal with the problem under consideration. Similarly, none of the Admiralty Research Stations had used centrifuges of larger capacity.

The authors' wrote in reply that Mr. Mackegg claimed first that "it was possibe to split the emulsion", and later made it clear that the emulsion was not split but merely concentrated by centrifuging. They had stated explicitly in the paper that for coalescence of emulsified droplets of water, it was first necessary that they should collide and that the frequency of collisions depended, among other factors, upon their mean distance of separation; i.e. upon the concentration of the emulsion. They were therefore quite aware that in dilute emulsions in which the water droplets were separated by considerable distance some sort of concentration of the emulsion would be advantageous for the rapid operation of a breaking process. They did not consider that the expense of installing centrifugal separators of the capacity needed was justifiable, when the necessary concentration could be achieved by settling under gravity in a sullage tank with moderate heating. Centrifugal separation had the inherent defect that, if fast enough to have any advantages, it must cause breaking up of the coarsely dispersed water drops, and breaking them up thereby increased the difficulty of subsequently breaking the emulsion. These were the water drops which sedimented most rapidly under gravity. They knew of no case in which a sufficiently rapid rate of settling was not attained without the use of centrifuging.

JUNIOR SECTION

The Combustion Turbine

A lecture on "The Combustion Turbine" was delivered by Mr. J. Calderwood, M.Sc. (Member of Council) to a large audience at Acton Technical College on Monday, 15th March. Mr. R. W. MacAdam, B.Sc.(Eng.), the principal, occupied the chair, and the Council of the Institute was represented by Mr. S. B. Jackson (Member).

Mr. Calderwood gave a thoughtfully planned lecture dealing with the historical evolution of the combustion turbine, the present position and its prospects of future development, and the considerations governing its commercial application. The lecturer described the principal cycles in use, with notes on their advantages and disadvantages, and he broadly reviewed the metallurgical and thermodynamical considerations affecting the design of combustion boilers and turbines.

The lecture evoked a most valuable and interesting discussion, no doubt due to the local interest in the combustion turbine.

On the proposal of the chairman, a hearty vote of thanks was accorded to Mr. Calderwood, and Mr. Jackson expressed the appreciation and thanks of the Institute to the principal for the arrangements which had been made to ensure such a successful meeting.

Visit to Battersea Generating Station

On the afternoon of Thursday, 26th February 1948 a small party of junior members of the Institute, in conjunction with a number of students from the Polytechnic, visited the Battersea Generating Station.

The party, accompanied by one of the station engineers, made an almost complete tour of the station, but it was impossible to cover the whole in detail in the few hours available. Starting from the boiler house, where attention was drawn to the large Babcock boilers, the next call was at the turbine flats, where the engineers explained in detail experiences of every day running. It was in the main control room where one realized the magnitude of the part played by this station in the supply of London's electricity.

Thanks are due to the London Power Company for the excellent facilities afforded.

Visit to Barking Power Station

On Thursday, 4th March 1948 a party of junior members and students from the Royal Naval College, Greenwich, visited the Barking Power Station. The party was welcomed by the deputy station superintendent, and accompanied by two technical assistants throughout the tour of the station, which started at the coal jetty, where storage, handling, weighing, and conveying methods were inspected and fully explained. At this point also, the circulating water inlets, revolving screens, and pumping equipment were seen. Then followed a visit to one of the boiler houses, and considerable interest was shown in the Raymond fuel pulverizing mills. The method whereby the coal was fed to the mills and then to the storage bunkers for each of the pulverized fuel boilers, was given in detail.

The party was then taken on to the roof, where the induced draught and forced draft fans, cyclone grit arresters, and gravity bucket conveyors were pointed out. The firing floor of the boilers was then visited, and the pulverized fuel burners and control panels shown, then the water gallery where the economizer casings were inspected.

On reaching the "A" Station engine room the Parsons 30,000 kW. sets were seen, and a brief inspection of the reheat boilers was made, and at "B" Station one of the 75,000 kW. B.T.H. sets was opened up for overhaul. This presented a good opportunity of observing the various stages, and of inspecting the blading and construction of this type of set.

Thanks are due to the County of London Electric Supply Company for arranging the visit, and for the provision of the two technical assistants who conducted the party in such an able manner.

Southern Junior Branch I.N.A. and I.Mar.E.

At a meeting of the Branch held in the Civic Centre, Southampton, on Wednesday, 11th February 1948, Mr. J. Calderwood, M.Sc. (Member of Council) delivered a lecture on "The Combustion Turbine" before an audience of about 130 members and students.

Eng. Com'r W. A. Graham, O.B.E., R.N.R. (Vice-President, Southampton), who was in the chair, spoke of Mr. Calderwood's wide experience as an author and lecturer on engineering subjects, and in particular the combustion turbine.

The lecture began with a brief survey of the history of the combustion turbine from the early discoveries of Da Vinci to modern times. Within the last few years only have metallurgists been able to produce materials capable of withstanding the extremely high temperatures for a sufficient length of time to make the combustion turbine suitable for marine purposes. Unlike the aero-engine which has a lifetime of a few hundred hours, a marine engine, suitable for use in merchant vessels, must have a lifetime of about 100,000 hours. The biggest metallurgical problem is not in connexion with the blade material but in the rotor forging where the larger dimensions involve difficulties in the forging process. The need for the production of cheap high grade materials is essential to the development of the marine combustion turbine so as to reduce initial and maintenance costs.

By means of slides the lecturer presented diagrammatic and machinery arrangements of the various cycles on which investigations are being made. He favoured the high-pressure cycle with air circulation. It is hoped that the thermal efficiency of the Sulzer high-pressure combustion turbine which is based on this cycle will be about 35 per cent corresponding to a fuel consumption of about 0·41b. per b.h.p.-hr.

ADDITIONS TO THE LIBRARY

Presented by the Publishers

Technical Drawing

By D. F. Morris, Technical Department, The Academy, Ayr. Thomas Nelson and Sons, Ltd. Edinburgh. 1947. 218pp., profusely illustrated. 12s. 6d.

The author, who is a member of the Technical Department of The Academy at Ayr, has prepared this elementary book essentially for the individual student, and it is ideally suited for self-instruction in the first year on the subject of geometry and mechanical drawing. It has been especially written for scholars preparing for School and Higher School Certificate in England and for Junior and Senior Leaving Certificates in Scotland, although it can with advantage be used in the First Year of a Senior Evening Class.

The opening chapters on drawing equipment, lettering and figuring offer nothing fresh from the many books already written upon the subject except perhaps for a page of interesting geometrical patterns which the author suggests should be copied to give the student practice in the use of the various drawing instruments. From then onwards however, the author departs somewhat from the conventional and devotes fully two-thirds of the book to a number of chapters on the construction of geometrical shapes including triangles, quadrilaterals, polygons and angles although in this latter instance the use of the 60 deg. and 45 deg. set squares for setting out direct angles could have been extended to advantage. A useful chapter on the circle, ellipse and tangent is well illustrated with revelant problems. An introduction to orthographic projection is nicely followed up with exercises on the plan and elevation of a wire, a rectangular sheet of metal, the circle, the sphere and the cylinder.

Projected sections of these geometric shapes and their true areas are very fully discussed. An excellent chapter on conic sections is followed by a good chapter on interpenetration and the development of surfaces. Plane curves including cycloidals, involutes, the spiral scroll and the Archimedean spiral are drawn out with full instructions but perhaps the author might have ventured upon the practical significance of such curves which otherwise become mere exercises in geometry.

The last third of the book consists of a series of progressively graded practical exercises beginning with some very simple shapes. Unfortunately the author has not adhered to the British Standards Institution recommendation for dimensioning.

The book throughout is well printed and the drawings are very clear and well executed assisting study by its pleasing presentation. It amply fulfils the object of providing an interesting elementary introduction to the subject of technical drawing which should inspire the reader to more advanced work.

Electro-technology for National Certificate

Vol. I. By H. Teasdale, B.Sc., M.Ed., A.M.I.P.E. (Head of Department of Technology and Science of The Wakefield Technical College) and E. C. Walton, B.Eng., Ph.D. M.I.E.E. (Head of Department of Electrical Engineering and Physics at The College of Technology, Leeds). The English Universities Press, Ltd. London, 1947. 318 pp., 193 figs., eight tables, 9s. 6d. Press, Ltd. London, 1947. 318 pp., 193 Figs., eight tables, 9s. 6d.

This is the first of three volumes planned to cover the work usually done in an Ordinary National Certificate course in electrical engineering.

In the preface it is stated that the volumes are arranged to correspond very roughly with the work normally done in the three years of the course. One feels that perhaps it is a pity that the authors committed themselves even to that limited extent, because much that is in the volume is beyond the capacity of the normal average S1 student. That is not to say that the matter in itself is not good—very good in certain sections. But he is a good S1 student who in addition to the rudiments of the subject generally dealt with, and the inter-relationships of the various units, can also absorb temperature coefficient of resistance, Kirchhoffs Laws, the theory and operation of the Post Office box and the chemical equations which purport to illustrate fully the electro-chemical changes which take place in secondary cells, both lead-acid and alkaline, during charge and discharge.

A useful feature of the book is the large number of exercises given at the end of each chapter, but here again, many of these would considerably exercise the average S2 student and be quite beyond the S1 student. The text is liberally illustrated with excellent line diagrams and a number of photographs. The value of the latter is always problematical and a number of them could well have been omitted without loss to the book or the student. They increase the size of the book and put up the price without any special benefit being derived.

At the end of the book is an addendum containing proofs of many electro-magnetic formulæ which require the calculus for their derivation. The authors frankly admit that at a first reading the S1 student will not know any calculus but state that they have included them for future reference. It would be better to include them in the text of the third volume which it is presumed that the student should be reading when taking his S3 year.

As so much of the early fundamental matter is so admirably treated one feels that in setting out to produce three volumes ostensibly to cover severally three years of a course it would have been better in this volume to limit oneself strictly to the matter which is common to practically all S1 courses even if this meant omitting one or two items which, it is possible, do appear in stray S1 syllabuses. This would have resulted in a smaller and presumably cheaper book for the young first year student, though in justice it must be added that the price of this book is very moderate for a text-book to-day. The alternative would be to produce one comprehensive volume to cover the Ordinary National Certificate course in a progressive manner.

The format of the book is excellent. It is most attractive in general layout and appearance.

Second Year Engineering Science (Mechanical)

Third Edition. By G. W. Bird, Wh.Ex., B.Sc., A.M.I.Mech.E., A.M.I.E.E. Revised by Struan A. Robertson, B.Sc., B.Com., A.M.I.Mech.E. Sir Isaac Pitman and Sons, Ltd. London, 1947. 256 pp., 230 Figs., tables, 8s. 6d. net.

Due to some alterations and additions, this edition enhances the already established reputation of a book which is designed to cover the second year of the National Certificate course including heat engines.

The substitution of B.Th.Us for C.H.Us may appear a retrograde step even though it is made necessary by recent changes in the syllabus of the London University examinations.

The method of numeration adopted which omits the commas subdividing numbers into groups of three digits is at first disconcerting and one is sometimes debating whether the missing punctuation is a comma or a decimal point.

The increased attention paid to the relationship between the units of mass and force is welcomed by at least one erstwhile student, who for years hid a haziness regarding this fundamental knowledge behind a fluent use of $\frac{W}{g}$.

The work is necessarily somewhat condensed in order that it may be contained in a single volume of modest size which also incorporates a considerable number of topical examples already set by prominent examining bodies but it is nevertheless confidently recommended for its avowed purpose.

Bentley's Machine Shop Companion

(Eleventh Edition). The Bentley Publishing Company, London and Manchester. 1947. 174 pp., profusely illustrated with diagrams and tables, 2s. 6d. net.

In the revised edition of this well-known little book many chapters have been re-written and several new ones included so that it is truly an up to date aid to machine shop practice.

The subjects covered include drilling, turning, milling, planing, templates and jigs, various drives (including electric), materials, hardening and tempering, and simple workshop arithmetic and in all cases the subject is simply and clearly dealt with.

Whilst it should prove a true "companion" to apprentices, there is much in it of value to the experienced machinist.

Ventilation and Air Conditioning

(Second Edition). By E. L. Joselin, A.C.G.I., A.M.I.C.E.. M.I.H.V.E. Formerly Lecturer in Heating and Ventilation, Borough Polytechnic, London. Edward Arnold & Co. London. 1947. 320 pp., 249 Figs., 41 tables, 21s. net.

There have already been several American publications dealing with the subject which this book covers but the need for a book written in accordance with British practice for the use of ventilating engineers and students in this country has long been recognized. The author appears to have been conscious of this and the 1947 edition goes a long way to meet this need.

The book starts with a consideration of air and its properties and after laying down the standards required, proceeds to discuss natural ventilation and, when this fails, the more positive mechanical ventilation with notes on the three main methods adopted in its practice. This, in turn, leads to chapters on air flow and the design of ducts, and two chapters on types of fans and their characteristics. These two chapters contain much valuable information which is not widely appreciated by engineers and while the argument is confined to problems of ventilation, a study of the characteristic curves will suggest to the engineer the advantages and disadvantages of different types of fans for other purposes, such as mechanical draught in boiler plants. This is, in fact, touched upon by the author in concluding the second chapter on fans. Here, he refers to the necessity for induced draught fans to be of suitable construction to withstand corrosive effects and deformation but he omits to state the equally important requirement that the fan wheels must be self-cleaning to prevent soot and grit deposits throwing them out of balance.

Mechanical ventilation combines easily with heating and a chapter on this contains useful information on different types of steam-air heat exchangers and calculations on heat transmission. Unit heaters are fully covered and their relationship to ventilators is discussed. A chapter on air purification leads into the subject of air conditioning and refrigeration for air cooling, with descriptions of the apparatus involved.

The book closes with a series of tables giving relevant information and a good index makes reference easy.

Gears, Gear Production and Measurement

By A. C. Parkinson, A.C.P.(Hons.), F.Coll.H., F.I.E.D., etc., and W. H. Dawney, A.M.I.E.I., etc. Sir Isaac Pitman and Sons, Ltd. London, 1948. 255 pp., 231 Figs., 12 tables, 25s. net.

This is a useful elementary text-book which covers all the types of gears in common use. The subject of gear measurement is handled well, particularly in regard to methods employing balls and rollers for which the necessary formulæ are worked out in considerable detail.

The action and production of spur, helical, spiral, bevel and worm gearing are adequately treated.

The more theoretical parts of the book dealing with involute and cycloidal curves contain all the relative information but could be considerably improved. It would be preferable to describe, first of all, the theoretical requirements of gear tooth profiles and then to indicate how the geometrical properties of the curves considered satisfy these requirements. Instead, the properties of these curves are described at some length and geometrical constructions are given for drawing them, but it is not at all clearly demonstrated why their properties should render them suitable for use as gear tooth profiles.

For a first edition, the book is comparatively free from inaccuracies, only three being noted. On p. 90 the width of gap in hobbed double helical gears is given as approximately equal to the pitch whereas, in practice, it is invariably much larger depending, as it does, on the diameter of the blank and the diameter and setting of the hob; on p. 152 the remarks on the use of a fly-cutter are rather misleading as this tool is invariably used with a cross-feed head in cutting a worm wheel; while on p. 165 the remarks on wheel dressing by crushing are not quite accurate, in particular the crushing roller could not have relief as indicated in the illustration. These are, however, relatively minor matters.

A useful feature of the book are the numerous references to current British and American standards and care has been taken to keep the notation and definitions in the text consistent with these standards.

Form Tools

By William F. Walker, A.M.I.P.E., A.M.I.I.A. Edited by Eric N. Simons, M.I.A.M.A. Hutchinson's Scientific and Technical Publications. London, 1948. 296 pp., 10 charts and 318 line drawings, 25s. net.

This book is probably the first of its kind to treat comprehensively the wide and varied applications of form tools. The book is principally designed to assist those directly connected with machine shop practice and tool design, whilst the large number of extremely informative illustrations will be of considerable help to those looking for new methods by which to increase production.

The author has first dealt with the general principles underlying form tool design and the materials used in their construction, after which detail consideration is given to the basic types of form tools, including flat, circular, tangential and dovetail form tools. Each section begins with a precise definition of the type of tool to be discussed, and continues with useful hints regarding methods of application and basic principles of design.

The mathematical treatment given to this subject, is negligible and extremely simple and should not prove confusing to the practical reader.

In the latter part of the book the author devotes a considerable amount of space to both the methods of holding the tools and of manufacture. Although some of the points covered—especially in the latter section—are rather elementary, many useful and practical suggestions are given.

The book is well produced, profusely illustrated and should prove a useful reference book to both the production engineer and the student.

Heat Engines

(Third Edition). By S. H. Moorfield, M.Sc.(Manch.), A.M.I.Mech.E. and H. H. Winstanley, B.Sc.(Lond. 1st Cl. Hons.), A.M.I.Mech.E. Edward Arnold & Co, London, 1947. 326 pp., 136 Figs., 8s. 6d. (not net).

This volume is a third edition of "Heat Engines" by the same authors, suitably revised to include all steam calculations and data in the Fahrenheit temperature system. The steam tables at the end of the book have also been brought into line with the latest Callendar steam information also in the Fahrenheit system. The C.H.U. system is still adhered to in this book when dealing with the gas laws and internal combustion engine cycles.

The chapters on entropy give the student a very good introduction to the more advanced work in this subject, although it is still confined to applications of steam problems.

There is a very useful section on the steam turbine, which deals mainly with impulse machines with some blade velocity diagrams and horse-power calculations.

There is a printer's omission in the heavy oil engine section on p. 207, where Y is given as $\frac{Cv}{p}$, which of course should read $\frac{Cp}{Cv} = Y$.

The book is still an admirable work on heat engines for the Ordinary National Certificate Student to absorb. If the student works through the examples at the end of each chapter it should be more than an introduction to the more advanced work required for the Higher National Certificate and the various institutions' associate membership examinations in this subject.

General Electrical Engineering

Edited by Philip Kemp, M.Sc.(Tech.), M.I.E.E., A.I.Mech.E. Head of The School of Engineering, The Polytechnic, Regent Street, London. Odhams Press Limited. London, 1947. 448 pp., with over 500 photographs and specially drawn illustrations. 9s. 6d. net.

This book surveys in a general manner the broad field of electrical engineering, progressing from the most elementary theory to the design and use of elaborate equipment and complex machinery in steps which the more academically minded may well find rather startling. It is not a text-book designed for the purpose of assisting students to pass any specific examination, but students and others engaged in the electrical industry will probably find it very useful in enabling them to have a practical understanding and intelligent appreciation of their work.

The book has ten chapters written by specialist experts which deal with the explanation of electrical phenomena, simple apparatus and measurements, generation of power, installation work, electric motors, utilization of electricity, applications of electricity including a section on electricity aboard ships, telegraphy and telephony, radio communications and television.

To cover so much ground in 448 pages means of necessity that the treatment must be condensed. There are over 500 photographs and illustrations. This reviewer has rather a "bee in his bonnet" about photographic reproductions in more formal text-books, but in a work of this nature they are not out of place, and they have been admirably chosen. The line diagrams are very good. Having regard to the scope of the work and its lavish content of helpful illustrations the price is extremely moderate.

Flow through Standard Nozzles, Orifice Plates and Venturi Tubes

By J. R. Finniecome, M.Eng., M.Inst.C.E., M.I.Mech.E., M.Soc.Ing.C., M.Inst.F. ("Mechanical World", Monograph No. 39). Emmott & Co. Ltd. Manchester, 1948. 84 pp., 56 Figs., 3s. net.

This book is a collection of articles which were published in the "Mechanical World". The articles are based on the published data of I.S.A. and D.I.N., particularly of D.I.N., covering the metering of the flow of air, steam, gases and water through nozzles, orifices and venturi tubes. It provides a fairly comprehensive reference to Continental data and, in particular, German data on the measurement of the flow of gases and liquids.

Engineers interested in this subject would find the booklet useful for comparison with similar information available from American and British sources.

Diesel Operation and Maintenance

By Orville L. Adams, Sr. Lieutenant Commander, United States Naval Reserve; Associate, American Society of Mechanical Engineers; Formerly Research Engineer, Guiberson Diesel Engine Co. Prentice-Hall, Inc. New York, 1946. Chapman & Hall, Ltd. London, 1947. 357 pp., profusely illustrated, 28s. net.

As it is stated in the preface this book will be useful to anyone concerned with the operation and maintenance of Diesel engines. Students attending technical colleges, will also benefit by reading the technical problems involved and discussed.

The author has certainly a sound knowledge of his subject, no doubt as the result of his duties in the U.S.A. Navy in charge of Diesel powered naval auxiliary craft and the divisions of the book, part dealing with fundamental principals and part covering operation and repair procedure describing causes of failures makes conclusive reading. At the end of each chapter questions relative to the contents will if answered, assist in mastering the study.

The book chiefly covers high-speed Diesels, but its scope would hardly cover the broad technical knowledge of various types required for British B.o.T. certificates. Criticism of the book however must be limited, for its reading shows the author has taken great care to substantiate his statements when dealing with metallurgical, fuel mixing, fuel and combustion and lubrication problems. It is a pity the work is more or less limited to U.S.A. Diesels for descriptions of British and Continental makes are required by British students.

It is felt that starting problems could probably have been given further space. The author's proposal to grind worn pistons to true roundness and instal liners of reduced bore is certainly an unusual practice. When dealing with firing compression pressures the author mentions the "Bacharach" pressure indicator. The "Okill" indicator could also have been referred to. The author limits his "burning" problems to high-speed machinery mentioning that special equipment is provided to burn heavy oil grades in large plants.

In conclusion this book in the main is full of sound practice and can be recommended to all interested in Diesel machinery.

Entropy as a Tangible Conception

By S. G. Wheeler, Eng. Lieut.-Commander, R.N. The Technical Press Ltd. Surrey. Re-issued 1948. 76 pp., 13 Figs., 7s. 6d. net.

In addition to the five characteristics of steam, or for that matter, of any substance, pressure, temperature, volume, internal energy and total heat, there is a sixth, namely entropy. This is not susceptible to any direct physical or mechanical definition. It may be defined as a function of state of the substance not capable of measurement, which increases when heat is added to it from its surroundings and which decreases when heat is abstracted from it by the surroundings.

Entropy may be regarded as a purely thermo-dynamical definition of an unknown and unmeasurable property expressed in terms of energy leaving a body by heat transfer (Q_{out}) and of the influence causing the heat transfer (T).

Thus

$$d\ (\text{property}) = \frac{d\,Q_{out}}{T}\ \text{units.}$$

However, the negative of this is used and the new property which is called entropy and is denoted by ϕ is defined as

$$d\phi = -\frac{d\,Q_{out}}{T} = \frac{d\,Q_{in}}{T} \text{ units}$$

which shows that it is numerical only.

Considering steam, Callendar's fundamental equation is

$$\phi = 1{\cdot}09876 \log \frac{T_s}{671{\cdot}58} - 0{\cdot}25356 \log \frac{p}{14{\cdot}689} - \frac{689{\cdot}3 \times 10^6\, p}{T_s \frac{13}{3}}$$

where T_s is the absolute temperature in deg. F. and p is the absolute pressure in lb. per sq. in., wherein it is confirmed that the relation is a numerical rather than a physical value.

The suggestion that entropy is explicable in terms of sense-perception implies that nothing can exist which the least of us cannot directly perceive, and is a premise which is presumptuous. Only a few of us are capable of apprehending the less tangible manifestations in our world, but this is not what is expected of us in the conception of entropy.

It is therefore difficult to believe that a book purporting to give a tangible conception of entropy can succeed in its purpose. It would be better to leave the student of thermodynamics to retain the mathematical concept rather than attempt tangibility in forms which are impermissible.

Transactions of the Liverpool Engineering Society
Vol. LXVIII. Session 1946-47.

The following publications of the British Standards Institution:—

B.S. 1121 : 1948—Methods for the Analysis of Steel—
- Part 2: Nickel in Permanent Magnet Alloys, 7pp., 1s. net, post free.
- Part 3: Tungsten, 7pp., 1s. net, post free.
- Part 4: Aluminium in Permanent Magnet Alloys, 7pp., 1 illus., 1s. net, post free.
- Part 5: Copper in Permanent Magnet Alloys, 6pp., 1s. net, post free.

P.D. 736 Amendment No. 2: January, 1948 to B.S. 1113 : 1943—Water Tube Boilers and Their Integral Superheaters.

Presented by the Engineer-in-Chief of the Fleet

BR.1335—Boiler Corrosion and Water Treatment, 1945
(Admiralty). H.M. Stationery Office. London, 1947. 33pp., 43 Figs., 6s. net.

The presentation of this treatise by the Admiralty for general publication will undoubtedly be greatly appreciated as a standard reference by those concerned with the construction, operation and maintenance of steam generators.

The most up to date knowledge of the various types of external and internal corrosion encountered under service conditions, their cause, frequency, importance and means of avoiding or arresting them, are dealt with comprehensively.

The treatise also includes information on the formation of scale, priming, the chemistry of distilled, shore and sea waters, and the basis of the modern method employed in the chemical treatment of boiler water.

Information relating to the circulation and heat transfer in boiler tubes, also explanation of the terms pH values, ions and buffer solutions are contained in appendices at the end of the treatise.

Presented by the Author

Viscount Pirrie of Belfast
By Herbert Jefferson. Wm. Mullan & Son (Publishers) Ltd. Belfast, 1948. Distributors: J. C. Doran & Co. Belfast. 318pp., illustrated, 12s. 6d.

The author has devoted himself with Boswellian love of detail to the compilation of this biography. It is evident that Lord Pirrie was his hero, and no circumstances which could be regarded as having a bearing on the life of the great shipbuilder has been omitted. The large amount of biographical detail about persons with whom Lord Pirrie was connected will, perhaps, be of interest chiefly to those who knew him or them personally; the carefully collected accounts of the shipbuilder's own career will appeal to a wider circle.

There are several very good illustrations of Lord Pirrie and his associates, also some of the houses owned by the late peer, and of the famous ship "Olympic" which represented the highest achievement of his creative work. The inclusion of some views of the great works as they are today would have enriched the book and increased its interest for the younger generation, who are not so deeply interested in biography as their fathers were. The true memorial of this famous man is the immense concern which he did so much to establish and invigorate, and by the absence of any illustrations of the great yards, drawing offices and workshops the book loses something of its appeal to the general reader; this may be due to the fact that the author is not familiar with the more technical aspect of the shipbuilding business. A few typographical errors have escaped the proofreader's attention, but the book will be appreciated by all those who were associated with its subject and with the great firm with which his name will always be identified.

MEMBERSHIP ELECTIONS

Date of Election, 12th April 1948

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Sherif Mohamed El-Safty, Sub. Lieut.(E), R.E.N.
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Saad Rizkalla Hanna, Sub. Lieut.(E), R.E.N.
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Hosny Mahmoud Hussein, Sub. Lieut.(E), R.E.N.
Leslie Alan Lee
Ahmad Metwally, Sub. Lieut.(E), R.E.N.
Abdel Aleem Abdue Nafie, Sub. Lieut.(E), R.E.N.
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Samwel Missak Wasif, Sub. Lieut.(E), R.E.N.

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Neil Welbourn Jephcott
David George Ousey
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Transfer from Associate Member to Member

Ronald Magill
Edwin Charles Plastow
Lewis David Trenchard

Transfer from Associate to Member

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George Reginald Hawkins
Humphrey Richard King
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Edward Robert Frank Lee
Parviz Nowroji Rabady
John Scott
Frank Southern
Gerald Valentine Sowter

Transfer from Student to Associate

William Millar McKerracher, B.A., Sub-Lieut.(E), R.N.V.R.

Transfer from Student to Graduate

Omer Medeni Akman

Corrigendum for March issue of the TRANSACTIONS—p. 60, col. 1, line 70, for H_2O_2 read H_2O.