A structured metric approach to compare marine collision avoidance algorithms

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Synopsis

The rapid advancement of technologies enabling autonomous ship capabilities has outpaced the development of corresponding legislative and regulatory frameworks, creating a bottleneck in the global application of autonomous ships as defined by the four MASS degrees of autonomy. A significant issue is the absence of a well-defined process for certifying new algorithms and systems to be installed on board. Recently, a reliable, open-access structured set of scenarios, including several challenging COLREG encounter situations, has been published, intended for testing the numerous path-planning algorithms developed over the years. When connected to the testing framework, the modules must accomplish two primary tasks: determining the applicable COLREG rules (COLREG classification) and computing an evasive route. This paper focuses on defining an approach to evaluate the performance of collision avoidance algorithms through dedicated metrics. These metrics are formulated to quantitatively compare escape manoeuvres according to relevant performance indexes, helping a human-based compliance evaluation assess COLREG adherence. The operation of the comparison metrics is demonstrated by testing an existing collision avoidance algorithm developed by the authors. This demonstration underscores the effectiveness of the analyzed algorithms in efficiently managing the challenging scenarios proposed in the literature. Finally, the paper provides valuable suggestions for modifying and improving the testing scenarios to enhance the robustness of the comparison metrics.

Keywords: Autonomous Navigation, COLREG, Path Planning, MASS, Simulation

1 Introduction

The international shipping industry transports around 90% of world trade. Over the past decade, reported shipping losses have significantly dropped (up to 50%) due to technological advancements, improved ship design, and implementing regulations and risk management systems (Allianz, 2021). The post-pandemic consumption pattern is also bound to create surges in trade volumes, giving rise to vessel traffic, especially near coastal areas. As ships continue to increase in size and the amounts of cargo onboard, one single incident, such as a collision, can result in the loss of several precious human lives, and incidents like oil spills from tankers can have catastrophic and long-term consequences for marine ecosystems, the environment, and local economies. However, despite noteworthy improvements in shipping safety, navigational accidents remain frequent and almost daily occurrences (EMSA, 2023). The need to improve safety, reduce the environmental footprint, and upgrade the quality of the seafarer's welfare pushes the maritime community to start the journey towards autonomous navigation. At the heart of this endeavour lies the development of collision avoidance systems that can navigate the complexities of maritime environments with precision and reliability. As vessels increasingly transition towards autonomous operation, the importance of these systems cannot be overstated, as they serve as the linchpin for ensuring safe passage and mitigating collision risks in dynamic maritime scenarios.

Collision avoidance falls under the broader problem of path planning, i.e., the determination of an optimal path automatically based on information about the surrounding environment. In the robotics field, path planning is commonly divided into two levels (Choset et al., 2005; Filotheou et al., 2020), the off-line or global level, in which the path is determined based on a priori known information, such as fixed obstacles or weather forecasts

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Raphael Zaccone has a BSc, MSc and PhD in Naval Architecture and Marine Engineering. He received his PhD in 2017 from the University of Genoa, Italy. After two years of post-doc, he joined the DITEN Department of the University of Genoa as Assistant Professor in 2019. His main research interests deal with ship autonomous navigation, collision avoidance and route planning, as well as ship propulsion control and simulation. He published over 35 peer-reviewed articles in international scientific journals and conferences.

Silvia Donnarumma received the Ph.D. degree in Mathematical Engineering and Simulation from the University of Genoa, Genoa, Italy, in 2016. After a period at the University of Trento, Trento, Italy, she got a Postdoc Position with the University of Genoa, where she is currently an Assistant Professor. Her research interests include the study and application of nonlinear control techniques, based on hybrid approaches (with resets) and on convex optimization techniques based on LMIs for the synthesis of feedback control systems, dynamic positioning system, control with actuator saturations, and automatic steering.

Michele Martelli received his B.Sc., M.Sc. and PhD degrees in Naval Architecture and Marine Engineering from Genoa University (Italy) in 2006, 2009 and 2013. From 2014 to 2016, as a post-doc, he worked on several research projects dealing with the autonomous capabilities of ships and small crafts. He joined as an Assistant Professor at the DITEN Department, University of Genoa, in 2016. In 2019, he was appointed as an Associate Professor in the same Department. Since the beginning of his PhD, he has been involved in several national and international projects either as a researcher or as the principal investigator; he has published over 90 peer-reviewed articles. He is a reviewer for high-ranked journals and part of multiple international scientific committees.

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(weather routing), and the local or reactive level, in which subparts of the path are re-planned online in reaction to changes in the environment detected by sensors, such as moving or unexpected obstacles. The scientific literature proposed various approaches to reactive collision avoidance of marine vessels, including A* (Seo et al., 2023), Dijkstra's algorithm (Singh et al., 2017, 2018), visibility graphs (D'Amato et al., 2021), rapidly-exploring random trees (Chiang and Tapia, 2018; Zaccone and Martelli, 2020; Enevoldsen et al., 2021), Artificial Potential Field methods (Zhu et al., 2022; Li et al., 2021), Randomly-Exploring Random Trees (Zaccone and Martelli, 2020), Dynamic Programming (Zaccone, 2024), and various population-based heuristics (Ito et al., 1999; Kang et al., 2018; Ning et al., 2020; Gao et al., 2023).

However, the effectiveness of collision avoidance systems extends beyond their technical sophistication, as it encompasses their ability to adhere to international regulations, particularly the International Regulations for Preventing Collisions at Sea (COLREGs). With autonomous and human-crewed vessels navigating the same waters, seamless integration and compliance with established maritime regulations become essential for harmonious maritime operations.

Verifying and validating systems engineering projects is crucial for ensuring the designed systems meet their requirements and perform as intended. The international regulatory framework still needs more precise and comprehensive testing procedures and scenarios. To the author's best knowledge, only one paper in the scientific literature deals with this aspect (Pedersen et al., 2023). The paper suggests 55 scenarios for testing collision avoidance systems, which are used in this paper for verification purposes. Against this backdrop, rigorous verification and validation processes are necessary to assess collision avoidance system performance. However, evaluating the performance of these systems is a multifaceted endeavour, requiring the establishment of comprehensive comparison metrics that enable the quantitative assessment and comparison of different collision avoidance strategies.

These comparison metrics are the yardstick against which collision avoidance systems are measured, encompassing various performance aspects such as collision avoidance capability, manoeuvre characteristics, and responsiveness in dynamic environments, as reported by Filotheou et al. (2020). By defining and utilizing such metrics, researchers and practitioners can systematically evaluate the efficacy of different collision avoidance algorithms, thus facilitating informed decision-making in system selection and refinement.

To address the complexities inherent in evaluating collision avoidance systems, researchers have developed simulation frameworks that provide controlled environments for testing and validation. These frameworks are useful tools for assessing system performance under diverse maritime scenarios. Within these simulation frameworks, the integration of dedicated comparison metrics allows the researchers to quantitatively analyze and compare the performance of multiple collision avoidance algorithms.

This paper presents a comprehensive framework for evaluating collision avoidance systems in autonomous ships. By emphasizing the importance of comparison metrics, the paper demonstrates an approach for the systematic assessment and comparison of different collision avoidance algorithms.

2 Methodology

This paper proposes a methodological approach for the comparison of collision avoidance algorithms. The method requires comparing algorithms against a dataset of scenarios and evaluating their performance using synthetic indices or metrics. This section describes the proposed metrics, the logic scheme of the comparison, and some aspects of scenario definition.

In the proposed pipeline, the tested algorithms are fed a set of testing scenarios they try to solve. The resulting solutions consist, for example, of a set of waypoints and/or travel speeds or a set of machine, rudder, and telegraph commands. A motion control system that simulates the actions on a dynamic reference model can then take over these actions by providing the resulting kinematics. In this framework, it is helpful to identify a set of metrics to concisely evaluate the performance of the algorithms under test and make comparisons. Such metrics can be helpful to the human operator in addition to a qualitative assessment of the resulting kinematics of the solved scenario, whose COLREG compliance must be evaluated a posteriori.

2.1 Definitions

Describing a ship's route or manoeuvre through a sequence of waypoints is a common approach in the literature, not only at the global planning level but also at the reactive planning level. Such an approach features some relevant advantages for the application to large human-crewed ships, both in a fully automatic collision avoidance system and within a MASS Level 1 decision support framework, as the one Figure **??** illustrates. The representation of a manoeuvre by waypoints and legs is intelligible to seafarers, and the decision support system can propose it to the officer on the watch, who can understand it and decide whether to acknowledge it. Then, the new sequence of waypoints is then taken over by a motion control system, for example, based on Line of Sight **??**, which determines, based on GNSS localization, the necessary propulsion and steering actions to track the course according to the ship's dynamics with reasonably low track error.

Within this paper, a generic manoeuvre or route R is represented as a sequence of consecutive waypoints:

$$R = (\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_N) = (\mathbf{x}_i)_{i=0}^{N} \tag{1}$$

A sequence of two consecutive waypoints $s = (\mathbf{x}, \mathbf{y})$ is called route leg. If $s_i = (\mathbf{x}_{i-1}, \mathbf{x}_i)$, and "$\oplus$" is the sequence concatenation operator such that $(\mathbf{x}, \mathbf{y}) \oplus (\mathbf{y}, \mathbf{z}) = (\mathbf{x}, \mathbf{y}, \mathbf{z})$, R can be represented as:

$$R = (\mathbf{x}_0, \mathbf{x}_1) \oplus (\mathbf{x}_1, \mathbf{x}_2) \oplus \ldots \oplus (\mathbf{x}_{N-1}, \mathbf{x}_N) = s_1 \oplus s_2 \oplus \ldots \oplus s_N \tag{2}$$

The notation $\vec{s} = \mathbf{y} - \mathbf{x}$ represents the vector connecting the start point of the leg to the endpoint. As a consequence, the course change between two consecutive legs s_i is expressed by the function θ defined as follows:

$$\theta(s_i, s_j) = \arccos\left(\frac{\vec{s}_i \cdot \vec{s}_j}{|\vec{s}_i||\vec{s}_j|}\right) \tag{3}$$

If there are M obstacles with known kinematics in the scenario, $\mathbf{a}_m(t)$ is the instantaneous position of the m^{th} obstacle. For this study, the motion of the obstacles is approximated as a straight line, constant speed motion:

$$\mathbf{a}_m(t) = \mathbf{a}_m(t_0) + \mathbf{w}_m t \tag{4}$$

Where $\mathbf{w}_m$ represents the speed vector of the m^{th} obstacle. The *CPA* is defined as the minimum distance between the centre points of the own ship $\mathbf{x}(t)$ and an obstacle, whose position over time is denoted by $\mathbf{a}_m(t)$, for $t \in [t_0, t_N]$. In particular, the $CPA_m(s_i)$ is a function defined as follows:

$$CPA_m(s_i) = \min_{t \in [t_i - 1, t_i]} |\mathbf{a}_m(t) - \mathbf{x}(t)| \tag{5}$$

2.2 *Performance metrics*

The selection of a proper collision avoidance algorithm, among all the present in the literature, requires a comparative evaluation. Since algorithms return complex output that is difficult to compare, it is necessary to define quantitative synthetic metrics that can help make a selection. Various approaches have been proposed in the literature related to robotics (Filotheou et al., 2020) and, more specifically, to ship collision avoidance (Zaccone, 2024). In this paper, the following metrics have been considered:

- Path smoothness;
- The path elongation;
- The speed reduction;
- The minimum CPA;
- The maximum track error;
- The computation time.

The path smoothness $\sigma(R)$ expresses the mean heading changes squared, expressing how challenging a manoeuvre could be for a steering system. Manoeuvres involving smoother course changes can be implemented with lower track error and overshoots, ensuring that the actual path adheres more closely to the planned path. The smoothness σ can be defined as follows:

$$\sigma(R) = \frac{1}{N-2}\left(\sum_{i=1}^{N-1} \theta^2(s_i, s_{i+1})\right)^{\frac{1}{2}} \tag{6}$$

Notice that, if defined as above, σ is the lower the better.

The path elongation $L(R)$ is the non-dimensional length of the avoidance manoeuvre:

$$L(R) = \frac{\sum_{i=1}^{N} |\vec{s}_i|}{|\mathbf{x}_{end} - \mathbf{x}_{start}|} \tag{7}$$

Manoeuvres with shorter total lengths normally lead to smaller CPAs.

Speed reduction is a parameter for evaluating the algorithm, not the scenario. In other words, complicated scenarios may not admit solutions allowing the ship to maintain its initial speed. Given that it is not usually appropriate to avoid a collision by increasing speed, in many cases, a reduction to a safe speed allows potential collisions to be avoided. It is, therefore, appropriate to monitor the speed reduction suggested by the algorithm to deal with the manoeuvre.

The minimum CPA during the manoeuvre $CPA_{min}(R)$ expresses the minimum distance at which the own ship avoids a target during the collision avoidance manoeuvre:

$$CPA_{min}(R) = \min_{i\in\{1,N\}} \left(\min_{m\in\{1,\ldots,M\}} CPA_m(s_i) \right) \tag{8}$$

The value of CPA is expected to be above a minimum threshold, but different algorithms or parameterizations can generate more or less conservative manoeuvres.

The maximum track error $T_{max}(R)$ represents the non-dimensional maximum transversal deviation from the original track:

$$T_{max}(R) = \max_{i\in\{1,\ldots,N\}} \frac{\vec{s}_i \cdot \mathbf{u}}{|\mathbf{u}|} \tag{9}$$

Where $\mathbf{u} = (\mathbf{x}_{end} - \mathbf{x}_{start}) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. In general, staying as close as possible to the original track is desirable whenever possible.

Lastly, the computation time to compute an evasive manoeuvre is crucial when real-time applications are required. Reduced computation time enables the deployment in real-time applications and on fast vessels.

The proposed metrics make it possible to compare different algorithms on the same scenario, whereas comparing an extensive set of scenarios can be difficult because the absolute value they take depends on the scenario setup. Defining a metric normalization approach to compare algorithms by aggregating or averaging data obtained in different scenarios is useful.

If $\mu(k,a)$ denotes a generic metric measuring the performance of algorithm a over the testing scenario k, the normalized metric $\mu_n(k)$ over a set A of algorithms is defined as follows:

$$\mu_n(k) = \frac{\mu(a,k) - \min_{a\in A}\mu(k,a)}{\max_{a\in A}\mu(k,a) - \min_{a\in A}\mu(k,a)} \tag{10}$$

It is worth noting that, using a normalized metric, the best algorithm over a scenario scores 1, and the worst scores 0. In the presence of a significant scenario dataset, normalized metrics allow one to understand how often one algorithm is better than the others on a per-scenario basis.

2.3 Scenario definition

Different approaches to defining scenarios for comparing collision avoidance algorithms exist in the literature. First, it is possible to differentiate between scenarios characterized by fixed obstacles, typical of robotic motion planning problems, in which the autonomous vehicle must find a collision-free path through a static map. In such cases, it is reasonable to refer to complex maps characterized by tortuous manoeuvres generated a priori or by more or less random placement of obstacles. In contrast, regarding marine vehicle collision avoidance, more attention is paid to moving obstacles, especially when considering COLREG compliance, a case in which scenarios must refer to typical navigational cases.

In the context of COLREG-compliant collision avoidance, many studies refer to so-called cornerstone scenarios, which represent the typical encounter conditions described by COLREG: head-on, crossing and overtaking. Some examples are proposed by Zaccone et al. (2019). Zaccone and Martelli (2020) presented navigation scenarios constructed a priori, characterized by dynamic obstacles with sudden course and speed changes. For testing and comparing a collision avoidance algorithm, Zaccone (2024) proposed an approach based on complete randomization of the positions of fixed and moving obstacles to assess the performance of the algorithms with a statistical metrics-based approach on a vast number of random scenarios. Eventually, Pedersen et al. (2023) systematized a methodology for generating navigation scenarios for testing collision avoidance algorithms, proposing an open source tool (DNV, 2023) and a dataset of 55 scenarios, built from cornerstone scenarios by combining them into more complex scenarios with two, three or more vessels.

3 Collision avoidance algorithms

The presented approach is applied to a set of algorithms to demonstrate its possible applications. Three algorithms, Dynamic Programming, Genetic Algorithm, and RRT*, are compared to this end.

Dynamic Programming (DP) is a practical approach for solving multi-stage optimization problems. From its introduction by Bellman Bellman (1954, 1966), DP has been successfully generalized and formulated to describe

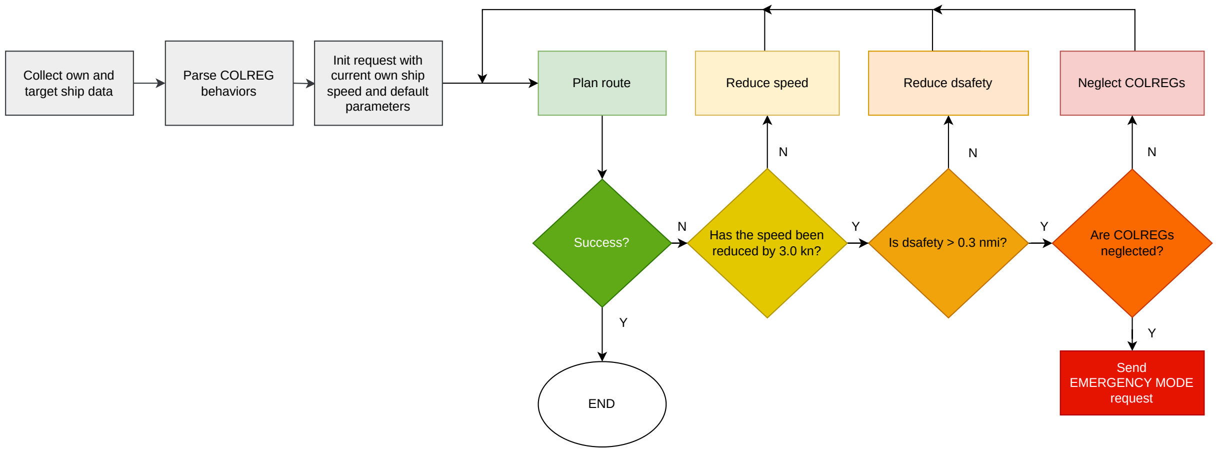


Figure 1: The proposed collision avoidance fallback strategy to deal with complex scenarios.

path planning problems Jones and Peet (2021). An application to marine collision avoidance was presented by Zaccone (2024), who also introduced a greedy-approximate solution scheme to reduce the computational complexity. The greedy-approximate DP is the first of the three algorithms featured in the present comparison.

Genetic Algorithm (GA) is a widely used Evolutionary algorithm that mimics the selection of genes in a population. Each candidate solution, referred to as a "phenotype" or "creature", is modelled as a sequence of parameters known as the "genotype" or "chromosomes". The first iteration generates a population of creatures with a random genotype. Then, the best creatures are selected according to a specific cost function. A new population for the next iteration is generated by quasi-randomly shuffling their chromosomes and introducing random mutations. This process is repeated until the population heuristically converges to one solution. GA allows for evaluating the minimum number of very complex functions and is generic enough to fit almost every optimization problem. In the proposed application, GA is used to find the best sequence of waypoints within the same gridded framework proposed by Zaccone (2024).

Eventually, the RRT* is a state-of-the-art algorithm in robotics and path planning: it is a random sampling algorithm designed to explore domains using tree structures and determine collision-free manoeuvres quickly. RRTs (LaValle, 1998) iteratively generate tree-like structures, initiating from a root node and terminating when a node is close enough to the desired goal. The "*" (star) variant, or Optimal RRT (Karaman and Frazzoli, 2011), includes local optimizations of the tree topology within a neighbourhood of each newly generated node, leveraging a cost function to generate heuristically optimal trajectories. The implementation of the RRT* used for this comparison has been presented by Zaccone and Martelli (2020).

The tree path planning algorithms can compute a collision-free path at a constant speed, keeping a minimum distance from the obstacles. The algorithms have been integrated into a collision avoidance strategy. The implemented strategy is represented in Figure 1 and operates based on the following steps:

1. The route is evaluated assuming that the own ship maintains its initial speed, meets a minimum CPA of 1 nautical mile and complies with COLREGs.

2. If no solution is found, the speed is reduced by up to 3 knots less than the initial speed. This speed reduction is small enough to occur in negligible time and be assumed instantaneous.

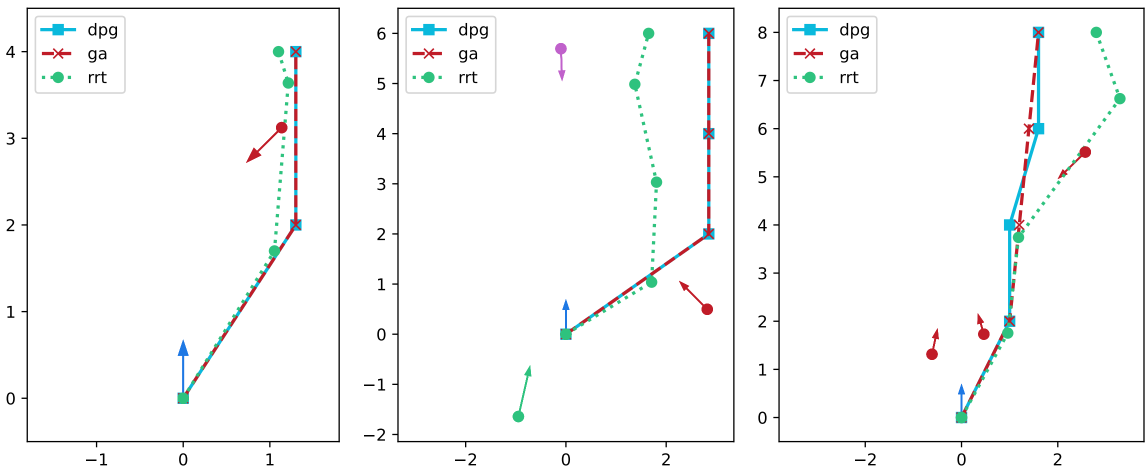
3. If no solution is available, the minimum allowable CPA is reduced to 0.3 nautical miles.

4. If no solution is found, the algorithm evaluates the possibility of violating COLREGs.

4 Results

Three collision avoidance algorithms are tested in this case study, specifically a greedy-approximate dynamic programming algorithm proposed by Zaccone (2024), an implementation of RRT* proposed by Zaccone and Martelli (2020) and a genetic algorithm. The three algorithms were used to minimize the same cost function (control energy) and with the same set-up of constraints, as described in Zaccone (2024).

The three algorithms were tested against the 55 scenarios proposed by Pedersen et al. (2023). These scenarios include cornerstone encounters, such as crossing, head-on, and overtaking, with only one target, as well as numerous seemingly dead-end situations characterized by multiple targets converging on the own ship. In such scenarios, compliance with COLREGs dictates that one's course should not vary from the give-way targets, leading to zero



(a) Scenario 2 (b) Scenario 29 (c) Scenario 43

Figure 2: Three of the 55 scenarios proposed by Pedersen et al. (2023).

CPA. For this reason, give-way targets were not included in the minimum CPA analysis since evasive manoeuvring is expected from them.

Figure 2 presents some examples of the 55 scenarios proposed by Pedersen et al. (2023), specifically scenario 2, scenario 29 and scenario 43. The figure shows the solutions of the three tested algorithms. Given its scattered and random nature, RRT* always presents a more irregular trajectory. GA, on the other hand, presents a heuristically optimal solution that often coincides with dynamic programming.

Figures 3 to 8 present the aggregated performance parameters of the 55 scenarios. Violin plots have been used to evaluate and compare the algorithms: these plots (Hintze and Nelson, 1998) show how each quantity is distributed over the dataset by highlighting the average value.

Figure 3 presents the results related to smoothness: it is observed that both in a relative sense, DP and GA provide smoother trajectories than RRT*. Figure 4 shows the elongation of manoeuvres compared with the reference trajectory. It can be seen that although the values are similar in absolute terms, RRT* systematically provides longer manoeuvres than the other algorithms. From Figure 5, it is observed that all the algorithms succeed in most cases in solving the scenarios without reducing the speed. Figure 6 shows that all algorithms perform similarly in terms of CPA in an absolute sense, but by normalizing the values, it is observed that RRT* allows slightly higher CPAs to be maintained due to the greater freedom the algorithm has in placing waypoints. Looking at Figure 7, we observe that the DP algorithm is the best in moving away from the original track, while the RRT* is the worst, given its heuristic nature. Eventually, Figure 8 shows that the DP and RRT* perform significantly better than the GA in terms of computation time, the former in particular guaranteeing times of less than 100 milliseconds.

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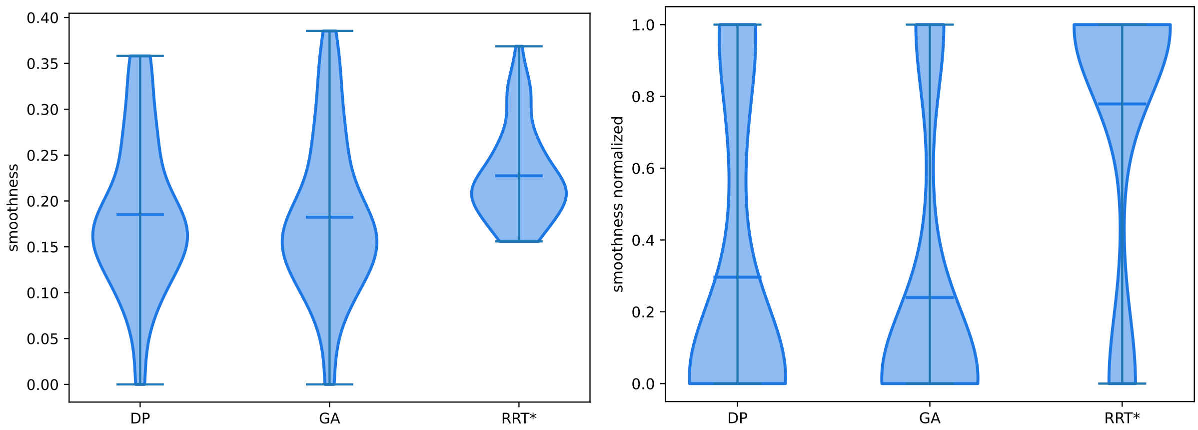
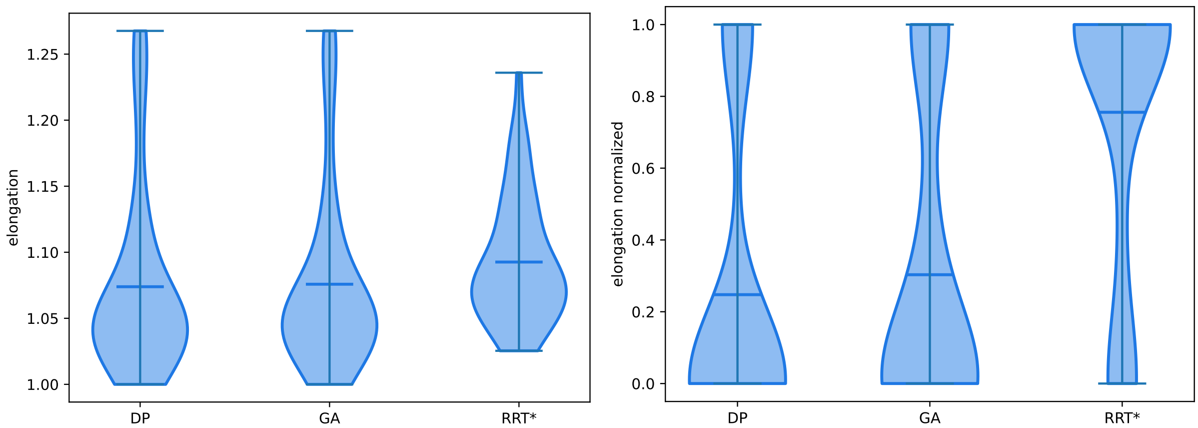


(a) Smoothness (b) Norm. Smoothness

Figure 3: Smoothness violin plots on the 55 scenarios.



(a) Elongation (b) Norm. elongation

Figure 4: Percentage path elongation represented with violin plots on the 55 scenarios.

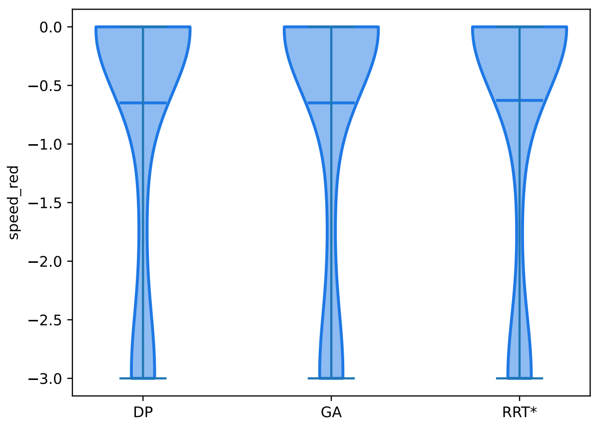
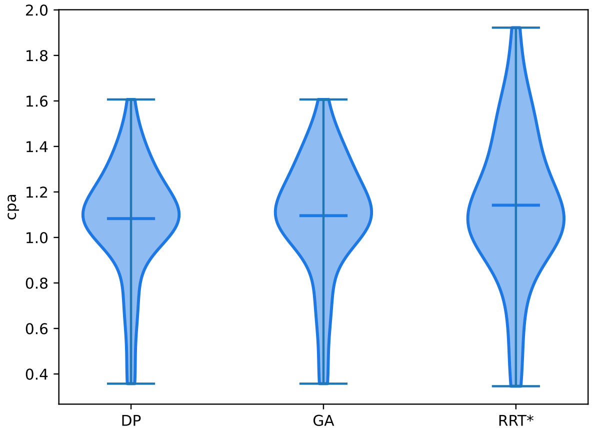
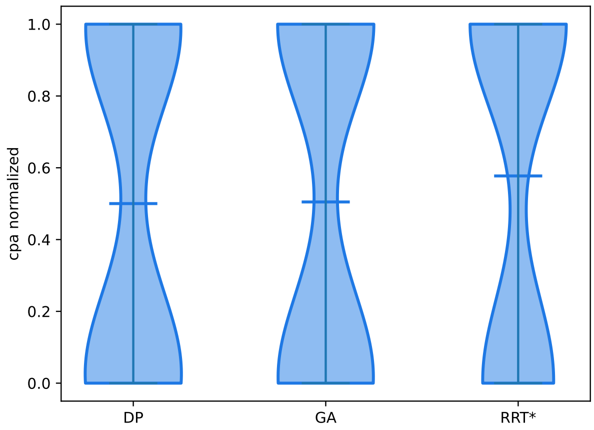


Figure 5: Speed reduction in Knots represented with violin plots on the 55 scenarios.

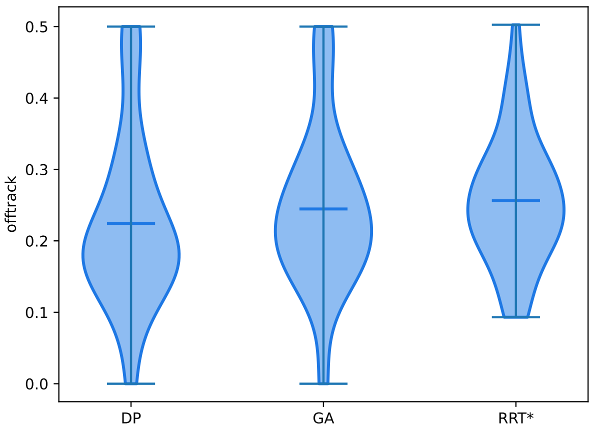


(a) CPA (nm)

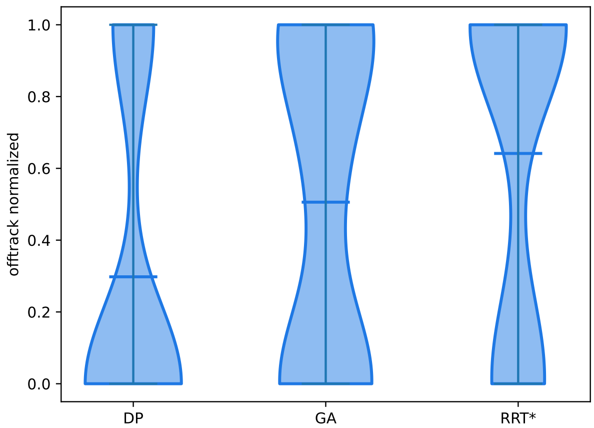


(b) Norm. CPA

Figure 6: CPA in nautical miles represented with violin plots on the 55 scenarios.

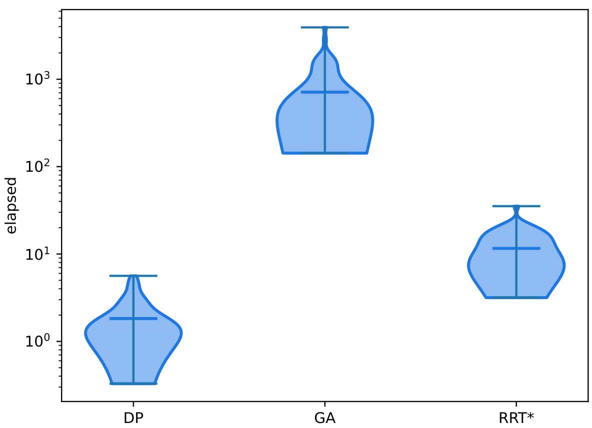


(a) Track err.

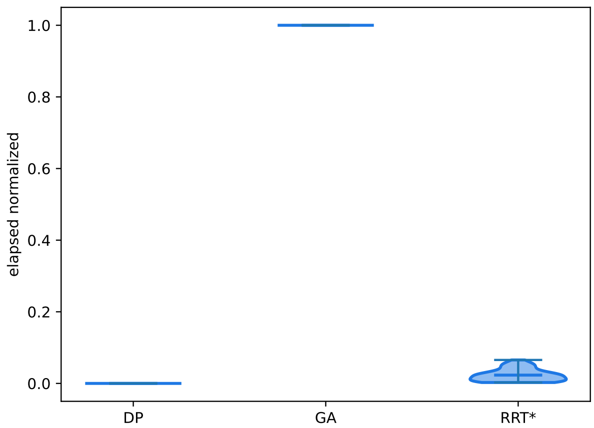


(b) Norm. track err.

Figure 7: Track error violin plots on the 55 scenarios.



(a) Elapsed time (ms)



(b) Norm. elapsed time

Figure 8: Elapsed time in milliseconds represented using violin plots on the 55 scenarios.

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5 Conclusions

Numerous algorithms for path planning, collision avoidance, and decision support have been presented in the autonomous navigation field. However, limited attention has been given to evaluating algorithms' performance for comparison purposes. This paper presented an approach for comparing collision avoidance algorithms based on appropriate performance metrics. The approach has been applied for demonstration purposes to three collision avoidance algorithms, comparing their performance on encounter scenarios proposed by third parties in the literature, showing how comparative evaluation of different algorithms on a defined set of scenarios can be performed.

However, the approach has some limitations. First, the performance metrics are constructed assuming the algorithms provide structured suggestions regarding waypoints. In principle, an algorithm can provide results in another form, which would require a different definition of some metrics. In addition, the viability of the proposed solutions was assumed in terms of the dynamic response of the ship, which could instead be verified and evaluated by an appropriate index. Finally, the automatic evaluation of COLREG compliance with a complex manoeuvre remains an open problem which cannot be released from expert judgment.

The scenarios used in this paper were proposed by third parties, and the analyses highlighted some critical issues. In particular, some particularly complex scenarios are of little practical and operational interest since, in navigational practice, they would be subordinate to the actions of target ships under COLREGs.

A potential future development could be to define scenarios in which the target ships have a plan of intended movement defined but not known a priori to the own ship. In this way, it could be assessed whether the own ship behaves according to the rules. Alternatively, the own ship could be called upon to interact with autonomous target ships commanded by a reference collision avoidance system for evaluation.

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