VIBRATION OF ROTOR DISCS IN IMPULSE TURBINES

A very considerable development in the size and numbers of impulse turbines took place during the few years previous to 1920 both in Europe and America mainly in connection with large electrical power stations. On the Continent and in America a large increase in the peripheral velocity of the turbine blades was also made with a view to reducing the first cost of the turbines by reducing the number of stages required to expand the steam. It is of interest to note that these high speeds were accompanied by an increase in rotational losses, and the turbines were not so efficient as those having more stages and running at moderate speeds. Failures due to breakage of the turbine discs occurred in many cases, nearly always resulting in completely wrecking the machine and often accompanied by serious loss of life.

These complete failures occurred for the most part to Continental and American designs, the English designers being generally more conservative with regard to operating speed. The failures were attributable to vibrations in the discs which set up alternating stresses of sufficient magnitude to cause eventual fracture by " fatigue."

Investigations into the nature of these vibrations were made by Stodola, and also by Lamb and Southwell, and a little later a very thorough investigation of a practical nature was carried out by the late Mr. Wilfred Campbell, of the General Electric Company of America.

In the Navy the peripheral speeds of the turbine discs are very moderate and the stresses comparatively low, and we have had no cases of burst discs, but there is evidence to show that in a few cases impulse turbines have suffered from vibrations of the type to be described.

Most vibration problems fall naturally into two parts :—

(1) The investigation of the natural frequencies of vibration of the structure in the various modes.

(2) The investigation of the disturbing forces likely to synchronise with the natural frequencies and set up vibration.

In this particular investigation part (1) may be sub-divided into :—

(*a*) The investigation of the natural frequencies of the disc at rest.

(*b*) The effect of centrifugal force in modifying these frequencies when the disc rotates.

The Vibration of the Disc at Rest.—The manner in which an elastic disc vibrates is as follows :—

If the edge of the disc is displaced from its normal position, the centre of the disc being clamped, it will be noticed that the disc exerts a force tending to return the edge to its normal position due to the bending of the material. For reasonably small displacements the force exerted will be proportional to the displacement, so that on releasing the edge, it will vibrate with harmonic motion. On observing the type of vibration set up it will be found that the metal along two diameters at right angles is stationary and the remaining metal vibrating always in phase in each 90° segment, but out of phase in adjacent segments, the amplitude of the vibration varying to a maximum half way between the stationary or nodal diameters. This mode of vibration is called a " two nodal diameter " vibration and is the easiest to produce, since it is the " gravest " mode (*i.e.*, that with the lowest natural frequency) in which the vibration is self-contained within the disc. If a force of suitable frequency (higher than that for the 2 node mode) be applied it will be found that the disc will respond by vibrating with 3, 4, 5 or 6 nodal diameters, and with one or two equidistant nodal circles, and so on for higher and higher modes with higher frequencies, these being as a rule successively more difficult to produce.

An analysis of vibrations of this type will show that they are equivalent to setting up two travelling waves of displacement in the disc, one rotating in one direction and one in the other, at the same speed.

Where the travelling waves are in phase one with the other the vibration is a maximum, and where they are 180° out of phase then they cancel one another and we have the stationary nodes ; in the case of a rotating disc the wave travelling in the direction of rotation is termed the progressive wave, and that in the opposite direction the retrogressive wave.

If there are h diametrical nodes in the mode of vibration considered, there will be h waves in the circumference of the disc. It follows that if the frequency of vibration (the number of times each wave crest comes up in a given interval) is n the speed of rotation of the wave is n/h revolutions in unit time.

The natural frequencies of vibration of a turbine disc at rest can be calculated by trial and error methods but the calculation is long and complicated, and it is usually found that the results do not agree with experiment, since the residual stress left in the disc after manufacture affect the natural frequency of vibration by altering the stiffness of the disc, and these residual stresses are unknown and cannot be taken into account in the calculations.

In consequence it is necessary to determine these " Static " natural frequencies by experiment ; fortunately the experiment is comparatively simple, and requires no expensive apparatus.

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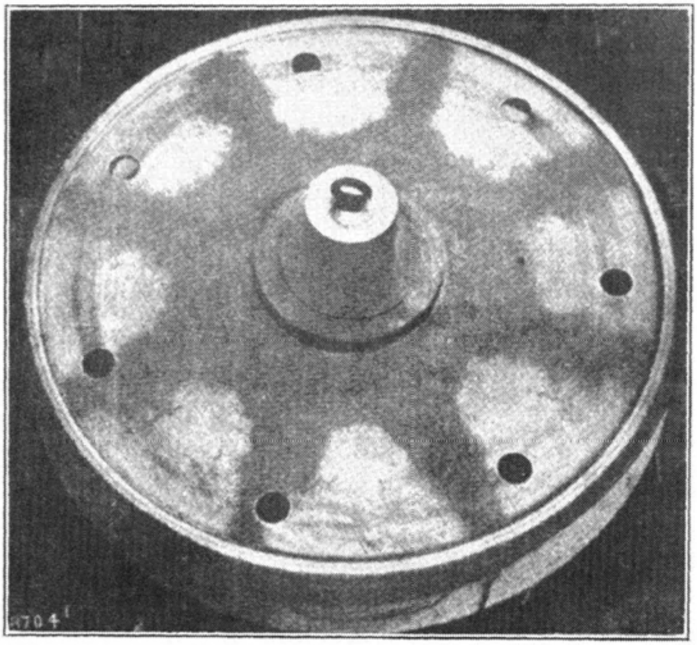

FIG. 1.

The disc is mounted horizontally, and its surface sprinkled with fine sand. An electric magnet is mounted close to the edge of the disc and energised from a source of alternating current whose frequency is under control and can be measured. When the frequency of the disturbing force set up by the magnet coincides with one or other of the natural frequencies of the disc, considerable vibration occurs, and the sand commences to dance up and down on the vibrating parts of the disc and eventually settles and remains at the nodes, leaving a sand pattern which marks the nodes so that the mode of vibration at the particular frequency can be seen. By the use of this type of apparatus the natural frequencies of vibration in the various modes can be obtained, the chief precaution required being to see that the control of the frequency of current applied to the magnet operates smoothly and that the current can be held at a steady frequency for prolonged periods to enable the vibration to build up. A photograph of a typical sand pattern is given in Fig. 1.

The Influence of Speed of Rotation.—Consider a disc made of some very flexible material, for instance, wash leather. If it is stationary there is no resistance to bending, and it has no natural frequency of vibration. If it revolves the centrifugal forces bring the particles out until the disc runs in a plane at right angles to the axis of revolution, and if the edge of the disc be disturbed from this path of revolution then the centrifugal force will tend to bring it back to its mean path, and we have the conditions for harmonic motion. Since the centrifugal force on a given particle will be proportional to the square of the speed, the $\frac{\text{acceleration}}{\text{displacement}}$ will also vary with the square of the speed, that is to say, that for this effect the natural frequency will be expressible by a formula of the form:—

$n^2 = BN^2$ where n = frequency of vibration.
B = a constant depending on mode of vibration.
N = Speed of rotation of disc.

For the revolving metal disc the centrifugal forces are applied in addition to the bending forces, and since the mass is not increased the frequency of vibration for the various modes is raised. Provided that the profile of vibration is not far removed from that when stationary, the frequency will be given by a formula of the form—

$n^2 = n_0^2 + BN^2$ where n_0 = static frequency for the mode considered and n is the frequency of the waves relative to the revolving disc.

The Coefficient " B."—To determine the frequencies of vibration of the running disc in the various modes at any given speed of rotation, it is therefore necessary to determine the " static " frequency by the experimental method described, and the coefficient " B " in the equation given above.

In the case of the disc of uniform thickness B. has been calculated by Lamb and Southwell, who give the following equation $B = \frac{1-\sigma}{4}h^2 + \frac{3+\sigma}{4}h$ where σ is Poissons Ratio and h is the number of nodal diameters in the mode considered. σ is about 1/3 for steel, and therefore $B = 1/6h^2 + 5/6h$.

It has been found by experiment that this formula gives correct results for a uniform steel disc. For a rim rotated unsupported except by flexible spokes, it can be shown mathematically that $B = h^2$.

For discs which are not of uniform thickness the value of B is very difficult to determine by calculation. It can be determined experimentally by a method first devised by Wilfred Campbell, but the apparatus required is somewhat costly and the time taken over each disc would be prohibitive if it were necessary to test every disc. At least two large firms in this country possess apparatus for determining the natural frequencies of the rotating discs.

The apparatus is shown diagrammatically in Fig. 2 and described below.

A turbine or electric motor drives the disc to be tested, which is shrunk on to a shaft and revolves within a case under a vacuum. A stiff disc is also mounted on the shaft adjacent to the wheel. The disc and the case carry one or more magnetic induction " search coils " mounted close to the rim of the test wheel. Axial vibration of the wheel increases or decreases the " air-gap " of the magnets and consequently the magnetic reluctance of the circuit and the impedance of the coils. The variation in current is amplified by thermionic valves and applied to oscillographs which give visual and photographic representations of the vibration of the wheel. The speed of the disc is measured by the frequency of the current delivered by an alternator directly coupled to the shaft. The oscillographs record a timing wave of standard frequency in addition to the vibrations relative to space and to the rotating wheel. The mode of vibration can be determined by counting the number of peaks between successive repetitions of the pattern, assuming the pattern repeats itself once a revolution, on the oscillograph record of the moving coil. The observer looking at the waves in the visual oscillograph takes photographs when he notices vibrations of large amplitude arising. The oscillograph method is capable of giving dependable results in the hands of experienced operators, but there are difficulties of interpretation of the records, due mainly to the fact that the waves are seldom or never pure sine waves. Both the progressive and retrogressive waves are superposed and recorded, and in addition " space harmonics " are set up as in an induction motor.

The experimental method of determining the centrifugal effect is more difficult and involves more costly apparatus than the static

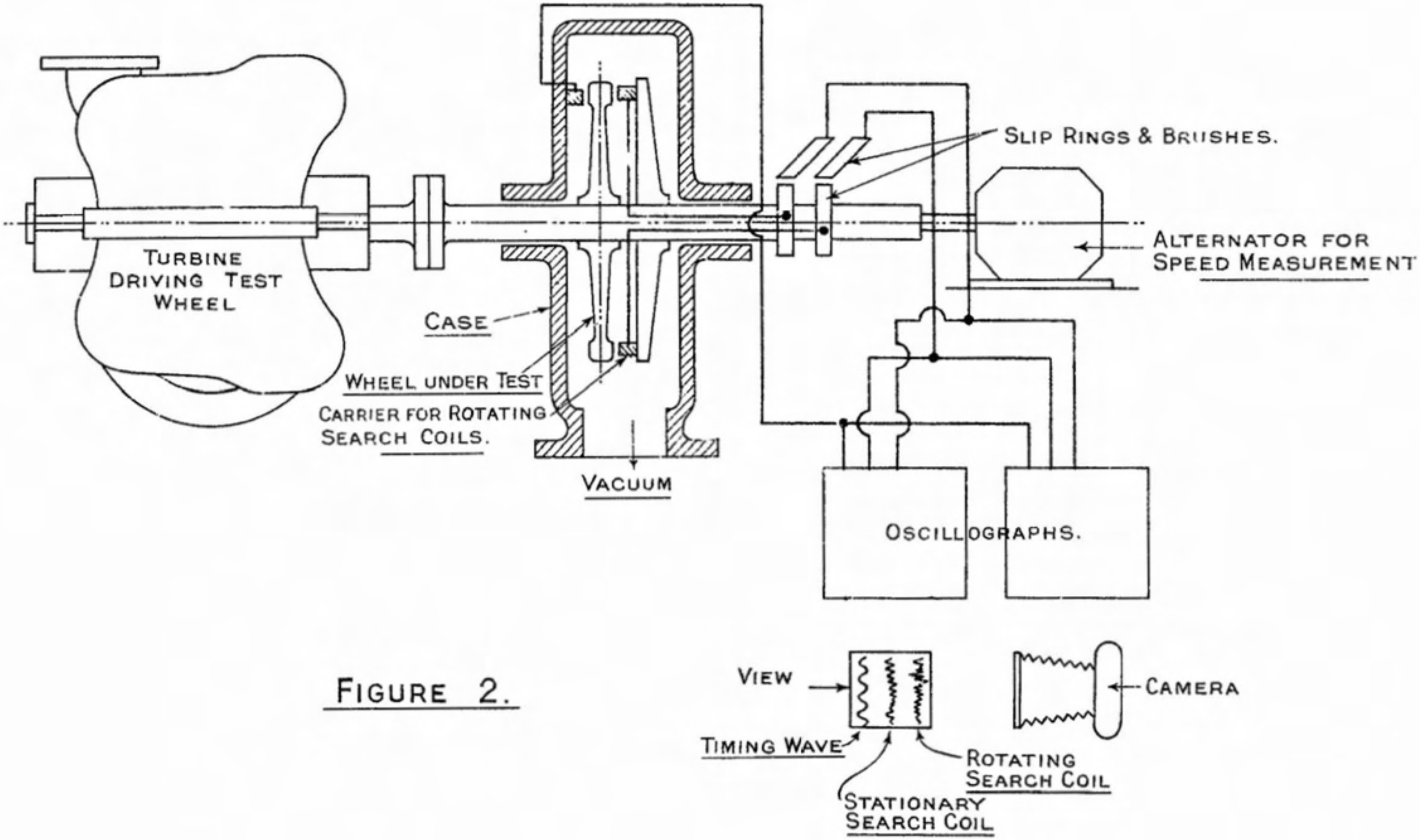

FIGURE 2.

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test, and the need for experiment is not so great as there are no unknown factors such as residual stresses which affect the result, and also fairly large errors in the estimation of B will not produce correspondingly serious errors in calculating the natural frequencies since as a rule the predominant term in the equation is the static frequency.

There is, therefore, a need for an empirical formula which will give the approximate values of B in any given disc, in order to avoid the expense of the rotating tests.

A close connection exists between the stress produced in the disc by the centrifugal force and the increase of the frequencies of vibration, as it may be said that it is these stresses which increase the stiffness of the disc and so raise the natural frequencies. Now the hoop stress produced at the rim of a rotating disc varies with the density of the material and the square of its peripheral speed, the radial stress at the rim being zero. We may write hoop stress $f = X \rho v^2$.

The value of X for a uniform disc $= 1/6$ and for a rotating rim with flexible spokes is unity.

It is tentatively suggested by the writer that the following empirical formula connecting B with the stress factor X is likely to give approximately correct results, viz., $B = X h^2 + h(1-X)$. Experimental confirmation of this formula is lacking, and it should be used with great caution, but it may be observed that the formula is correct for the extreme cases of the uniform disc and the rim, and in this respect is somewhat analogous to the well-known Rankine formula for struts which is correct for very long and very short struts.

The rim stress factor X is, of course, quite calculable for a disc of any shape.

The curves of figure 3 give the estimated B values as calculated from the above formula for the modes where $h = 2$ to $h = 5$, and the usual range of the stress factors found in turbine discs is delimited on the curves.

Frequency of Vibration relative to a Stationary Observer.—The natural frequencies of vibration so far considered have been those that would be noted by an observer mounted on and rotating with the disc. To a stationary observer the apparent speed of the waves would be given by $n/h \pm N\frac{(\text{revs.})}{\text{time}}$ and the apparent frequency by $n \pm hN$, $n + hN$ being the frequency for the progressive wave and $n - hN$ for the retrogressive wave, the latter wave travelling backwards, that is in a direction opposite to the disc until the speed is such that $n = hN$ when it will be stationary in space, after which as the disc speed is increased it will travel in the same direction as the disc, but at a slower speed.

Fig. 4 shows the natural frequencies plotted relative to the disc and those of the progressive and retrogressive waves relative to a stationary observer in a typical case.

It should be noted, however, that it is not always the case that the two nodal retrogressive wave comes to rest first.

The Disturbing Forces. (1) *Critical Speeds.*—When the retrogressive wave in any mode comes to rest the vibration may be set up by a stationary disturbing force, such, for instance, as a steam jet falling on one part of the disc and due perhaps to an uneven distribution of nozzles round the periphery. The speeds at which this occurs were termed by Wilfred Campbell the critical speeds, and he produced a considerable volume of evidence to show that these critical speeds were, in fact, very dangerous in the turbines built at that date by his company. There is also evidence that in other turbines considerable vibration of the turbine wheels has resulted from running them at their critical speeds.

(2) *The Windage Theory.*—Reverting to the rotating wash leather disc, if the speed of rotation be increased beyond a certain limit it is observed that the disc starts to vibrate with large amplitude. On examination by stroboscopic or other means, it will be found that the vibration is in the two nodal mode and that the wave is travelling forward in space at an angular velocity equal to a quarter of that of the disc.

(NOTE:—For wash leather $\sigma = \frac{1}{2}$ and when $h = 2$, $B = 1/8h^2 + 7/8\ h = 2\frac{1}{4}$. The frequency of vibration is then given by $n = \sqrt{B}N = 1 \cdot 5\ N$ relative to disc, relative to space the frequency is $2\ N - 1 \cdot 5\ N = 0 \cdot 5\ N$, and hence the rotational speed of the wave $= 0 \cdot 25\ N$ in a forward direction.)

If the wash leather disc be rotated in a partial vacuum instead of in air, the speed reached before the disc breaks out into vibration is increased and experiments indicate that a wave breaks out when the product of the speed of rotation, the density of the air, and some linear dimension, divided by the viscosity of the air (in the usual symbols $\frac{vd\ \rho}{\mu}$) reaches a given figure. It is also noticeable that the speed required for a wave to break out is somewhat greater than that at which a wave dies down on reducing speed, that is to say, there is a region of instability. Both in this respect and in respect of the relationship between density, velocity and viscosity at which the wave breaks out the phenomenon is analogous to the critical conditions required to change viscous flow into turbulent flow in the case of fluid flowing through a pipe.

It is considered that the wave of vibration is blown up by the windage of the disc in a manner similar to that in which waves are raised by the wind at sea. The state of affairs is indicated in Fig. 5.

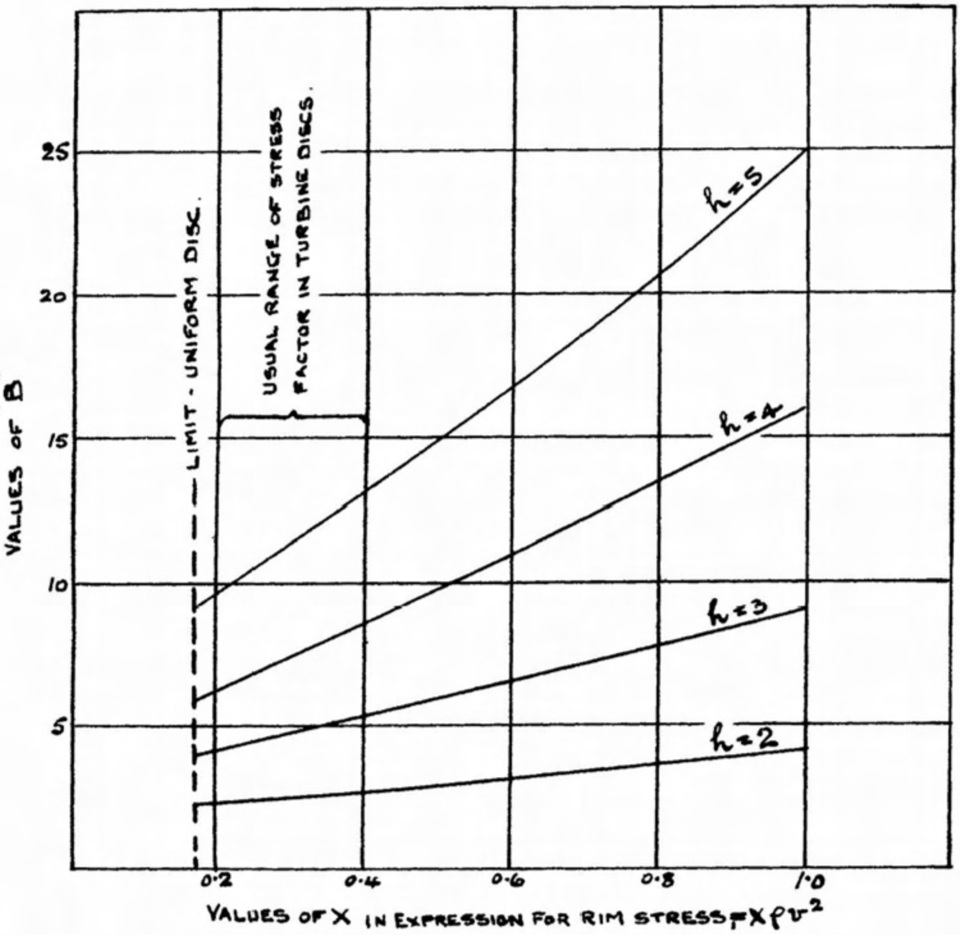

25
20
15
10
5
VALUES OF "B"
0·2
0·4
0·6
0·8
1·0
VALUES OF X IN EXPRESSION FOR RIM STRESS = X ρ v²
LIMIT - UNIFORM DISC.
USUAL RANGE OF STRESS FACTOR IN TURBINE DISCS.
h=5
h=4
h=3
h=2

FIGURE.3.

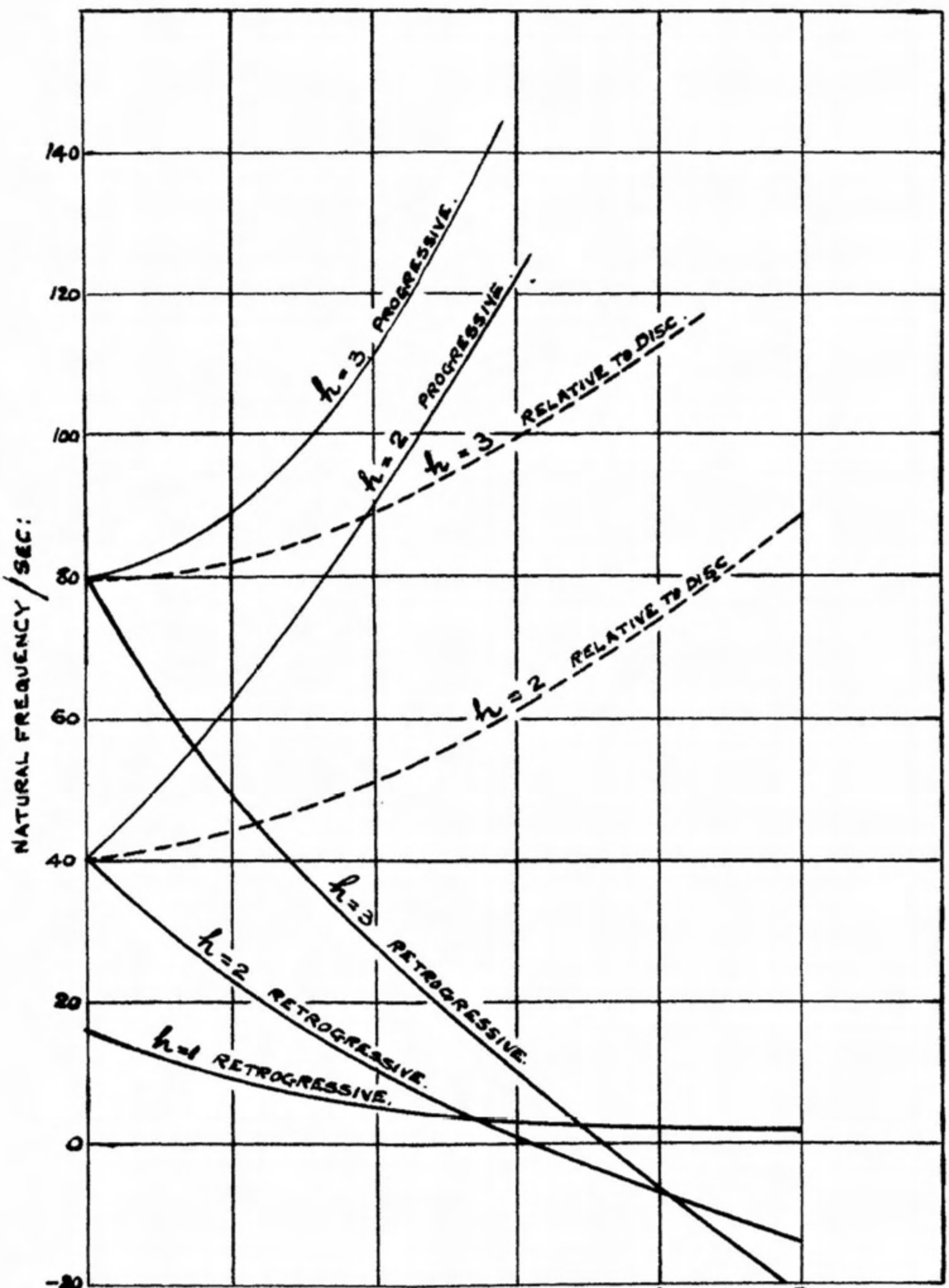
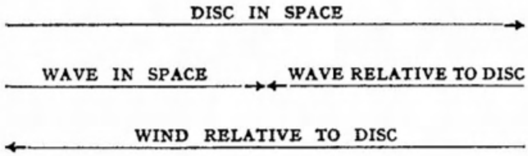

140
120
100
80
60
40
20
0
-20
10
20
30
40
50
NATURAL FREQUENCY / SEC:
REVOLUTIONS PER SECOND
h = 3 PROGRESSIVE.
h = 2 PROGRESSIVE.
h = 3 RELATIVE TO DISC.
h = 2 RELATIVE TO DISC.
h = 3 RETROGRESSIVE.
h = 2 RETROGRESSIVE.
h = 1 RETROGRESSIVE.

FIGURE. 4.



VELOCITY DIAGRAM

Fig 5.

The air surrounding the disc produces a wind effect relative to the disc which is blowing at a speed greater than that of the wave and if a ripple be once started on the disc surface the wind will provide the force necessary to produce the waves.

Similar 2 nodal vibrations have been observed in the case of a steel disc running at such a speed that the waves are moving forward at about one-fifth of the disc speed or more.

It should be noted that the wind will always tend to damp backward moving waves and not to blow them up.

Windage effects cannot be observed in the Wilfred Campbell testing apparatus as the wheels are run in a vacuum.

Wilfred Campbell quotes a case in which oscillograph coils were placed in two large turbines during operation and several stages developed wave phenomena of the same general characteristics which disappeared when the load was reduced. It appears evident that the waves were produced by windage and the reduction of the steam density which accompanied the reduction of the load caused the phenomena to disappear.

A Particular Case.—A special mode of vibration not hitherto mentioned is the vibration of the disc about one nodal diameter. Vibration in this mode exerts a couple on the shaft, whereas the others are balanced in their reactions. For this reason the one nodal node is somewhat difficult to reproduce in the static experiment unless the hub of the disc is very firmly clamped.

" B " is unity for this type of vibration in discs of all shapes and the retrogressive wave is backward at any speed and never comes to rest. There is some evidence, nevertheless, that turbine discs occasionally vibrate in the one nodal mode when the backward speed of the wave is equal to the forward speed of the rotating disc. The cause of the vibration is obscure, and as far as is known there have been no troubles from this source in naval vessels.

Design Practice.—The majority of builders of large impulse turbines for electric generators in this country now design their discs so that the critical speeds are not reached and the retrogressive wave is always backward. Under these conditions a vibration can only be produced by a suitable pulsating force and the occurrence of resonant vibration is very rare; in practice the wheels may be regarded as safe. To accomplish this it is necessary to make discs thick compared to their diameter and the tendency is to fit discs of comparatively small diameter and long blades at the exhaust end, the ratio of diameter of disc to height of blade being sometimes as low as 2·5 : 1. One large firm tunes the discs so that the designed speed of the turbine does not coincide with any of the critical speeds, but the discs may run above one or other of their critical speeds. This tuning is not an easy matter as the frequency of vibration is affected by temperature gradients which may exist in the discs when running and, moreover, as previously seen, there is a danger that windage may cause vibration, especially in the early stages when the steam is dense, if forward moving waves can be blown up. For marine practice where the turbines may be called upon to run continuously at any speed within the designed limits it appears necessary to ensure that there are no critical speeds within the running range and the present Admiralty specification calls for this.

In certain cases it has been necessary to replace defective discs by others of more suitable design both in shore and marine practice ; in this connection it is of interest to note that the natural frequency of discs of similar diameter and profile can be raised in proportion to the square of the thickness by thickening the discs and it appears that for the lower modes, with a small number of nodal diameters, the thickness of the body of the disc below the rim has the chief influence, but for the higher modes the thickness of the rim itself.

The windage theory was first propounded to the writer by Dr. P. B. Haigh, D.Sc., M.B.E., Professor of Applied Mechanics at the Royal Naval College, Greenwich, and the experiments mentioned above with the steel and leather discs were carried out under his direction.