TORSIONAL OSCILLATIONS

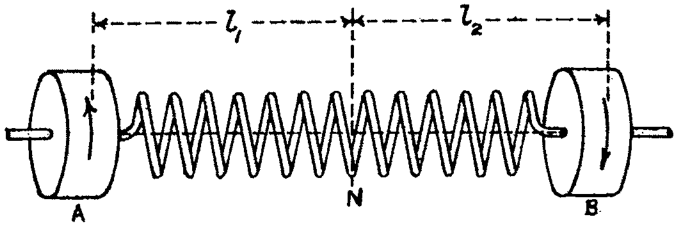

FIGURE 1.

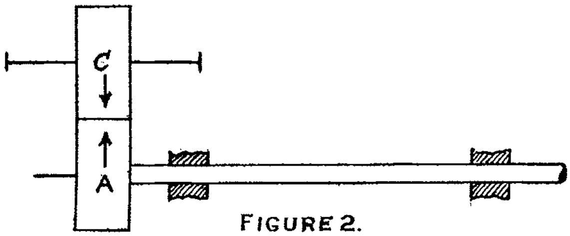

FIGURE 2.

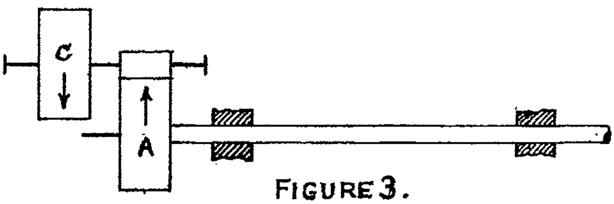

FIGURE 3.

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TORSIONAL OSCILLATIONS OF MARINE PROPELLER SHAFTING.

Torsional oscillations may be set up in a shaft whether it is initially at rest or rotating with a steady mean angular velocity. In the latter case points on the shaft will oscillate about a mean position, while this mean position itself revolves uniformly about the axis of the shaft.

A propeller shaft can be considered as a particular case of a long stiff coiled spring. If each end of such a spring carry a cylindrical mass, and if these are made to oscillate freely about the axis of the spring in opposite directions as in Fig. 1 it is obvious that at some point N there will be no torsional movement of the spring. Impressing on the whole system a steady angular velocity will not affect these oscillations, which will still occur as the spring revolves about its axis.

The position of the point N will depend on the relative moments of inertia (I) of the masses A and B, and this point N where no oscillation occurs is called a " node " or " nodal point."

If the moment of inertia of the mass A is equal to that of B, the position of N is evidently such that $l_1 = l_2$ in the Fig., but if it were twice that of B then l_1 would be half l_2. In fact the position of the nodal point is given by the relation :—

$$\frac{l_1}{l_2} = \frac{\text{Moment of inertia of B about its axis}}{\text{Moment of inertia of A about its axis}} = \frac{I_B}{I_A}$$

Hence, if the moment of inertia of the mass A be very large compared to that of B, the nodal point is quite close up to A and practically the whole of the spring is oscillating about its axis to an increasing degree as B is approached.

This is the case of a geared turbine driven shaft, where A corresponds to the gear wheel and B to the propeller, for though the masses of the gear wheel and propeller may be somewhat similar, yet due to the speed and mass of the turbines driving the gear wheel, their *equivalent* mass, which may be considered concentrated at the wheel itself, is very considerable.

To prove this statement, consider a mass A at the end of a shaft, driving by gearing an exactly similar mass C, the latter turning between centres as in Fig. 2. Then if I_A and I_C are their moments of inertia any torsional oscillation given to A will cause C to oscillate at exactly the same speed, but in an opposite direction. The effect of adding the mass C is equivalent therefore to increasing the moment of inertia of A, so that it is now equal to the sum of I_A and I_C, *i.e.* $= 2\ I_A$.

Now suppose the same mass C is geared to A by a massless pinion, so that it rotates, say, K times as fast as A, as in Fig. 3, any oscillation of A is then retarded by C to a greater extent than before, for C will now be speeded up much faster for a given movement of A.

Under such circumstances the equivalent angular momentum at A, of A and C acting together, will be the sum of the separate angular momenta of A and C, when the momentum of C is referred to the shaft A. This can be allowed for by increasing its value in proportion to the gearing ratio between A and C.

Angular momentum is measured by moment of inertia × angular velocity, *i.e.*, $I.\omega$, just as linear momentum is measured by mass × linear velocity, *i.e.*, m.v. Hence in this case we can write—

Resultant moment of inertia $\times\ \omega = I_A.\ \omega_A + I_C.\ \omega_C\ \times$ ratio of gearing.

And since due to the gearing $\omega_C = K \times \omega$.

$$\text{Resultant moment of inertia} = \frac{\omega_A}{\omega}(I_A + K^2\, I_C).$$

This means that if the whole mass of A and C be considered concentrated at A, such a mass must have a moment of inertia equal to $(I_A + K^2\, I_C)$ when it is moving with an angular velocity given by $\omega = \omega_A$.

In an actual installation there are usually two turbines driving one gear wheel, but the moments of inertia of each can be dealt with in this manner, and from the same reasoning the equivalent moment of inertia at the gear wheel can at once be written down, viz. :—

$$I_{\text{equivt.}} = I_G + (K^2_1 \times I_{\text{H.P.}}) + (K^2_2 \times I_{\text{L.P.}})$$

$$\text{where } K_1 = \frac{\text{Revs. per min. of H.P. turbine.}}{\text{Revs. per minute of Gear Wheel.}}$$

$$K_2 = \frac{\text{Revs. per min. of L.P. turbine.}}{\text{Revs. per minute of Gear Wheel.}}$$

And I_G, $I_{\text{H.P.}}$ and $I_{\text{L.P.}}$ are the moments of inertia of the gear wheel and two turbines respectively.

Since the revolutions of the turbines are large compared with those of the propeller shaft, it is evident that the equivalent moment of inertia at the gear wheel will be considerable, and the preliminary explanation thus justifies the assumption that such a shaft has its nodal point very close to the gear wheel. The shaft may therefore be considered as a spring fixed at the gear wheel end with the mass of the propeller oscillating to and fro at the other end.

Two corrections are necessary, however, at this stage. In an actual case the propeller is oscillating in water and the blades will accelerate or decelerate water in contact with them. This is equivalent to saying that the mass of the propeller must be increased by the mass of a certain amount of water. It is not possible to estimate what this amount will be in any particular case. Various authorities have stated that the moment of inertia of the propeller should be increased from 25 per cent. to 10 per cent. from this cause.

A further addition has to be made to the mass of the propeller due to the axial moment of inertia of the shaft itself. In long shafts this is not negligible, but it can readily be taken into account since the effect of the moment of inertia of a shaft fixed at one end is the same as if it were considered massless and a mass having one-third of its moment of inertia were placed at the free end. Thus the final total moment of inertia to be considered due to the propeller is given by—

$$I_{\text{total}} = I_{\text{propeller}} + \tfrac{1}{3} . I_{\text{shaft}} + \alpha I_{\text{propeller}}$$

where α is a coefficient that may vary from 10 to 25 per cent. or possibly even more.

Assuming I_{total} is known, however, it is easy to calculate the natural period of oscillation of such a shaft, *i.e.*, the time in seconds in which it will oscillate to and fro when given a small displacement and left to itself. This time (t) is given by—

$$t = 2\pi\sqrt{\frac{\text{Mass moment of inertia of oscillating weight.}}{\text{Static torque to cause unit angular deflection.}}}$$

$$= 2\pi\sqrt{\frac{l. I_{\text{total}}}{g.C.I_a}}$$

where I_{total} is as above, I_a is the geometric moment of inertia of the shaft's cross section about its axis, C is the coefficient of rigidity for steel, l the length of the shaft, and g is $32 \cdot 2$ ft. sec.^{2}.

For a given propeller the number of oscillations occurring per second will consequently be much greater for a short shaft than for a long one of similar diameter.

The next point to consider is how these oscillations can be set up in an actual ship. If a propeller be working behind a ship's stern-post in a varying wake, it is probable that the pressure on the blades will vary according to their position. Hence in such a case it may be expected that a periodic force will act on the propeller, and that the number of times this occurs per second will be directly proportional to the revolutions per second of the shaft and to the number of blades of the propeller. It can readily be shown that if this number of forced oscillations coincides with the number of natural or free oscillations of the shaft, the oscillations will go on increasing in size until the shaft ultimately breaks, unless there is some damping effect.

The fact that the shaft is mounted in bearings is not lost sight of, but this does not affect the problem because friction, unless abnormal, does not materially affect the rate of free vibration.

It is necessary, therefore, in the design stage to arrange the revolutions and dimensions of the shafts so that with a given propeller the period of the forced oscillations is nowhere in the vicinity of the natural period of oscillation of the shaft, due regard being given to the extreme possible values of α in the particular case. Even if the shaft did not break very large pressures might be brought on the gear-wheel teeth causing undue wear and tear.

From a paper read before the Institute of Naval Architects in 1921, it appears that satisfactory results have generally been obtained as regards the wear of the teeth of turbine gears when the ratio of maximum torque to mean torque at full power does not exceed 1·4, and that to ensure this the frequency "n" per second of the forces causing oscillation (*e.g.*, revs. per second × number of blades per propeller) should lie outside limits given by the expression—

$$2 \pi n \sqrt{\frac{l.\ I_{total}}{g.\ C.\ I_a}} = \cdot 5 \text{ to } 1 \cdot 3$$

where the symbols have the same meaning as before.

If near, or at full power the number of forced oscillations per second of a shaft coincides with any value of n lying within the limits given by the above formula, some alteration in revolutions or dimensions of the shaft is necessary to prevent pressures on the gear wheel teeth exceeding safe values.

Due to the long lengths of shafting used in H.M. ships there is usually little difficulty in complying with the requirements of the above formula in such cases, and typical designs are well outside the upper figure 1·3, but where the engines are placed well aft in a ship and the shafts are short, dangerous oscillations may occur unless precautions are taken in the design stage.

An apparently clear case of the effect of torsional oscillations was seen in a large mercantile oil-tanker fitted with double reduction gearing, the machinery being fitted right aft. After trials, indications were observed of excessive pressure at definite angular positions around the gear wheel. The frequency of the torsional oscillations was calculated and was found to be about 280 per minute corresponding in the case of the four-bladed propeller to 70 revolutions per minute. The actual revolutions during the full speed trial were 65 to 67. The shafting was therefore running very close to the critical speed and this was remedied to some extent by fitting propellers of larger pitch and thus reducing the shaft revolutions. Although this still left matters in a state which would in the design stage generally be considered undesirable, this vessel has run satisfactorily on service. In the case of a sister vessel the four-bladed propeller was exchanged for one of three blades the revolutions remaining unaltered, but the frequency of the impulses was changed to about 210 per minute as compared with a critical frequency of 280.

The naval designs, in addition to being outside the range of limits given by the preceding formula, are also multiple screw and the periodic variation of forces on the shafting induced by the blades moving in a varying wake are relatively of less amplitude than those in a single-screw ship, thus further lessening the tendency to bring excessive forces on the gears.

By suitable design, therefore, it is quite possible to prevent excessive pressures being brought on gear wheel teeth due to this cause, and in some cases where defects have been attributed to violent torsional oscillations, it is far more probable that any unsatisfactory working of turbine gearing was due either to—

(*a*) the use of unsuitable or defective material;
(*b*) imperfect gear cutting, or unsuitable design of the gear teeth;
(*c*) incorrect alignment of gear wheels or pinions; or
(*d*) insufficient lubrication.

The general question of torsional oscillations in geared turbine installations has recently been given considerable prominence owing to the trouble experienced with the gears for double-reduction fitted in merchant ships. The mathematics of the subject has been ventilated in a paper read at the 1922 meetings of the Institution of Naval Architects entitled "Nodal arrangements of geared drives." The necessity for a full investigation of this question arose from a series of troubles experienced in the ss. "Melmore Head," one of the first vessels fitted with double-reduction gearing, the history of this ship being fully described in a contemporary paper read at the same meetings.

The failures in this vessel, in which excessive noise and vibration followed by rapid wear of the gears were experienced, did not appear to be definitely attributed to any of the causes stated previously, but in forming an opinion on this matter it has probably not been realised that the necessities for accuracy in tooth form in reduction gearing generally, are much more pronounced as the number of reduction stages increases. An error in tooth form or other irregularity in a gear wheel or pinion in a single-reduction gear, necessarily causes or tends to cause a relative departure from uniform motion between wheel and pinion proportional to the irregularity. But if the same irregularity is considered between the gear-wheel and the intermediate reduction element in a double-reduction installation, then the effect is considerably magnified at the pinion teeth of the first reduction. There appears to be little doubt that this feature has not been fully appreciated, and the high standard of accuracy required for a single reduction gear should, therefore, in general, be equalled or excelled in a double-reduction installation if equal efficiency and reliability under the same conditions of tooth pressure, &c., are to be obtained.

In investigating the case of the "Melmore Head" it was appreciated that in addition to the oscillations of the main shaft and gear-wheel, those of the intermediate and primary elements of the reduction system in addition to those of the turbines should be investigated. Disturbances arising from the variations in torque of the propeller may conceivably be transmitted to the primary elements and induce dangerous oscillations in these details. Such oscillations bring about a hammering between the teeth and cause much larger pressures and stresses to be set up than are induced by a uniform transmission of torque. The investigation was found to be exceedingly complex but certain deductions were arrived at which indicated that great improvement could be effected if the system could be so "tuned" as to reduce the number or critical speeds that might be set up in the various parts of the system within the running range of the machinery of the ship. Important differences of opinion on the general question were expressed during the discussion on the papers referred to, and it is still a difficult matter to pass judgment on the method that was eventually adopted for overcoming the difficulties in the "Melmore Head." Still the method adopted is worthy of description in view of the great interest created. By "tuning" was meant, the proportioning of the turbine and pinion shafts so that the time of natural oscillation of each turbine about its pinion extremity was made equal to the natural time of oscillation of the propeller and shaft about the gear-wheel extremity. The reason for designating such a gear a "nodal" arrangement can be understood, for the system as a whole now has a natural period of oscillation equal to that of the propeller and shaft alone, and the node or stationary point of the system is at the gear-wheel. Under the conditions most favourable therefore for setting up resonance the gearing elements are no longer subjected to oscillations and all wheels, &c., in the gear case, apart from errors in the teeth, rotate uniformly although turbines and propeller may be oscillating.

To carry out the idea of "tuning" the turbine and its driving shaft to the same periodicity of the propeller and propeller shaft, the pinion shaft was removed and replaced by a hollow bored pinion shaft with an internal flexible driving shaft as illustrated. The collars on the internal shaft were made a sliding fit in the bore of the pinion shaft and were introduced to prevent any possible whipping of the internal shaft. (*See* Fig. 4.)

The modifications made appear to have had a marked improvement on the running of the gear in this ship, but as previously stated it is impossible to pass judgment in so far as to state that such an arrangement was necessary. There are several gears on service of somewhat similar design to those fitted in the "Melmore Head," and which have not given trouble. It must be appreciated that apart from "tuning" the modifications entailed new pinions and intermediate members of the gearing being fitted and the provision of a measure of flexibility between pinions and turbines. Such changes must be borne in mind and considered as having some effect on the improvement in conditions, but as a scientific application of certain theories the modification is of great interest and continued experience of this vessel and others will be valuable.

As far as naval experience with single reduction gears go, there are gears of all sizes and powers in different classes of vessels which have at times to run constantly at any speed within the working range. There has been no marked evidence in any of the types and sizes fitted that, providing the workmanship is good, the running of the gears has been affected

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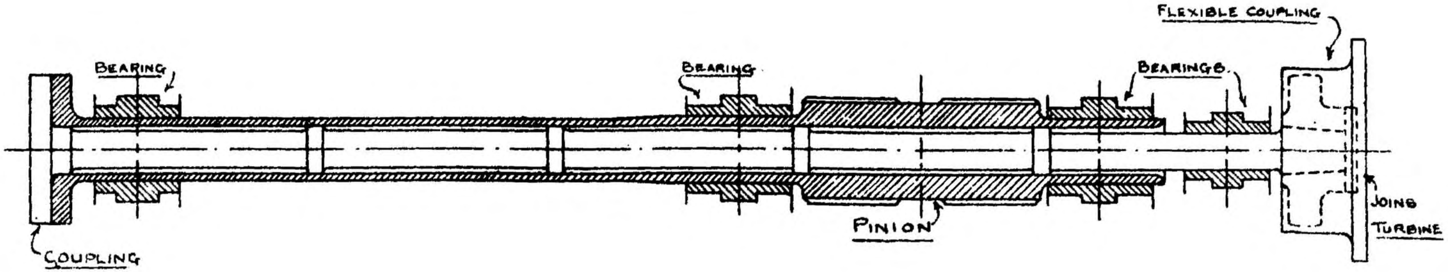

ARRANGEMENT OF NODAL GEARED DRIVE.

FIGURE 4.

by torsional oscillations. In the case of a few vessels the original gears have been very noisy and wear has been experienced on the teeth. Such defective gears have been replaced by exactly similarly proportioned sets which have run satisfactorily so that torsional oscillations pure and simple could not account for the initial failures.

It is, however, recognised that if inaccuracies in the gears are present the running conditions may set up oscillations which then play an important effect in producing hammering and wear.