THE MECHANISM OF TOOTHED GEARING.

In the preceding issue the subject of mechanical reduction gearing in warships was dealt with in its broad aspect. It is intended in this article to consider more closely the actions that take place at the surfaces of the mating wheels and illustrate how the required conditions in the teeth are obtained.

Even at the present time, although experience has resulted in an intimate acquaintance with the practical working of such gears, it seems probable that imperfect ideas exist in the minds of many of this feature of gearing, and the following is intended to clarify these ideas, the mathematical aspect being in this issue purposely, and to a large extent, avoided.

The principal problem to be solved in the design of a pair of toothed wheels is to impart such a shape to the working faces of the teeth, that the motion of the driving shaft shall be transmitted to the driven shaft steadily and without fluctuation. This condition is necessary if smooth and quiet working is to be obtained, and it attains special importance when dealing with large masses rotating at the high speeds current in turbine installations. When underway these masses, in virtue of their weights and speeds of running, possess considerable inertia, and any changes of speed incurred by inaccurate formation of the teeth would require very large forces to bring them about, which forces must arise at the tooth surfaces. The large masses in fact resist any variation in velocity, and to a certain extent any such fluctuation would have to be absorbed in the strain of the teeth themselves. The necessity for uniform angular transmission is therefore at once apparent, and, apart from a true kinematic form to satisfy this condition, the necessity of ensuring that the theoretical form is accurately produced during manufacture and that the gears are in correct alignment readily follows.

The further problems to be solved are that the teeth shall be of sufficient strength to transmit the required force and that they shall possess other proportions so as to enable them to do this with the highest possible efficiency.

It is also desirable, for general purposes, that toothed wheels should be constructed with standard proportions of teeth, so that wheels having any number of teeth of a given pitch will work correctly. This consideration is perhaps of minor significance in turbine reduction gears, where interchanges except as a whole are not required, but it has a bearing in that the cutters used for the production of such gears can be standardised, special tools for various sized gears being not therefore necessary.

Returning to the question of tooth form, theoretically a variety of forms can be given to the teeth of one wheel from which complementary teeth can be determined for the other so that the required ratio of speed reduction can be given. In general,

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however, the cycloids have been used almost exclusively except for special mechanisms. A cycloid is a curve traced out in space by a point on a circle as this circle rolls without slipping on another fixed circle. If the rolling circle becomes very large a portion of this circle is practically a straight line and the curve traced out by a point on this line is of the special epicycloidal form known as the involute, which is the only form of tooth of practical interest at the present time owing to the essential advantages it possess over other forms. An involute to a circle is generally described as the curve traced out by the free end of a cord as the cord is wound on to a stationary disc, the cord being kept taut. This definition it will be seen is synonymous with the preceding, but the first definition enables the fact that the involute is a special form of epicycloid to be appreciated.

The advantages of involute teeth are :--

(1) The flank and addendum form a continuous curve and the tooth has its maximum thickness at the root.

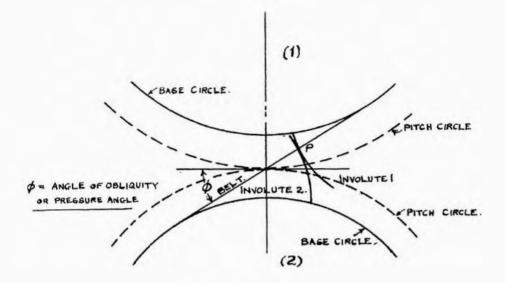
(2) The line of action of the resultant force on the tooth apart from frictional effects is constant, thus giving steadier conditions on the wheels and bearings.

(3) The distance between the centres of the two wheels can be varied so long as the teeth remain in contact without disturbing the constant relative angular velocity of the two wheels. The importance of this can be appreciated for marine gearing where small changes in alignment consequent on the necessary bearing clearances and wear in the bearings cannot be entirely avoided.

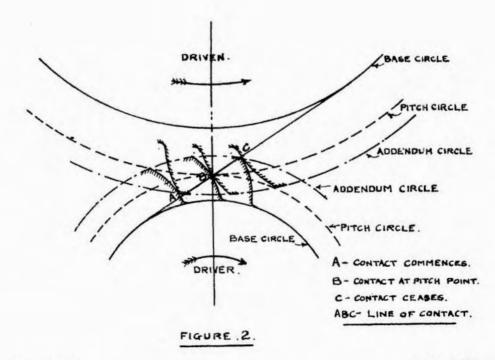
(4) The teeth can be automatically generated in a machine with cutters having straight cutting edges. These cutters can therefore be accurately made and so with due care teeth having the correct theoretical form accurately produced.

The last two conditions in themselves are sufficient to displace other forms of teeth which would satisfy the condition of transmitting uniform velocity. The advantage indicated at (2) is seen by considering the problem in the following simple manner.

Suppose the dotted circles in Fig. 1 represent the pitch circles of two toothed wheels which are required to revolve with angular velocities in a constant ratio, proportional to the pitch circle diameters; imagine two pulleys, shown by the full circles, to be put on the same centres, and to drive one another by a crossed belt; if the diameters of the pulleys be made the same proportion as the diameters of the pitch circles, it will be evident with a non-slipping belt that the pulleys will turn with a constant velocity ratio equal to the ratio desired between the two pitch circles. A point, P, on the belt will by definition describe involute curves relatively to the respective pulley circles, here-after referred to as the base circles, and if the teeth of the respective wheels be made of the same outline as these involutes respectively, the two teeth will always be in contact along the line of the







imaginary belt. The normal to the two tooth surfaces in contact will always also be along this line and the elementary kinematic condition for uniform motion, which is that this common normal should pass through the pitch point at the point of contact of the imaginary rolling discs, will be satisfied.

The inclination of the line of force to the common tangent of the pitch circles at the pitch point is referred to as the "angle of obliquity" or "pressure angle." Retaining the pulleys and belt analogy, advantage (3) above can be appreciated. If the pulleys be moved further apart consistent with the belt remaining tight, the same involute curves are traced out by a point on the belt and the meshing of the teeth would be unaffected as regards the uniformity of transmission, but the inclination of the line of action of the normal force is altered.

The various terms used in connection with toothed wheels should be well-known and will not be described here, but it is only desired to point out that when definite heights have been assigned to the teeth, it will be obvious that contact will commence between the root of a driving tooth and the point of a driven tooth where the addendum circle of the driven wheel cuts the line of the imaginary belt. Similarly, contact will cease where the addendum circle of the driving wheel cuts this line. See Fig. 2.

It will be seen that the diameters of the base circles in comparison with those of the pitch circles depend on the angle of obliquity selected. This angle should from various considerations lie between somewhat narrow limits, however, and in normal cases it ranges from $14\frac{1}{2}^{\circ}$ to 20° , the latter figure being exceeded only in very special cases. The angle $14\frac{1}{2}^{\circ}$ is a generally adopted standard angle, a departure from it being only found in gears where some special features have to be satisfied. It is obviously desirable to keep the angle as low as possible from consideration of the forces on the teeth. The actual force between two teeth is, neglecting friction, normal to their faces, and the component in the circumferential direction is the part doing the useful work in the transmission of power. The normal force, however, has to be taken into account when considering the strength of teeth, surface of teeth required, bearing pressures, and the friction between the teeth which must always exist to some extent. From these considerations the necessities of a small angle are seen. The smaller the angle of obliquity, however, the nearer the base circles approach the pitch circles, and the respective addendum circles of the teeth will eventually cut the base circles of the opposite wheels. From the definition of the involute, however, it must finish on the base circle, and any portion of the tooth lying within the base circle must be cut away to give clearance to the involute face of the engaging tooth to avoid "interference." This leads to weakening of the tooth and the lessening of the surface of the tooth that can be utilised. The foregoing in its broad aspect gives the main reasons for selecting

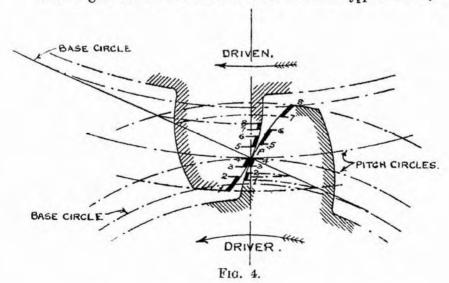
an angle of $14\frac{1}{2}^{\circ}$ as a standard angle. The reason for $\frac{1}{2}^{\circ}$ lies in the value of a simple trigonometrical function that the angle $14\frac{1}{2}^{\circ}$ possesses, its sine being $\frac{1}{2}$, thus simplifying design calculations. The diameter of the base circle with this angle is always .968 of the pitch circle diameter. With any chosen angle and particular tooth proportions, there is always a limit to the number of teeth that may be cut on the wheels to avoid the "interference" referred to above. With this particular angle and standard proportions the number is about 24. If circumstances compel wheels with a less number than 24 teeth to be used, either the undercutting and interference must be accepted or corrections applied in other ways. Two ways are possible :—

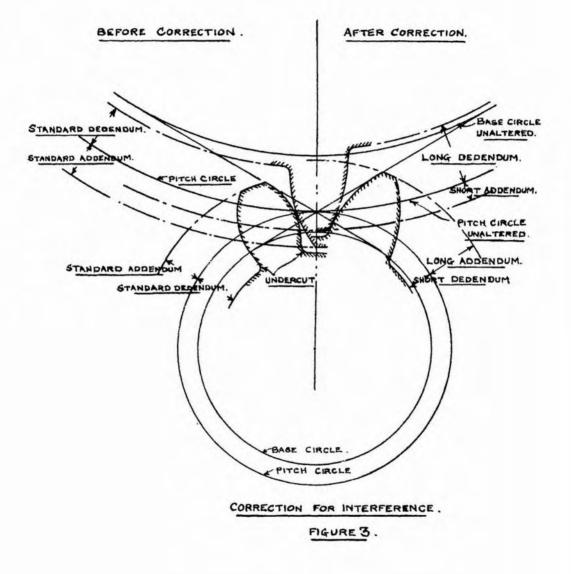
(1) By increasing the angle of obliquity, or

(2) By altering the root and outside diameters of both pinion and wheel.

The effect of the first alteration is obvious, the base circles being now further removed from the pitch circles, thus enabling the involute portion of the tooth to be increased.

With the second alteration the diameter at the root of the teeth on the small wheel is increased in proportion to the amount of correction required, thereby reducing the length of that portion of the tooth which was not of the involute shape (Fig. 3). The outside diameter is then increased in exactly the same proportion. This will give the pinion teeth long addenda and short dedenda. If with this modification the pinion teeth do not intersect the base circle of the wheel to an extent to cause appreciable undercut, the true involute form may be preserved in the main on that member, the teeth of which will now have short addenda and long dedenda, the pitch and base circles remaining unaltered. If this method is not entirely practicable,





it may be combined also with the first modification and a necessary compromise obtained.

Nature of Tooth Contact.

The nature of the action between two mating teeth can be seen by Fig. 4, which shows diagrammatically two involute teeth in contact at the pitch point P. Contact will commence where the point of the driven tooth intersects the line of obliquity, i.e., at this instant the points 1 and 1 will be in contact. Divide the driving tooth into four equal parts on each side of the pitch circle as shown by the figures on this tooth 1 to 8. The corresponding points of contact on the driven tooth can be easily constructed when it is remembered that contact always takes place on the line of the imaginary belt. Thus it will be found that the points, 1, 2, 3, and 4 on the pinion tooth contact at 1, 2, 3 and 4 on the wheel tooth. Similarly with the corresponding points 5, 6, 7 and 8. If the distances between corresponding points on the respective teeth be compared the sliding and rolling between the faces can be followed, e.g., 1 to 2 on the pinion is short and as 1 starts on 1 and 2 eventually arrives on 2. the difference in length of 1 to 2, and 1 to 2 on the respective wheels must be a measure of the sliding that has taken place during the action between these two parts of the faces. In the same way the shorter distance 1 to 2 must represent the distance made up by rolling contact.

The greater the amount of sliding, the greater will be the loss by friction and probable wear of the gears. From this consideration rolling contact should be aimed for to approximate to conditions as obtain in roller and ball bearings. The proportions between rolling and sliding are to a certain extent variable by the particular choice of tooth angles and proportions. This is the aim of the manufactures of the Maag gears used for special purposes in the commercial world, the gears in these cases being accurately ground after hardening. Rolling contact, with a minimum of sliding which cannot be entirely avoided, form the main feature of the design and these gears are run without oil, the only lubrication of the teeth arising from the oil vapour in the case of oil obtaining access to tooth surfaces from leakage of the bearings. There is something however to be said for sliding contact. As is well known, the principle of action involved in the lubrication of a bearing or in the Michell Thrust, is the creation of an oil film between two surfaces, one of which is sliding over the other. If sliding takes place, the velocity of sliding for maintenance of an oil film must be above a certain very low value. This low value is normally well exceeded in gears and therefore if subjected to lubrication, the general conditions are such that an oil film is possible. This oil film serves a double purpose, it decreases the frictional losses and also lessens the intensity of pressure on the tooth faces. Under dry conditions the gears if accurate and without elasticity would be in

contact on a line. Due to elasticity of the materials, however, flattening takes place and contact is on a narrow strip. With the oil film the strip is potentially widened and the load, therefore, distributed over a greater area. At a pitch point where rolling contact takes place, the oil film is possibly at times pierced and the intensity of pressure is greatest. This possibly explains one of the causes of the pitting to be seen on the teeth of gears. During the sliding action while in contact off the pitch circles, the slight irregularities on the teeth are flaked off and worked down towards the pitch point and forced into the surfaces of the teeth. It has been found that the action is most pronounced near the pitch line, and as it usually ceases after a certain period of working as might be expected from the above theory when the irregularities are removed, this would appear to be the most probable cause of the action.

The nature of the conditions between two teeth has its influence on the permissible pressures that may be allowed in As pointed out, the action either takes place on a strip practice. of metal or on an oil film. If metal to metal contact takes place, the permissible load should vary as the radius of curvature of the surface, whereas if an oil film up to a certain thickness be assumed capable of standing the pressure being transmitted, the load will be spread over a surface depending on the square root of the radius of curvature. (In each case the combined radius of curvature of the surfaces must be considered.) In reduction gears as encountered for ship use, the wheel teeth are sensibly flat and may be assumed to possess no curvature, the surface of the pinion teeth only being therefore considered. It is still a matter of opinion, owing, perhaps, to want of intimate knowledge of the action between teeth of lubricated gears to say whether the law varying as the radius or the square root of the radius of curvature should be worked to, although general opinion tends to the latter. With an angle of obliquity of 14¹/₂° the mean radius of curvature of pinion teeth is one-quarter the radius of the pinion. The load should therefore vary either as the pinion diameter or the square-root of the pinion diameter.

The ratios
$$\frac{\text{Load per inch}}{\text{Diameter}}$$
 or $\frac{\text{Load per inch}}{\sqrt{\text{Diameter}}}$ are often referred

to as the respective "load constants." One advantage of working to the second law is that if a value of the load constant is fixed, then the larger the gear becomes and therefore the more valuable, and more difficult to manipulate and replace if necessary, the load per inch on the gear does not go up so rapidly as if the first law were worked to, thus giving possibly a greater margin of safety the larger the gear.

The tooth surface is more important in its effect on the efficiency of gears than the section at the root of the teeth, as although the necessary strength must be given, it happens that with the standard designs of teeth worked to and with the permissible limits of pressure at the surfaces, the stresses on the roots are quite small in view of the materials used. This, of course, in view also of the serious consequences that may follow a tooth failure is as it should be, especially, too, as the designed pressure and stress may at times be subject to large increases due to deviations from correct workmanship, alignment, errors due to torsional strains, and temperature effects, apart from shocks.

So far the problem of tooth contact has been considered in relation more to straight cut spur wheels, but the same general arguments apply also to helical gearing. In straight cut spur gears with the usual proportions, the whole load may at times be taken on one tooth, and in any case is not usually shared by more than two. The load on the teeth therefore comes on suddenly, and may suddenly increase at a certain stage of the contact. with the same sudden decreases of load when passing out of In helical gearing, however, only a portion of a tooth contact. is under load at any instant, and this load comes on more or less gradually, passes along the tooth, and then passes off gradually. Other conditions being the same, the teeth of helical gearing may therefore be subjected to a greater intensity of load than straight cut spur gearing, enabling smaller teeth with attendant advantages of smoother running, and higher efficiency to be obtained. In helical gearing, the involute tooth form, as generated by the cutter, is found in a plane at right angles to the tooth, the section perpendicular to the axis giving a more inclined tooth form, or a greater angle of obliquity. In determining the total length of tooth surfaces in contact therefore the new angle of obliquity must be calculated to enable this contact to be accurately determined.

The construction for the line of contact in helical gearing will be understood from Fig. 5, and what has gone previously. When this procedure is carried out for all the teeth in contact the total length of contact is obtained and has been given for typical gears in the article in the previous issue.

It will be appreciated from the figure that the contact depends on tooth proportions, helical angle, angle of obliquity and length of working face in the axial directions. When comparing helical gears, the load is often expressed in terms of the load per inch measured on the axial length of the pinion, the force on which this is based being that giving the required torque. Such a load must be used guardedly, as owing to the effect of the other factors, it does not convey a true idea of the normal load on the teeth per inch of contact which load governs running efficiency and tooth strength. The total normal load on the teeth is greater than the torque load, and depends on the helical and pressure angles. A further comparison with the tables given in the preceding number will enable the whole of these details to be appreciated. It should be noted in these gears that the law $p = 200\sqrt{D}$ where p is load in lbs. per axial inch, and D is the diameter of the pinion is closely approached, but the actual pressure with the 30° gear is much less than with the 45° gear. The reason the 30° gears when adopted were not made smaller was based on the preservation of interchange-ability as a whole with the 45° gears, giving at the same time a greater margin of safety on the same overall proportions and reducing the number of spare sets required.

It should be noted that in the early days of turbine gearing there was not much experience available bearing on helical gearing for comparatively high powers, and in the first Naval designs the gears were arranged with a helical angle of 23° in conjunction with a pitch normal to the line of the teeth of $\frac{3}{4}$ in. This design was rather noisy, and in consequence the angle was increased to about 45°, which value was known to have proved satisfactory in De Laval installations of small power. The 45° design was continued for some years with generally satisfactory results, but in the meantime, with the improvements in gear cutting processes, it had gradually been established that the noise and tremor experienced in the early installations, which in any case is anticipated as the gear approximates to a straight cut spur gear, was due more to irregularities in cutting than to the small helical angle itself. An angle of 30° which possesses advantages in the way of efficiency and tooth strength, in that the effective contact between the teeth is increased as compared with 45°, while the normal load on the teeth is decreased, has now been generally adopted by the Admiralty.

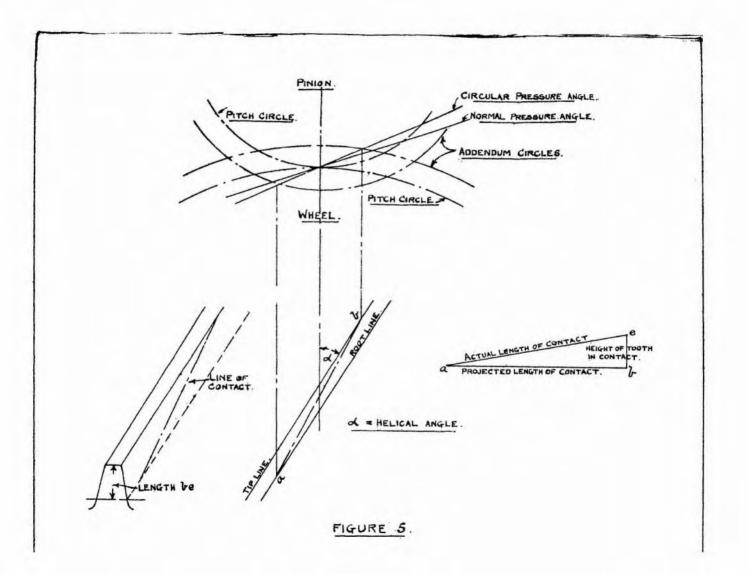
As has been already indicated, the intensity of pressure assumes that the contact between all teeth is accurate, but certain considerations arise in practice which tend to upset the contact and these must be carefully considered during the design and construction stages. Causes of an incorrect contact assuming the actual tooth form is correct are :—

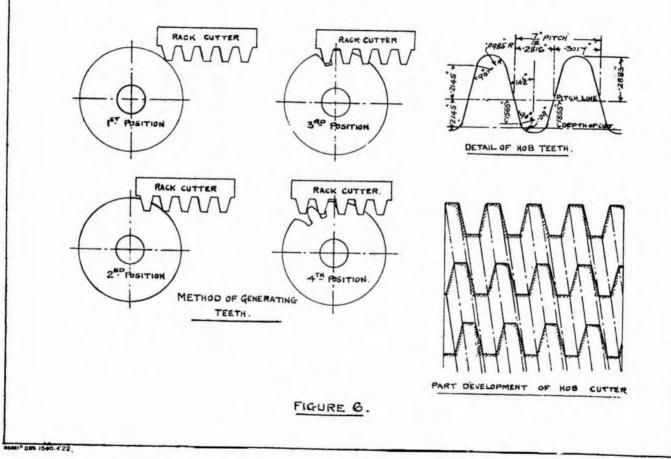
(1) Helical angle of pinion and wheel being slightly different.

- (2) Torsional strain of pinion.
- (3) Transverse bending of pinion.
- (4) Faulty alignment.
- (5) Temperature effects.

The effect of (1) is obvious and must be guarded against during the cutting of the gears, no subsequent correction being possible. It depends not on the accuracy of the cutter so much as the right proportion between the speed of rotation of the job in the machine, and its correct relation with the feed motion of the tool in the axial direction. Great accuracy is required, the allowable difference being only such as will still preserve an oil film the whole length of contact, and it should not, therefore, amount to more than a few thousandths of an inch at the most.

As regards (2) and (3) when under load the driving end of the pinion advances circumferentially with respect to the other end and the helical angle of the pinion tooth is, therefore, altered.





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The gear wheel is a very stiff structure and its torsional strain may be neglected. Due to the transverse forces the pinion is bent between the bearings and with it the teeth, and the alignment of the pinion teeth and gear-wheel teeth is consequently altered, the effect of the forces on the gear-wheel in this case being also negligible.

The gearing is divided into two portions of opposite hand in order to balance the axial thrust. The torque is necessarily taken equally by the two portions, for the effect of any one portion taking more than the other could only arise with a disturbance of the normal and axial forces on each portion. The discrepancy in the axial forces would cause displacement of the pinions till these forces balance and other forces also equalise. The effect of the torsional deflection of the shaft between the pinion elements, which would have the tendency to induce the near pinion element to take all the load is similarly discounted. while its axial stretch or compression due to the axial load is corrected by a small automatic rotary movement of the pinions with reference to the wheel. The torsional deflection of the pinions themselves, which tends to cause a concentration of tooth pressure at each driving end cannot, however, compensate itself in rigid gearing. It is necessary, therefore, to limit the ratio between the length and diameter of the pinion elements in order to keep this deflection down to an amount which can be absorbed by variation in the thickness of the oil film on the teeth or by the strain of the teeth. This amount is usually not greater than .0015 inches. The width of face obtained on this basis will usually safeguard the design from undue transverse deflection if as in Naval designs the pinion is carried on three bearings, the necessity for these three bearings mainly arising from this consideration.

As regards (4) this is a question of workmanship combined with a due appreciation of the effect of the forces on pinions and wheel under the running conditions. This point was dealt with fully in the preceding article.

As regards temperature, due to the more frequent contact of the individual pinion teeth as compared with those on the wheel, and also owing to the fact that the wheel presents a greater surface for radiation, the heat generated by the frictional effects on the teeth will, as regards rise of temperature of the parts, affect the pinion much more than the wheel. In view, however, of the rise of temperature being minimised by the ample supply of oil delivered by the sprayers, any slight change of helical angle will be of negligible quantity, as an increase of length should also be accompanied by an equally proportionate increase in diameter. The effect of the rise of temperature will be more pronounced on the tooth form than on the helical angle, and as this is a more serious fault owing to its effect on smooth working and life of the gear, the ample oil supply for carrying away the heat is more important in this particular aspect.

Cutting of Teeth.

The modern methods of cutting the teeth are intimately associated with the mechanism aspect and will be briefly considered.

Until a few years ago the only method of cutting involute wheels was on a dividing engine using milling cutters formed to the shape of the spaces between the teeth. In making the cutters, a draughtsman made a drawing of the space, a fitter then made a template from the drawing and a toolmaker made a cutter to the template. There were thus three possible sources of error, apart from subsequent errors in the actual process of cutting the teeth. For a given pitch the spaces are different for every different number of teeth, so that for absolute accuracy it would be necessary to have a separate cutter for each number of teeth. As this was not always practicable, it was customary to make a series of cutters, say about eight, for each pitch. Thus. when cut on this system, only eight sizes of wheel for each pitch could be cut accurately and all other sizes were more or less inaccurate.

Such a method of cutting teeth would obviously not be suitable for high speed gears where great accuracy is necessary, and the modern method is now almost entirely confined to machines of the involute generator type.

Wheels with involute teeth of the same general proportions, pitch and pressure angle, possess the property of interchangeability in that any pair of such wheels correctly mounted will mesh accurately with one another to satisfy the main condition of transmission of uniform angular velocity. The limiting case of such a set of wheels is that having infinite radius, or, in other words, a straight rack, the sides of the teeth of which consist of straight lines at right angles to the line of incidence.

If an involute rack of hard material be taken and geared with a wheel blank made of some plastic substance and the two be run together at the correct relative speeds, the rack will impress teeth on the plastic wheel blank. These teeth will be conjugates of the rack teeth and will be of true involute form (see Fig. 6). This is the principle upon which cutting machines of the involute generator type are constructed. In these machines a tool steel cutter in the form of a rack is provided. It has two motions, firstly, a cutting motion parallel to the axis of the wheel, and secondly, a feed motion tangential to the wheel rim. The tool and wheel blank are correctly geared together, and before each cutting stroke the cutter is advanced tangentially a short distance and the wheel blank is revolved a corresponding amount. The cutter thus forms the teeth by combing out the material between them.

This method of generating teeth by a planing process possesses disadvantages. The rack must, from the point of view of stiffness, be of limited length and would normally run out of engagement with the blank after cutting a number of teeth.

It is, therefore, necessary at intervals to index it back a distance equal to a multiple of the pitch, and this is a source of possible inaccuracy. The principle of cutting is obviously unaffected if. instead of employing one rack, we have a series of racks, each being in advance of the preceding a distance equal to the feed given to the single rack at each cut. If this series of racks be arranged round a cylinder so that one revolution of the cylinder. which is fixed in space, brings the first rack again into engagement with the blank when the latter has advanced one tooth space, it is obvious that this series of racks will mill out depressions in the blank such that, if at the same time it be given a very slow feed in the direction of the axis of the blank, involute teeth will be generated over the whole length without any further adjustments or indexing being necessary. This process is known as the hobbing process, the special milling tool being known as a hob, the general appearance of which resembles a large screwing To cut helical gears the process is exactly similar, the hob, tap. however, being inclined so that the line of cutting taking the relative motion of the tool and blank into consideration is in the line of the helix required.

The foregoing illustrates advantage (4) of involute teeth, the successive racks constituting the hob having straight sides so far as the part generating the working teeth surface is concerned, and these parts can be accurately gauged and ground. The further proportions of the cutting teeth are only such as to give a well defined radius at the root of the teeth being cut, the necessary clearance for the points and in some cases a slight rounding of the tooth point. This is to prevent, under working conditions, digging in of these points and the formation of a razor edge which tends to scrape the oil film from the tooth when engaging or disengaging. The tendency to dig in is accentuated by rise of temperature as this will produce its greatest effect at the point of the pinion teeth, due to the greater lineal increase of the pitch at this position.

The necessity for accuracy of tooth forms has already been indicated, and the foregoing description will enable the means by which errors may creep in to be appreciated and dealt with. The errors may arise in the gear-cutting machine itself, and such machines should, therefore, be checked for accuracy before commencing the cutting of a large wheel, as after cutting subsequent correction is impossible. It is necessary that all relative motions of the hob and blank should be strictly uniform, and a possible source of error arises in the worm-wheel attached to the table or face-plate carrying the job. Any error in this wheel will be copied on the job and give inaccuracy or inaccuracies parallel to the axis of the blank. It was this possibility that led to the adoption in earlier machines of a "creep" mechanism, which involves the addition of an epicyclic train of wheels causing the table to rotate at a speed a certain percentage faster than the worm-wheel. Any error existing in this wheel is thereby distributed in spirals round the job and its effect minimised by this distribution. With the more accurate methods of manufacture of these worm-wheels now current and with the exercise of due care to the lubrication of this detail during a cutting operation, this source of error has been practically eliminated and the necessity for the creep mechanism, with its possible small errors due to additional back-lash and lost motion, obviated, so that it is not now fitted to new machines.

The hob, besides being accurate as regards its cutting edges, must be correctly mounted so as to be free from axial and side motion and must be at its correct inclination to the axis of the job. Its feed motion must also be truly parallel to the axis of the wheel being cut. The blank must also be set truly central on the table or face-plate.

The cutting operation should be continuous so that all parts of the driving mechanism of the machine are under a steady strain with the back-lash of the change-wheels, &c., destroyed, the hob and face of blank being steadied at a practically uniform temperature by a copious lubricating supply. Due attention being paid to the foregoing main details in addition to minor points should lead to a wheel possessing the accurate form of tooth that will enable it under normal conditions to give the efficiency and durability required.