BACKLASH AND RESILIENCE IN GEARING AND STRUCTURES

BY

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In most engineering structures, strength is of more importance than stiffness, and provided there is sufficient volume of metal to carry, as strain energy, the kinetic energy resulting from any likely motions, the stiffness/inertia relationship is not of great import. But it is necessary to ensure that natural frequencies do not resonate with any likely cyclic disturbing forces, external or internal, although, in general, the precise relationships between the forces arising during operation and the resulting yields are not a determining factor in design.

Machine tool structures and gearing are cases where these relationships are very important, and in the remote control of heavy structures by servo motors this overall structural stress/strain relationship becomes the dominating factor in design. Typical extreme cases occur in naval gunmountings, radar aerial structures, and directors, and this article deals with some recent work on them. The results are of general application, and are likely to be of interest in other fields.

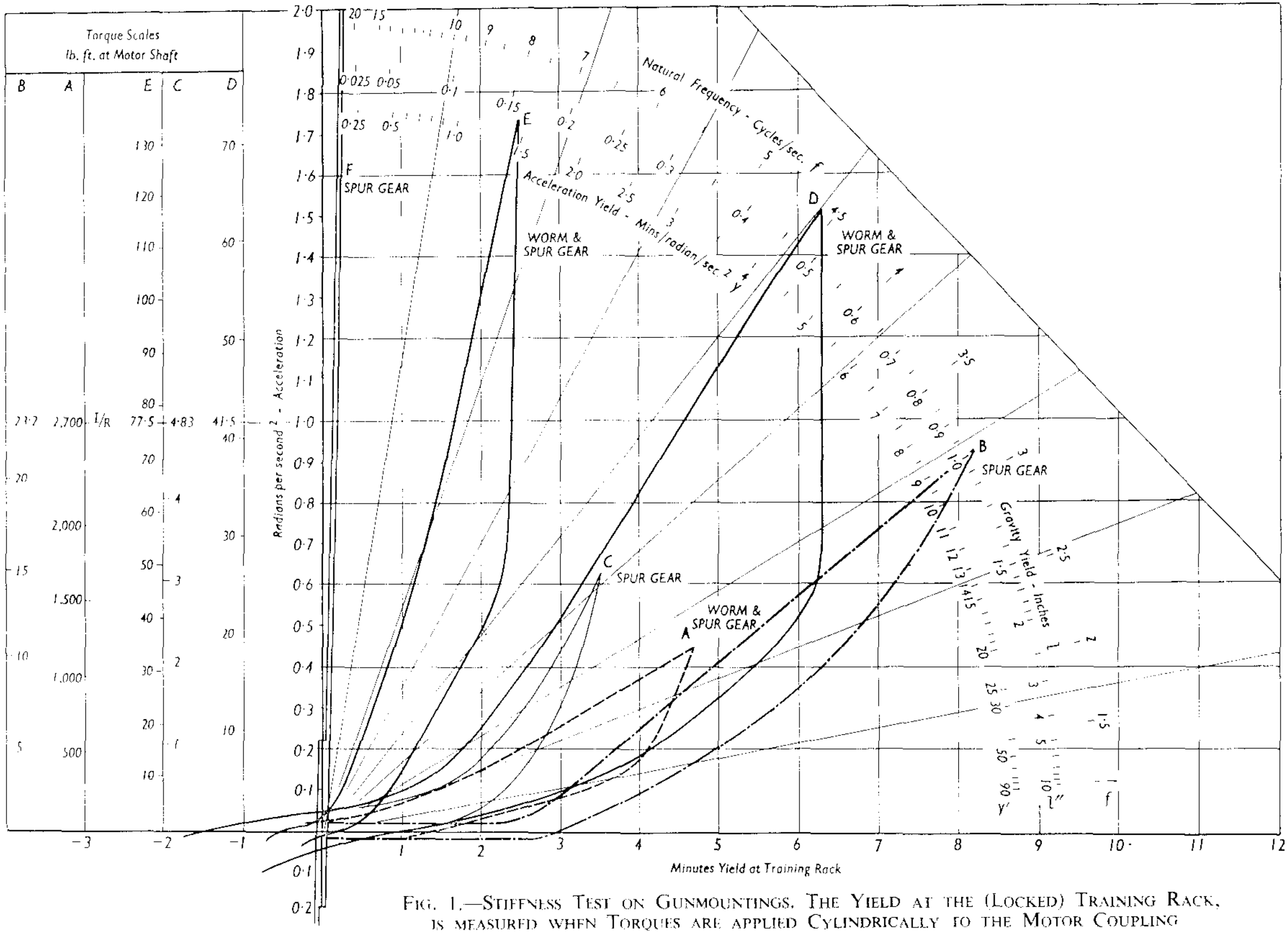

FIG. 1.—STIFFNESS TEST ON GUNMOUNTINGS. THE YIELD AT THE (LOCKED) TRAINING RACK, IS MEASURED WHEN TORQUES ARE APPLIED CYLINDRICALLY TO THE MOTOR COUPLING

Servo theory, dealing with backlash as a fixed quantity, and assuming that beyond the limits of backlash the stress/strain relationship between load and motor is more or less linear, indicates means for minimizing the effect of these structural defects by modifying the control system, as for example, by the well known 'divided reset', which is, in effect, resetting from an adjustable nodal point in the resilient drive between motor and mounting. It assumes that the torque/yield relationship is reasonably linear.

Some years ago, trials were started to find out what this relationship really was, and what its effects were, in various types of mountings in service. As experience grew, the method of carrying out the trials, and of presenting the results, developed into the form of a mechanical hysteresis loop, or a series of successive loops, on the precise analogy of a magnetic hysteresis test. The results are very illuminating, and their application has resulted in a revolution in servo performance.

For carrying out the stiffness test, the mounting is firmly locked at the training rack, and means are provided for measuring any yield of the locking arrangements. Torques are applied cyclically, in increasing amplitudes in both directions to the motor spindle, and the angular yield is recorded. Subtraction of the angular yield of the locking arrangements gives the yield, which, when plotted out as a hysteresis loop, enables backlash, plastic yield, and material stiffness to be effectively separated.

FIG. 1 shows a number of typical loops from tests of actual mountings, referred to as *A*, *B*, *C*, *D*, *E* and *F*, in the approximate chronological order of design date. The torque at the motor spindle is converted into the corresponding mounting acceleration. If I is the moment of interia in pounds, feet, seconds^2, and R is the overall gear ratio, I/R lb-ft torque is one radian per second^2 acceleration. Plotting the results to an acceleration/yield basis puts them in non-dimensional terms, where the performance of all different types of mountings can be directly compared. The examples shown cover a range of torque at the motor spindle, for the same mounting acceleration, of about 500 : 1. The loop shown at *F* is the only example whose design embodies the results of the work described in this article.

In a diagram of the type of FIG. 1, if the hysteresis loop can reasonably be represented by a straight line through the origin the mounting can be said to have the natural frequency represented by the slope of that line ; but where there are large ambiguities in the torque/yield relationship there is no real natural frequency at all. The diagram can be made to any suitable scales of acceleration and yield, and the scale for natural frequency comes out as follows :

$(2\pi f)^2$ = stiffness/interia = acceleration/yield, so y, minutes yield per radian per second^2 = $3440/(2\pi f)^2$ (f = cycles/second) = $87/f^2$. If own weight applied as a torque at own radius of gyration K gives l inches yield, g/K radians/second^2 acceleration gives $l \times 3440/12K$ minutes, or 1 radian/second^2 acceleration gives $l \times 3440/12g$, i.e. $9 \times l$ minutes of arc, and we get the following relationships between y, the acceleration yield (minutes per radian/second^2), l, the gravity yield in inches, and f, the natural frequency in cycles/second :–

$$y = 87/f^2 = 9 \times l$$

$$l = y/9 = 9{\cdot}7/f^2$$

$$f = 9{\cdot}35/\sqrt{y} = 3{\cdot}1/\sqrt{l}$$

These relationships are plotted out in FIG. 2 for ready reference from one to the other.

The quality desired in the constructions under consideration has been described as 'stayputness', and there seems no better word to describe the capacity of a structure to retain its integrity of form under the action of acceleration forces. 'Stayputness' is, of course, the ratio of stiffness/interia, and it can be defined in the term of 'natural frequency of oscillation', or of gravity yield, the extension of a simple spring mass system under its own weight (or, in the case of a rotary system, the linear yield, at the radius of gyration, due to a force equal to own weight applied tangentially at that radius), or in terms of the angular yield per unit of acceleration.

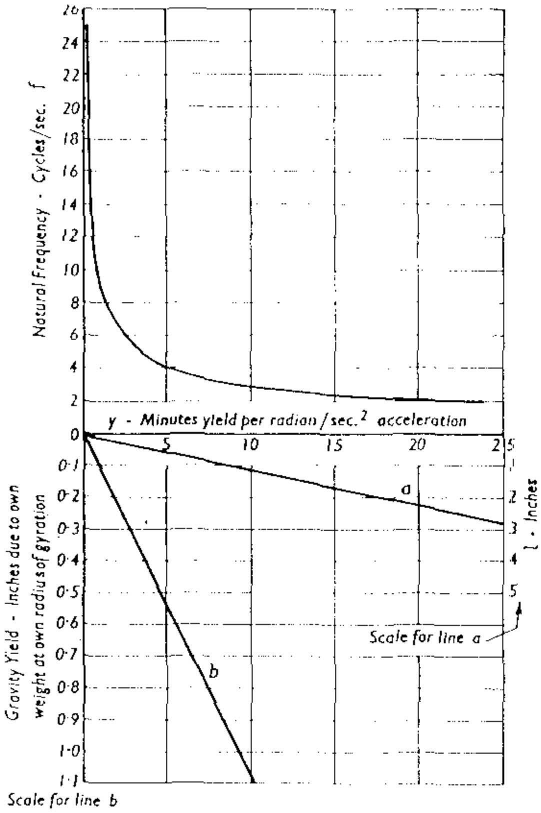

FIG. 2—RELATIONSHIP BETWEEN NATURAL FREQUENCY, GRAVITY YIELD AND ACCELERATION YIELD

As regards servo performance, the natural frequency, as such, has very little influence ; resonance effects are not material. The serious matter is the acceleration yield, and also the ambiguity of the torque-yield relationship, both of which must be kept low. This implies a high natural frequency, but below a certain figure of acceleration yield, no useful purpose is served by raising the natural frequency higher. Acceleration yield is the important matter ; natural frequency is perhaps the most familiar way of defining it, and gravity yield is often the most convenient way of measuring it.

Backlash in heavy gearing cannot usually be considered, as in instrument work, as a sum of mechanical clearances. Considerable force may be needed to overcome friction, and backlash is best defined as the motion, to and fro, on the input side, which will just— or just not—initiate motion on the output side. It is a single quantity, without sign, and if the first cycle of the hysteresis loop is only taken to slightly more than the slow creep torque it can be defined as shown in FIG. 3(*a*). If the next loop is taken down to a greater negative torque, as point *c* in FIG. 3(*b*), and then continued back to the first positive value, we get the distance '*a*' as the plastic yield due to the difference in torque between *c* and *b*, and so on. The whole loop may come out like FIG. 3(*d*), where friction is relatively high, or, as in FIG. 4, where the creep torque is too low to measure.

The largest loop defines an area of ambiguity in the torque/yield relationship, and any point within this area has the same validity as any other point, depending on the direction from which it has been approached. There are a surprising number of places in a composite structure where small yields may occur, if design is not carefully watched. The true material stiffness line will lie somewhere between the upper line of the loop, which gives stiffness plus friction, and the lower line, which is stiffness *minus* friction. An approximate stiffness line is shown dotted in FIGS. 3(*c*), (*d*) and 4.

The difference in character of the loops between worm and spur gearing is very noticeable. That between loops *D* and *F* (FIG. 1) is startling, and the

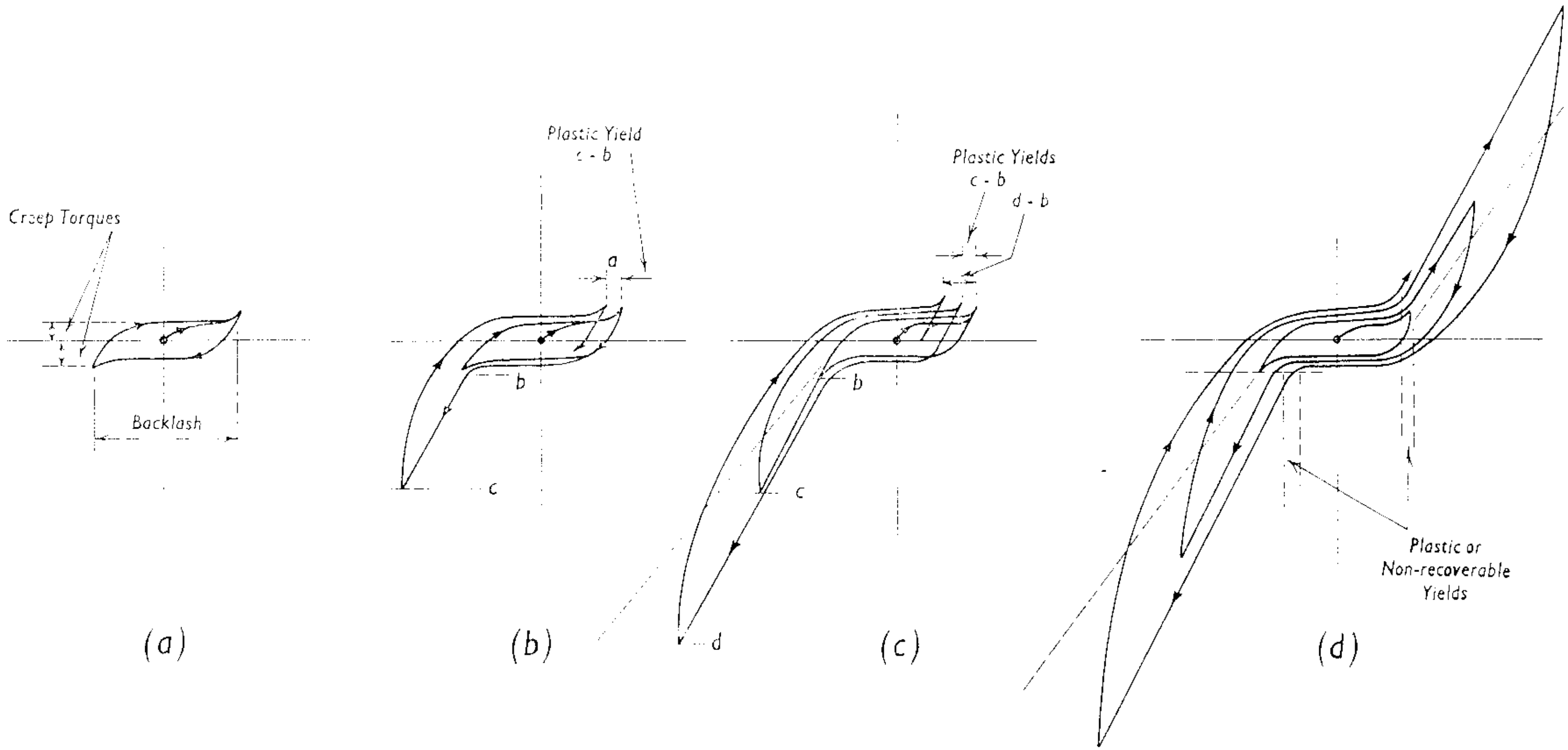

Fig. 3—Hysteresis Loops, showing Backlash and Plastic Yield in Heavy Gearing

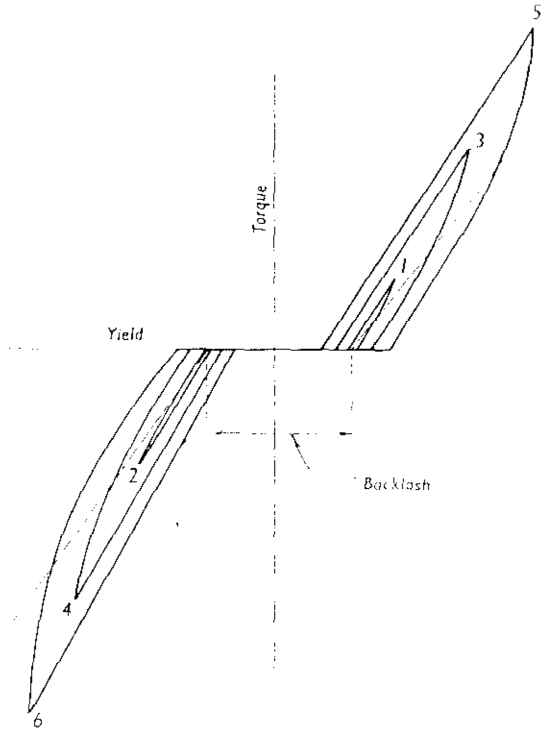

FIG. 4 TYPICAL HYSTERESIS LOOP WHERE THE CREEP TORQUE IS TOO LOW TO MEASURE

difference in reflected in the relative servo performance. *D* is a mixed spur and worm gear drive, with rather a low stiffness. The loop *F* is from a similar structure, but an all spur-gear drive was used and particular attention was paid to stiffness in the design. Both structures are round about 8 tons, rotating weight. The possibility of getting, in a structure of this size, a natural frequency of oscillation in training, with the driving motor locked, of between 20 and 30 c/s, is of some interest.

Contrary to what might be expected on first consideration, the friction of the worm gear of mounting *D* is of no benefit in improving servo stability. The energy loss per cycle at low amplitudes of motor oscillation is seen to be negligibly small, and the friction losses occur—most undesirably—only at high torques. The high-efficiency loop *F*, on the other hand, shows much greater energy loss per cycle at low amplitudes. Pre-loading gear meshes and bearings gives a useful frictional damping effect at small amplitudes without lowering the overall efficiency appreciably at high torques.

Experience of many types of drives shows clearly that the character of the stiffness/inertia relationship dominates the servo performance. If the 'stay-putness' is low the nominal power of the driving motor cannot be effectively used, and refinements in the servo control gear give only very limited benefits. With a high natural frequency, on the other hand, a good performance is possible with relatively simple circuitry, and the full benefit of improved performance is obtained from any refinements of the servo control system.

As an example of what one might aim at, 1 radian per second^2 is quite a high acceleration for a mounting of any size. One minute yield as a result of this acceleration would be a very small error. One minute yield per radian per second^2 is 9 c/s (see FIG. 2). There may be little need to try to go higher, and there will probably be some difficulty in reaching this figure in mountings of any size. Directors can be, and have been, made with much higher figures.

As a first approach to a design, any performance programme will give a programme of acceleration. The relationship between the required acceleration and the maximum tolerable yield will give an acceleration yield (minutes of arc per radian/second^2). Inspection of FIGS. 1 or 2 will give the corresponding natural frequency and gravity yield. If the weight of the rotating structure is assumed to be applied as a torque at the radius of gyration, with the driving motor locked, an estimate can be made of the resulting linear yield at that radius. Some large structures, such as swing bridges, would probably collapse completely under these conditions, but proportionality can be maintained, i.e. if 1/*n*th of the rotating weight be applied tangentially at the radius of gyration, either during a test or as an estimate, the resulting linear yield must be multiplied by n to give the gravity yield, from which the natural frequency, or 'stayputness' of the structure, can be obtained on inspection from FIGS. 1 or 2.

Hysteresis loops taken the other way round, by locking the motor and applying torque to the mounting, differ in shape from the normal ones to an extent depending on the efficiency of the gearing and the nature of the structure. Little work has been done on this as results do not seem to justify the increased difficulty of doing tests this way. By fitting strain gauges on the motor coupling shaft and electrical means of measuring the overall yield between motor and mounting and taking the two quantities to the plates of a cathode-ray tube, hysteresis loops have been taken under dynamic conditions of normal working. The changes in the shape of the loop are interesting, particularly in a case such as D of FIG. 1, when input frequencies make multiples of any natural frequencies in the system, but here, again, the results did not seem to justify much further effort. The main thing is to keep the loop narrow, as in F of FIG. 1—for example, where there is little or no room for changes in shape. This is a matter of good engineering design.

The range of designs involved in mountings, directors, radar aerials, etc., is so wide that it is not possible to show typical constructions, but the following elementary principles seem to be a useful guide where precision servo control is required :-

(1) The radius of the training rack should be about equal to, preferably larger than, the radius of gyration of the rotating mass, with a direct connection to the rotating masses at, or about, that radius.

(2) Gear wheels should be as large in diameter as possible and pinions as small as possible, so as to give the minimum number of gear meshes.

(3) Two separate training drives, arranged so as to give a pure turning couple, have many advantages over a single pinion drive, particularly where, as with some large roller paths, precision centring is not possible.

(4) Structural distortions, rather than 'backlash' or torsional yields, may be the dominating factor, and any unbalanced couples arising from forces applied in one plane and resisted by stresses in another plane must be carefully watched.

(5) Normal securing arrangements such as keys, splines or fitted bolts are usually quite inadequate at the heavy end of the gear train, and some form of preloaded securing arrangement is usually necessary, both for securing gear wheels to shafts and for holding gearboxes in place. There are many such constructions available, some of them being modernized versions of the wedge arrangements used by James Watt and his contemporaries.

(6) Gears, couplings, and the bearings which support and position the mass should be designed on a resilience basis rather than load carrying capacity.

The matters dealt with are of an elementary nature, but the results are important and of wide application. The design of almost any machinery or piece of mechanism can usually be substantially improved from the evidence shown by a hysteresis loop test as described.

Figures of natural frequency obtained from dynamic tests of completed mountings show reasonably close agreement with those from static hysteresis loops.