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Automatic Steering of Ships by Proportional Control*

LEONARD I. SCHIFF† and MARVIN GIMPRICH‡

This paper treats one phase of a continuous programme of study of the control characteristics of ships. The programme started with empirical studies of the kinematics of turning, but it soon became evident that a more comprehensive study of the motions involved, including steering, was necessary. Previous reports have set up a criterion for dynamic stability on course of an unsteered ship; the present work extends this to steered ships and, in particular, to a basic consideration of the all-important time factors involved in the actual process of steering. Some further clarification of this phase is in order.

The paper attempts to set up quantitative measures of both the directional stability and the course-changing ability of automatically controlled ships, in terms of which

(1) approximate requirements can be formulated for the control systems needed for particular ships.

(2) approximate limits can be placed on the properties of unsteered ships (rudder amidships) in order that they can be handled by particular types of control.

In a previous paper dealing with course-keeping qualities [1],¶ the quantity p_1 was introduced as a measure of the *dynamic stability* on course of an *unsteered* ship, $1/|p_1|$ being approximately the number of ship lengths travelled by a stable ship in the time required to reduce an accidental deviation from undisturbed motion in a straight line to $1/e$ of its initial value ($e \cong 2{\cdot}718$), the rudder remaining in the amidships position. In the present paper, the quantity q is introduced as a measure of the *directional stability* of a *steered* ship. It is defined in an analogous way, but presupposes corrective applications of the rudder, actuated by suitable automatic control, to return the ship to its original heading (point of the compass).

In the previous paper, it was inferred that the greater the dynamic stability (shown by greater negative values of p_1), the more easily a ship can be steered to maintain a given heading. In the present paper, this inference is proved by analysis, and the dependence of the quantity q on p_1 is demonstrated by sample calculations, summarized on Fig. 36. In general, it is also seen that a larger negative value of q (increased directional stability) implies a larger permissible time lag in steering the ship, to produce the same performance in course keeping.

The effects of proportional displacement control, of proportional displacement and rate controls combined, and of time lag in the control mechanism are investigated. Different techniques for studying the directional stability of steered ships are presented as well as the calculation of trajectories.

It is shown that, in general, dynamically stable ships (negative p_1) can be steered successfully by proportional displacement control alone, while, in general, dynamically unstable ships (positive p_1) need the addition of proportional rate control. For slightly stable ships (small negative p_1) or for slightly unstable ships (small positive p_1), rate control is desirable but not always absolutely necessary. It is seen also that both increased dynamic stability and the addition of rate control contribute to a more rapid change of course. It is shown further that the greater the dynamic stability of a ship, the greater the time lag in rudder response that can be tolerated, to obtain equal results in steering.

Quantitative indices are established, as far as possible at this time, and methods are described for calculating control parameters.

Basic work is needed on the effect of rough water on steering and ship control in general. It has been recognized from the start that rough water is the ultimate consideration governing the necessary control characteristics and that smooth water studies are only the first step in an over-all programme. The authors have made brief preliminary studies of rough water problems and more detailed studies are currently in progress as part of a programme under way at the Experimental Towing Tank, Stevens Institute of Technology.

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†Professor of Physics and Executive Head of the Physics Department, Stanford University, Cal.

‡Consulting physicist to the Experimental Towing Tank, Stevens Institute of Technology, and Hydrodynamics Laboratory. California Institute of Technology; also, Guest Lecturer at Stevens Institute of Technology Graduate School.

¶Numbers in brackets indicate references listed at the end of the paper.

Automatic Steering of Ships by Proportional Control

PART 1

GENERAL CONSIDERATIONS

The motion of a ship through still water and air with rudder amidships has been treated in detail previously [1] [2]. This paper deals with the motion of an automatically steered ship under the same conditions of still water and air. The words "still water" and "air" are used, in a practical sense, to mean the existence of only very small disturbances. Certain sections of the present paper presuppose external disturbances of substantial magnitude; where this is done, it constitutes a preliminary approach to a treatment of rough water. Further work on rough water is needed and is contemplated.

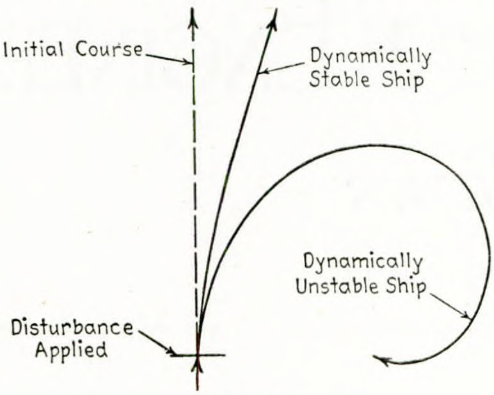


FIG. 1

One of the results obtained in earlier studies [1] [2] of the unsteered ship is the specification of course-keeping qualities with rudder amidships in terms of the dynamic stability index p_1. A negative value of p_1 means that the ship is *dynamically* stable: when a dynamically stable ship moving on straight course is disturbed slightly, it settles down on a new straight course that is close to the original course (see Fig. 1). The greater the negative magnitude of p_1, the more rapidly the ship settles on its new course and the closer the new course is to the original one. A positive value of p_1 means that the ship is dynamically unstable: when a dynamically unstable ship moving on straight course is disturbed slightly, it will wind up into a steady circling motion (even though the rudder is held amidships), as shown in Fig. 1. It is apparent that neither dynamically stable nor dynamically unstable ships can be relied upon to maintain their original courses or headings indefinitely in the presence of normal seagoing disturbances.

The primary purpose of an automatic steering device, just as of a human helmsman, is to compensate for disturbances, making the ship maintain a prescribed heading and thus making it *directionally* stable. A second purpose of an automatic control or of a helmsman is to bring the ship from one course to another in an optimum manner.

CLASSES OF CONTROL SYSTEMS

Broadly speaking, there are two general groups into which automatic steering devices may be classified: (1) continuous response devices, and (2) step response devices. In both groups, the control system translates a deviation from desired heading into a rudder motion that tends to decrease the deviation. A continuous response control makes the rudder motion some continuous function of the heading deviation, the rate of change of heading deviation, or possibly higher derivatives or integrals of this quantity. This continuous function is chosen usually to be a linear function and the device is then called a proportional control. A step response control places the rudder at one of two or more predetermined correcting angles, depending on the value of the heading deviation (and its derivatives).

It is apparent that a step response control in which there are a large number of closely spaced positions may differ little from a continuous response control. Also, step response systems may be combined with continuous response systems over part of the range of rudder settings. As an example, one automatic steering device applies rudder pulses of prearranged magnitude and duration, and can be regarded as a special kind of step response device. In general, continuous response controls are more difficult to design and construct, but usually produce smoother course lines for the ship than all but the most carefully designed step response controls. An extreme example of a simplified step response control is the "bang-bang" steering control used on some torpedoes. This is simply a switch that throws the rudder by a given amount in one direction or the other, depending on the direction of the heading deviation. With such a control, the body necessarily oscillates continually about its preset course, although, with careful design, the oscillations can be made rapid enough so that the heading deviations are not troublesome.

Whatever type of control is used, the nature of the information that is to be put into it (heading deviation with or without its derivatives or integrals) must be made definite for ships of varying degrees of dynamic stability. Properly, the decision as to type should depend more on the degree of perfection of the control (ability to use the information fed into it, magnitude of time lags, etc.) than on its classification in terms of continuous or step response. For instance, it is well known that a very high frequency bang-bang control can give performance that is practically identical with that of a good proportional control, in so far as the observable motion of the body is concerned. It should be possible, therefore, to place the choice between classes of controls at the level of engineering practice, where the influential factors are problems of design and construction, wear on steering engine, etc. General conclusions then can be drawn from a study of different classes of controls concerning the ease or difficulty of controlling various ships and the desirability of approaching the control problem analytically.

Only continuous, proportional controls are considered in this paper. This restriction greatly simplifies the mathematical procedures, making it possible to examine a variety of situations quantitatively without excessive computational labour.

Every actual control possesses lags since the control and rudder do not respond instantaneously. The kind of lag which most nearly seems to describe the rudder response of ships is an "exponential lag". It corresponds to the fact that the ship's rudder does not attain the angle called for instantaneously but moves at roughly a constant rate, which finally tapers off in reaching the specified angle. It is described in further detail in Part 2. Another type of lag often met in practice is a "constant lag". This type occurs, for example, if it takes a constant finite time to transmit a signal to the controls. In a ship, this type of lag is relatively insignificant, because of the relatively large time interval required to move the rudder; but in a small fast moving object, e.g., a torpedo or guided missile, a lag of this type may have an effect of great significance. Constant lag also is discussed in Part 2.

In order to keep the discussion general, both types of lag will be considered throughout the text.

PARAMETERS OF THE CONTROL FUNCTION

The parameters of a proportional control system are the numerical factors that determine the ratio of rudder angle to heading deviation and its derivatives and integrals, the time lag in response of the rudder to the information fed into the control, and the region over which the control operates (usually limited by the maximum rudder angle). The rudder angle is called δ and the heading deviation θ; both quantities are positive to starboard, as shown in Fig. 8. The simplest proportional control is one in which $\delta = -\gamma\theta$, with γ positive; this evidently tends to return the ship to course ($\theta = 0$). The larger γ is, the stiffer or more sensitive the control is, and the more decisively it corrects for a heading deviation. Thus, it would seem at first that a stiff control is necessarily desirable. However, if γ

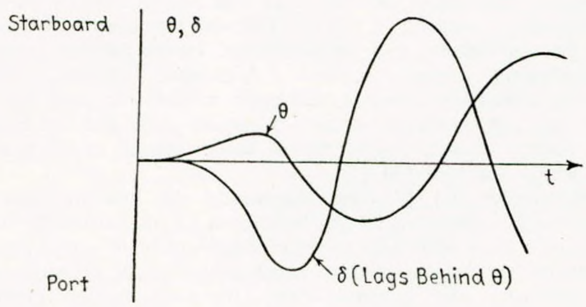


FIG. 2

is too large, the time lag, which is always present to some degree, can cause instability. This is because a heading deviation calls for a correcting rudder that, because of the time lag, continues to be applied even after the deviation has disappeared (see Fig. 2). Then the ship overshoots its course and swings in the opposite direction. If γ and the time lag are both excessive, oscillations of increasing amplitude can result.

The time lag can be compensated for to some extent by introducing control terms that are proportional to the first or higher derivatives of the heading deviation and that have an "anticipatory" effect. If the heading deviation starts to increase from zero, it is manifested as a positive value of $\dot{\theta} = d\theta/dt$ before θ itself has attained an appreciable value, as shown in Fig. 3. Similarly, when θ has reached a maximum and is starting to decrease, $\dot{\theta}$ is passing through zero from positive to negative values. Thus when θ starts to increase from zero, a control function of the form $\delta = -\gamma\theta - \bar{\sigma}\dot{\theta}$ (Fig. 4) calls for correcting rudder before the function $\delta = -\gamma\theta$ and when θ starts to decrease from its maximum toward zero, it slacks the rudder off more promptly. Higher time derivatives of θ increase the anticipatory effect. From a mathematical point of view, a Taylor's series expansion can be used to express the heading deviation at the time $t + \bar{t}$ in terms of the deviation at the time t and its derivatives:

$$\theta_{(t+\bar{t})} = \theta_{(t)} + \bar{t}\dot{\theta}_{(t)} + (\bar{t}^2/2!)\ddot{\theta}_{(t)} + \ \ldots$$

The prediction of θ at the later time requires, of course, an infinite number of terms in the series; this can never be attained in practice any more than a real prediction can ever be made in practice. However, the use of two or more terms in the series permits an extrapolation such as that made by gunfire computers in finding the position of a target in the near future from its past and present positions.

The use of an integral term (δ proportional to $\int\theta \cdot dt$) in the control function has the advantage of allowing course to be maintained in the presence of steady disturbances such as a cross wind. To illustrate this, consider the effect of a steady side force on a ship with the control function $\delta = -\gamma\theta$. The ship will have to carry some rudder in order to counteract the force; but no rudder can be carried unless there is a heading deviation, since δ is proportional to θ. Thus a compromise is reached in which the ship carries some rudder and is somewhat off course. If δ is proportional to $\int\theta \cdot dt$, however, any deviation at all, no matter how slight, soon produces an appreciable value of the integral and the rudder corrects accordingly (see Fig. 5). In the steady state, then, the ship carries the correct amount of rudder and has no heading error. An integral control term, being the opposite of a derivative term which makes the ship respond more quickly, would be expected to make the ship sluggish. This is actually the case, as can be seen by noting that an integral control is equivalent to making the time derivative of the rudder angle proportional to the heading deviation. The rudder attains a given value more slowly than it would if δ itself were proportional to θ (see Fig. 6). This, however, can be compensated for by introducing higher derivatives of θ into the control function in addition to the integral term.

SCOPE OF THE PRESENT PAPER

In this paper, the control function $\delta = -\gamma\theta - \bar{\sigma}\dot{\theta}$ is applied to three ships—"A", "B" and "C"—and account is taken of

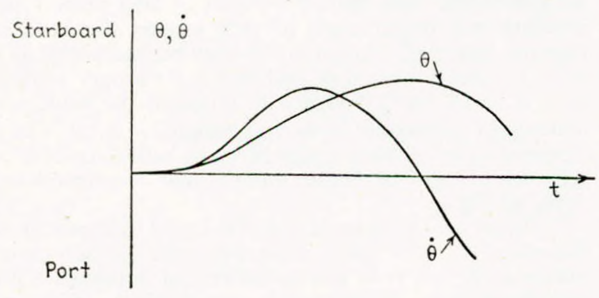


FIG. 3

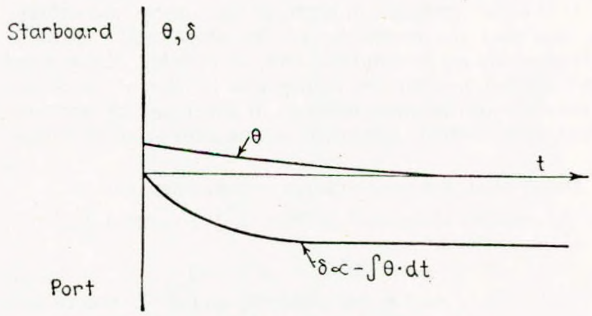


FIG. 4

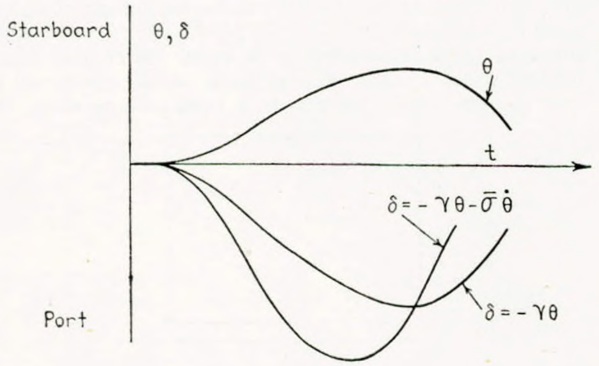


FIG. 5

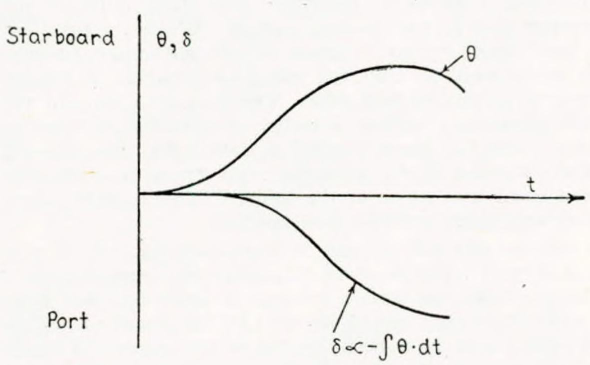


FIG. 6

time lag. Derivative terms higher than the first are omitted not only because they would add to the complexity of the calculations but also because it was found that sufficient flexibility is obtained with θ and θ terms alone. The integral control term is left out because it tends to make the ship sluggish.

The heading error that appears without integral control in a steady cross wind is not believed to be an important effect, and in any event helps to offset the lateral drifting which will be caused by the cross wind. For a particular ship and a particular centre of pressure of the wind, it is possible to choose the parameters γ and $\bar{\sigma}$ in the above control function so that the correct course is maintained in the presence of a steady cross wind. In this instance, the ship carries rudder and has a heading deviation. (See Appendix 2 for calculations on this point.)

The range of operation of the control system is of interest since the control must be able not only to handle large disturbances but also to produce quickly desired changes in course. It is assumed in this paper that the control system functions out to the maximum rudder angle permitted by the construction of the ship. No consideration is given to the choice of this angle, since there is no reason for departing from present practice because of considerations of automatic control.

EARLIER WORK

Most of the large body of literature on control problems and servo mechanisms [3] deals with industrial process control and related applications. Prior to the recent war, little analytical work appears to have been done in connexion with the automatic piloting of aircraft, and still less in connexion with the automatic steering of ships. The war stimulated interest in both these problems and, in particular, in the parallel problem of analyzing torpedo control. Automatic piloting devices actually constructed and in use prior to the war were for the most part responsive to heading deviation only, and not to rate of deviation as well, and empirical determination of parameters was largely the rule [4] [5].

Minorsky [6] [7] was apparently the first to make an analytical investigation of the behaviour of an automatic steering device on a ship and a preliminary study of some aspects of control systems. He considered proportional controls with the derivative and integral terms discussed in the previous section. His work is of limited usefulness because his assumption of incomplete equations of motion for the ship makes it impossible to relate given values of parameters with consequent ship motions, and thus prevents reaching quantitative conclusions. However, he constructed a proportional steering control, installed on the battleship *New Mexico* in 1923 [8], in which the rudder position was dependent on linear functions of the heading deviation and its first two derivatives, and (later) on the integral.

MacColl [3] gives a simple example relating to the automatic steering of ships; others have made considerable progress on the analysis of automatic control of torpedoes, both in depth and steering.

As far as is known, no earlier work has attempted to discuss ship motions, with the objective of determining suitable ship and control parameters, as is done in this paper.

PART 2

ANALYTICAL METHODS

A number of objectives can now be set up for the analytical work which follows. A directional stability parameter q for the automatically steered ship, analogous to the dynamic stability parameter p_1 for the ship with rudder held amidships, is to be defined. It is to be expressed in terms of the control parameters, the time lag, and the properties of the ship itself. Finally, course changes are to be studied, and, if possible, the control parameters chosen so that the course can be altered automatically in an optimum manner without ill effect on the performance of the same control system in steering on straight course.

ANALYTICAL REPRESENTATION OF RUDDER LAG

For the reasons discussed in Part 1, the control function is taken to have the form:

$$\delta^* = -\gamma\theta - \bar{\sigma}\dot{\theta} = -\gamma\theta - \sigma\Omega \tag{1}$$

where $\sigma = (V/l)\bar{\sigma}$, γ and σ are positive, and δ^* is the rudder angle that is called for by the control. It is not possible to have the ideal situation in which δ is equal to δ^*, since this would require that the rudder be placed instantaneously in the desired position. Actually, there are time lags both in the control system and in the steering engine. While the first can be made very small by good design of the automatic steering device, it seems unlikely that the speed of response of rudder engines will be improved radically. The detailed nature of the time delays in steering engines is rather complicated to describe analytically: when a given rudder is called for, the steering engine starts moving fairly promptly, and proceeds with constant speed until it is close to the desired rudder angle, when it slacks off and stops without overshooting.

For reasons that will appear, it is desirable to find (if possible) an analytical representation of the steering engine motion that is linear, since the entire system of equations for ship, control, and rudder then can be handled by the powerful mathematical methods that have been developed for systems of linear equations. There are two simple linear relationships between δ and δ^* that appear to approximate the steering engine delays reasonably well:

$$\delta_{(t+\bar{t})} = \delta^*_{(t)} \quad \text{or} \quad \delta_{(s+\bar{s})} = \delta^*_{(s)} \quad \text{(constant lag)} \tag{2}$$

and

$$\delta + \bar{t}\dot{\delta} = \delta^*_{(t)} \quad \text{or} \quad \delta + \bar{s}\delta' = \delta^*_{(s)} \quad \text{(exponential lag)} \tag{3}$$

where in both cases $\bar{s} = (V/l)\bar{t}$. Equation (2) states that the rudder angle δ^* called for at a particular time t is attained by the rudder at a time $t + \bar{t}$ that is later than t by $\bar{t}$; this evidently can be expressed in path lengths s as well, in which case the delay is $\bar{s}$. Equation (3) may be interpreted as follows: if $\delta^* = 0$ for a long time and at $s = 0$ changes suddenly from $\delta^* = 0$ to $\delta^* = \delta_0^*$, then δ will approach the value δ_0^* exponentially in accordance with the relation $\delta = \delta_0^*(1 - e^{-s/\bar{s}})$. Fig. 7 shows a plot of such a step function behaviour of δ^*, together with the changes of rudder angle δ that correspond to the two types of lag.

Equation (3) is a first order linear differential equation. Equation (2) is a linear equation since its solutions can be superposed, but it is not a differential equation. While this introduces some mathematical complications, they are not as serious as those that would be introduced by the assumption of a nonlinear relation (differential or otherwise) between δ and δ^*. It is interesting to note that equation (3) comprises just the first two terms in the Taylor's series expansion of equation (2), since $\delta_{(s+\bar{s})} = \delta_{(s)} + \bar{s}\delta'_{(s)} + (\bar{s}^2/2!)\,\delta''_{(\,)} \ldots$ This shows that equation (2) is equivalent to a linear differential equation of infinite order. Thus if $\bar{s}$ is quite small compared to the s intervals that are of interest in a particular problem, there will

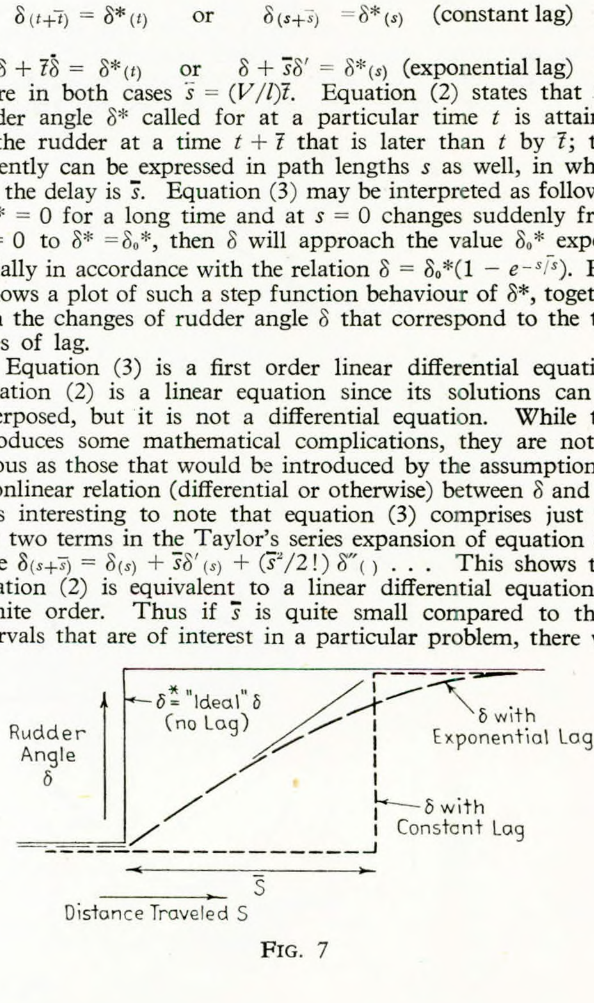


FIG. 7

be little difference between the results obtained from the two equations. In other cases, and in general, it appears likely from an inspection of Fig. 7 that the exponential lag of equation (3) provides a better approximation to the actual rudder motion of ships than the constant lag of equation (2).

It is now necessary to decide upon a suitable value for $\bar{t}$ or $\bar{s}$. Since the rudder moves with sensibly constant speed over most of its travel, it is clear that $\bar{t}$ is roughly proportional to the rudder angle that is called for suddenly. In heavy weather, when the ship suddenly may be thrown considerably off course, δ^* will change quickly from a small to a large value, and $\bar{t}$ will be roughly the change in δ^* divided by the constant rudder engine speed $\dot{\delta}_0$. During normal steering, on the other hand, δ^* changes relatively slowly and $\bar{t}$ will be correspondingly smaller. Also, an increase in the control parameters γ and σ will tend to increase $\bar{t}$, since the control will call for a larger change in δ^* for given changes in θ and $\dot{\theta}$. Now equations (2) and (3) are linear only as long as $\bar{t}$ and $\bar{s}$ are constants independent of δ^*. Thus it is necessary to choose a compromise value of $\bar{t}$ for a particular problem. Since this paper is concerned mainly with normal steering, $\bar{t}$ is chosen to correspond to a sudden change in δ^* of a few degrees. Then for various ships and speeds, $\bar{s}$ is proportional to $(V/l\dot{\delta}_0)$.

EQUATIONS OF MOTION FOR SHIP AND CONTROL

The analytical work now involves setting up the equations of motion for the ship and the control (including time lag), describing the various kinds of disturbances, and obtaining solutions that illustrate the desired results. The equations of motion of the ship have been presented and discussed in an earlier paper [1] and will not be derived again here. As far as steering a straight course is concerned, the linearized equations (15) of [1] are adequate, and have the form (see Fig. 8):

$$\left.\begin{aligned} &m\Omega - m_2\psi' - C_l\psi = -\delta C\lambda \text{ (equilibrium of transverse forces)} \\ &n\,\Omega' + C_k\Omega - C_m\psi = \delta C\mu \text{ (equilibrium of moments)} \end{aligned}\right\} \quad (4)$$

The independent variable in equations (4) is $s = (V/l)t$, which is the number of ship lengths l travelled at the speed V in the time t. A prime denotes differentiation with respect to s, and a dot, differentiation with respect to t; in particular, $\Omega = \theta' = d\theta/ds = (l/V)\dot{\theta}$. ψ is the yaw angle which, like θ, is measured in radians. $m = m_1 - C_f$; m_1, m_2, and n are dimensionless coefficients of longitudinal inertia, transverse inertia and rotational inertia, respectively. C_l and C_f are the derivatives of the transverse force coefficient with respect to yaw angle ψ and space angular velocity Ω, respectively; C_m and C_k are the similar derivatives of the moment coefficient. $\delta C\lambda$ and $\delta C\mu$ are the rudder force and moment coefficients, respectively, and can be taken to be proportional to δ, with sufficient accuracy for the present purpose.

Equations (1), (4), and either (2) or (3) comprise a complete system of linear equations for the motion of an undisturbed ship; i.e., four equations for the four unknowns ψ, θ (or Ω), δ and δ^*. Two types of disturbance are taken into account readily: impulsive forces and moments that are equivalent to initial conditions for the undisturbed motion, and steady forces and moments that are constant in time. Both types are considered in this paper. A third type of disturbance, perhaps the most interesting from a practical point of view, is that consisting of continuing non-steady forces and moments such as are encountered by a ship in a seaway. Although the mathematical methods for handling linear equations with inhomogeneous terms depending in an arbitrary way on the independent variable are well understood, it seems a little premature for the extra computational labour involved in lengthy calculations which require specification of these forces and moments. It should be noted, however, that when these terms are exponential or sinusoidal functions, the entire calculation can be handled analytically. No further mention of this type of disturbance will be made in this paper. However, a good qualitative picture of the behaviour in rough weather of an automatically steered ship can be obtained from results like those presented in Part 3 of this paper.

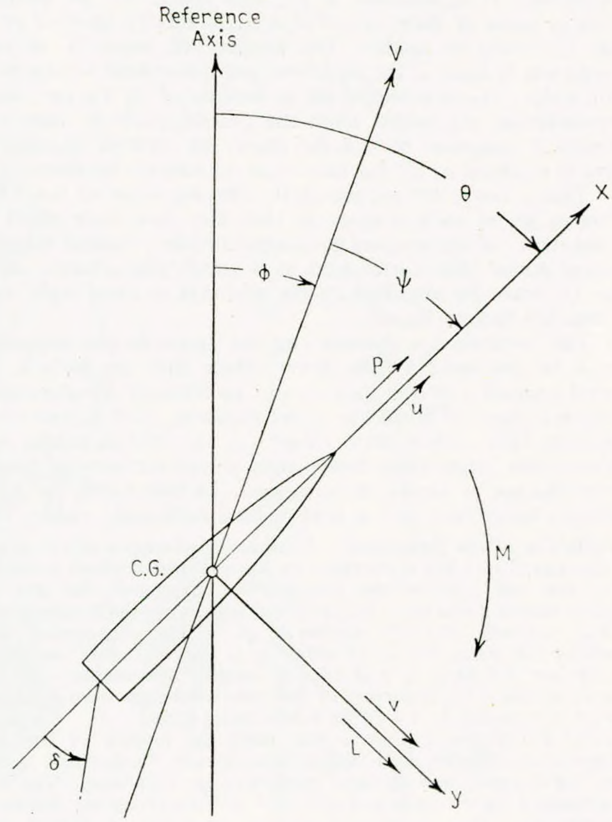


FIG. 8

Arrows indicate the senses in which the quantities are taken to be positive. Note that ψ, $\dot{\theta}$, and δ are all positive for a steady turn to starboard. This figure is identical with Fig. 16 of Reference [1].

In the previous two sections, the basic differential equations of motion have been set up. In the following two sections, two methods of solution will be presented:

(1) The method of characteristic exponents, leading to the Hurwitz criterion, and Routh's rule as the criterion of stability. This method gives a means of computing the path (trajectory) of the ship's motion.

(2) The Nyquist method [9], sometimes more suitable as a criterion of stability, though not suited to the determination of the path.

Both methods can be used with either assumption regarding time lags.

METHOD OF CHARACTERISTIC EXPONENTS

The general method for solving a system of simultaneous linear equations in which one or more dependent variables (ψ, θ, δ, δ^*) are functions of a single independent variable (s) consists in expressing each of the dependent variables as a sum of exponential functions of the independent variable:

$$\left.\begin{aligned} \psi_{(s)} &= \sum_{i=1}^{N}\psi_i e^{q_i s} \qquad & \theta_{(s)} &= \sum_{i=1}^{N}\theta_i e^{q_i s} \\ \delta_{(s)} &= \sum_{i=1}^{N}\delta_i e^{q_i s} \qquad & \delta^*_{(s)} &= \sum_{i=1}^{N}\delta_i^* e^{q_i s} \end{aligned}\right\} \quad (5)$$

It can be shown that the solutions of homogeneous equations of the present type can be expressed in the form (5), where the coefficients ψ_i, etc., are constants that depend on the initial conditions, and the characteristic exponents q_i are constants that depend on the equations but that do not involve the initial conditions. It is assumed in (5) that the q_i are all distinct; if two or more of them coincide, a slight modification of equations (5) must be made. The number of terms N in each summation is finite if the equations are differential equations of finite order. Since equation (2) is, because of its Taylor's series representation, equivalent from the present point of view to a differential equation of infinite order, an infinite number of terms is required in (5) for this assumed relation between δ and δ^*. This is not a serious practical difficulty since all but a few of the q_i are of such magnitude that they have little effect on the solution. If the original equations contain constant inhomogeneous terms that correspond to a steady disturbance, equations (5) must be modified by the addition to their right sides of constant factors ψ_0, etc.

The methods for determining the characteristic exponents q_i will be discussed a little later. Once they are known, the general character of the motion can be inferred by inspection. If the real parts of all of the q_i are negative, ψ, θ, δ, and δ^* all approach their steady state values ψ_0, etc., exponentially as s becomes very large; these steady state values are zero, of course, in the absence of steady disturbances. In this event, the automatically controlled ship is said to be directionally stable.

Definition of the Quantity q. A convenient measure of the degree of directional stability is provided by the quantity q. which is defined to be the real part of the characteristic exponent that has the smallest negative real part; i.e., of the exponent q_1 which corresponds to the dynamic stability parameter p_1. The exponential term involving the particular q_i of which q is the real part dominates the motion for large s, and $1/|q|$ is roughly the number of ship lengths in which the departure of the controlled ship from its steady motion is reduced to $1/e$ of its initial value ($e \cong 2.718$). If one or more of the q_i has a *positive* real part, the motion of the ship following an arbitrary small disturbance deviates farther and farther from the desired heading, until nonlinearities dominate. The ship is then called directionally unstable, and with the foregoing definition, q is positive. Thus positive q implies directional instability, and negative q, directional stability. The larger the negative value of q, the more stable the controlled ship, and the more rapidly it returns to its original course following an impulsive disturbance.

When the number of terms N in the summations (5) is finite, an algebraic equation for the q_i can be obtained by the same procedure that resulted in equation (21) of [1]. Substitution of equations (5) into equations (1), (3), and (4) gives relations between series of exponentials that also must hold between the coefficients of each exponential factor $e^{q_i s}$. In this way, the following simultaneous algebraic equations are obtained for each value of i:

$$\left.\begin{aligned} mq_i\theta_i - m_2q_i\psi_i - C_l\psi_i &= -\delta_iC\lambda \\ nq_i^2\theta_i + C_kq_i\theta_i - C_m\psi_i &= \delta_iC\mu \\ \delta_i^* &= -\gamma\theta_i - \sigma q_i\theta_i \\ \delta_i + \bar{s}q_i\delta_i &= \delta_i^* \end{aligned}\right\} \qquad (6)$$

Equations (6) are four simultaneous homogeneous algebraic equations from which q_i and the ratios of three of the four quantities ψ_i, θ_i, δ_i, δ_i^* to the fourth can be determined. The condition for the existence of a non-zero solution is that the determinant of the coefficients vanishes:

$$\begin{vmatrix} -m_2q_i - C_l & mq_i & C\lambda & 0 \\ -C_m & nq_i^2 + C_kq_i & -C\mu & 0 \\ 0 & \gamma + \sigma q_i & 0 & 1 \\ 0 & 0 & 1 + \bar{s}q_i & -1 \end{vmatrix} = 0 \qquad (7)$$

Equation (7) is a quartic ($N = 4$) that can be written:

$$\left.\begin{aligned} &a_0q_i^4 + a_1q_i^3 + a_2q_i^2 + a_3q_i + a_4 = 0 \\ &i = 1, 2, 3, 4 \\ &a_0 = \bar{s}nm_2 \\ &a_1 = \bar{s}(nC_l + m_2C_k) + nm_2 \\ &a_2 = \bar{s}(C_lC_k - mC_m) + (nC_l + m_2C_k) + \sigma m_2C\mu \\ &a_3 = (C_lC_k - mC_m) + \gamma m_2C\mu + \sigma(C_lC\mu + C\lambda C_m) \\ &a_4 = \gamma(C_lC\mu + C\lambda C_m) \end{aligned}\right\} \qquad (8)$$

Thus the characteristic exponents can be found by the solution of a fourth degree algebraic equation. In the event that the actual values of the q_i are not needed, but merely a determination of whether or not the motion is stable, it is sufficient to determine whether or not the real parts of all of the q_i are negative. A simple method for doing this is provided by Routh's rule, which is a special case of the Hurwitz criterion [10]: the motion is stable if, and only if, all of the a's in equations (8) are positive, and also $a_1a_2a_3 - a_0a_3^2 - a_1^2a_4 > 0$. From a study of this expression, it can be shown that increasing dynamic stability generally leads to increased directional stability, for the same control mechanism.

The same general procedure can be employed when equation (2) (constant lag) is used in place of equation (3) (exponential lag), for which N is infinite. The only change is that the last of equations (6) is replaced by $\delta_ie^{q_i\bar{s}} = \delta_i^*$. This changes the element in the fourth row and third column of the determinent of equation (7) from $1 + \bar{s}q_i$ to $e^{q_i\bar{s}}$. The quartic equation (8) is now replaced by the transcendental equation [9]:

$$\begin{aligned} q_ie^{q_i\bar{s}}[nm_2q_i^2 + (nC_l + m_2C_k)q_i + (C_lC_k - mC_m)] &+ \\ \sigma m_2C\mu q_i^2 + [\gamma m_2C\mu + \sigma(C_lC\mu + C\lambda C_m)]q_i &+ \\ \gamma(C_lC\mu + C\lambda C_m) &= 0 \qquad (9) \end{aligned}$$

which has an infinite number of solutions for q_i. The solutions of equation (9) can be obtained by putting $q_i = a + bi$, where $i = \sqrt{-1}$ and a and b are real, and equating the real and imaginary parts of the left side of (9) separately to zero. In this way, two simultaneous transcendental equations are obtained for a and b; these may be solved numerically or graphically to yield an infinite number of pairs of solutions. The assumption of exponential lag, which leads to (8) rather than (9), probably provides a better approximation to the functioning of a steering engine than does a constant lag. It has been shown, however, by D. Shanks of the Naval Ordnance Laboratory (see also [11]) that in practical problems, all of the solutions q_i of (9) have large imaginary parts (high frequency of oscillation) and large negative real parts (rapid damping), except for a few that are given in order of magnitude by using the first two terms in the expansion of the exponential.[4] (This corresponds to replacing (9) by (8).) However, the stability or instability of a system with constant lag can be determined without approximation in a simple manner by the Nyquist method (see the next section).

NYQUIST'S METHOD

The stability or instability of the automatically steered ship can be determined also in a quite different way, by means of Nyquist's criterion [9].[5] This criterion is expressed in terms of the response of the ship and control system to sinusoidal rudder oscillations of all possible frequencies. The connexion between the output of the automatic steering control and the input to the rudder engine (both of which have been denoted by δ^*) is imagined to be broken, and the rudder engine input is oscillated sinusoidally with a particular frequency and amplitude. The resulting rudder oscillation produces an oscillatory motion of the ship, which in turn produces an oscillation of the steering control output. The stability can then be inferred from a plot of the ratio of the amplitude of the steering control output to that of the rudder engine input against the frequency of the oscillation, together with a plot of the phase difference between these two motions against frequency. If the unsteered ship is dynamically stable (negative p_1), the steered ship is directionally stable if, and only if, the frequency at which the amplitude ratio is equal to unity is less than the frequency at which the phase difference is zero (the latter frequency may be infinite or may not exist). If the unsteered ship is dynamically unstable (p_1 positive) but directionally stable when steered, the two frequencies in the stability criterion just given are interchanged. (This statement of the Nyquist stability criterion is not completely general, but is sufficiently so for the problems studied in this paper.)

It is convenient to adopt the complex number notation

[4] While the approximation to a constant lag of retaining the first few terms in a Taylor's series expansion was suggested by Minorsky [7], the apparent contradictions produced by this method when an odd number of terms of the series are retained were incorrectly explained in [7].

[5] The Nyquist method provides only a criterion of stability, and is not suited to computation of paths.

used in alternating-current circuit theory. Each of the dependent variables is assumed to have a time dependence proportional to $e^{i\omega t} = e^{iks}$, where ω is the angular frequency of the rudder engine input oscillation and $k = \omega l/V$ is the corresponding "space frequency" ($2\pi/k$ is the number of ship lengths travelled during one period of the oscillation). Then:

$$\left.\begin{aligned} &\psi_{(s)} = \psi_k e^{iks},\ \theta_{(s)} = \theta_k e^{iks}, \\ &\delta_{(s)} = \delta_k e^{iks},\ \delta^*_{(s)} = \delta^*_k e^{iks},\ \bar{\delta}^*_{(s)} = \bar{\delta}^*_k e^{iks} \end{aligned}\right\} \quad (10)$$

Here δ^* represents the rudder engine input and $\bar{\delta}^*$ the steering control output when the connexion between steering control and rudder engine has been broken. (If the control is functioning, δ^* is of course equal to $\bar{\delta}^*$.) Each of the coefficients ψ_k, etc., is in general a complex number that depends on k but not on s. Each of equations (10) is so interpreted that the quantity on the left, which is necessarily real, is equal to the real part of the quantity on the right. Thus if the complex number ψ_k is written as $|\psi_k e^{ix}|$ where x is the phase of ψ_k and $|\psi_k|$ is the magnitude of ψ_k, then the first of equations (19) implies that $\psi_{(s)} = |\psi_k| \cos (ks + x)$. It is well known from alternating-current circuit theory that this interpretation of the solutions (10) is consistent provided that the original equations are linear.

Substitution of equations (10) into equations (1), (2), (3), and (4) gives, respectively:

$$\left.\begin{aligned} ikm\theta_k - (ikm_2 + C_l)\psi_k &= -C\lambda\delta_k \\ (-k^2n + ikC_k)\theta_k - C_m\psi_k &= C\mu\delta_k \end{aligned}\right\} \quad (11.1)$$

$$\bar{\delta}_k^* = -(\gamma + ik\sigma)\theta_k \quad (11.2)$$

$$\delta_k e^{ik\bar{s}} = \delta_k^* \quad (11.3)$$

$$(1 + ik\bar{s})\delta_k = \delta_k^* \quad (11.4)$$

Equations (11.1) can be solved for θ_k and ψ_k in terms of δ_k. Then (11.2) gives $\bar{\delta}_k^*$ in terms of θ_k and hence δ_k; either (11.3) or (11.4) gives δ_k in terms of δ_k^*. The first step yields:

$$\left.\begin{aligned} &\theta_k = \\ &\frac{(C_lC\mu + C\lambda C_m) + ikm_2C\mu}{k[-nm_2k^2 + (C_lC_k - mC_m) + ik(nC_l + m_2C_k)]}\,\delta_k \\ &\psi_k = \\ &\frac{mC\mu + C\lambda C_k + iknC\lambda}{-nm_2k^2 + (C_lC_k - mC_m) + ik(nC_l + m_2C_k)}\,\delta_k \end{aligned}\right\} \quad (12)$$

The first of equations (12) and equations (11.2) and (11.3) then give:

$$\frac{\bar{\delta}_k^*}{\delta_k^*} = \frac{-(\gamma + ik\sigma)[(C_lC\mu + C\lambda C_m) + ikm_2C\mu)]}{ike^{iks}[-nm_2k^2 + (C_lC_k - mC_m) + ik(nC_l + m_2C_k)]} \quad (13)$$

Plots of the magnitude and phase of the right side of equation (13) as functions of k determine the stability or instability of the automatically steered ship in accordance with Nyquist's criterion. Substitution of $1 + ik\bar{s}$ for $e^{ik\bar{s}}$ in the denominator takes account of the change from equation (2) (constant lag) to equation (3) (exponential lag) in the description of the behaviour of the steering engine.

COMPARISON OF NYQUIST AND HURWITZ METHODS

The Nyquist method can be used with relative ease when the steering engine and other delays are constant, for which case the Hurwitz criterion may be applied only approximately (though generally with a high degree of approximation). When only the stability of a hydrodynamic body fitted with various control devices needs to be examined, it is often simpler to use the Nyquist method. This may be seen from the expression

$$\frac{\bar{\delta}_k^*}{\delta_k^*} = \left(\frac{\bar{\delta}_k^*}{\theta_k}\right)\left(\frac{\delta_k}{\delta_k^*}\right)\left(\frac{\theta_k}{\delta_k}\right) + \left(\frac{\bar{\delta}_k^*}{\Omega_k}\right)\left(\frac{\delta_k}{\delta_k^*}\right)\left(\frac{\Omega_k}{\delta_k}\right)$$

For a given body $\left(\frac{\theta_k}{\delta_k}\right)$ and $\left(\frac{\Omega_k}{\delta_k}\right)$ are constant and independent of the control mechanism, while $\left(\frac{\bar{\delta}_k^*}{\theta_k}\right)$ $\left(\frac{\delta_k}{\delta_k^*}\right)$ and $\left(\frac{\bar{\delta}_k^*}{\Omega_k}\right)$ $\left(\frac{\delta_k}{\delta_k^*}\right)$ depend only on the control mechanism. In the relatively simple case of ships, there is little choice between the methods when different controls are applied to the same ship and when only stability is under consideration. Perhaps the Hurwitz method is somewhat simpler since it reduces to a quadratic equation in γ and σ. In using the Hurwitz method, the characteristic exponent which yields the entire motion may be found with ease. However, the Nyquist method is not suited to obtain the characteristic exponents and so cannot be used to study the trajectory.

An important use of the Nyquist method is that quantities such as $\left(\frac{\theta_k}{\delta_k}\right)$ and $\left(\frac{\bar{\delta}_k^*}{\delta_k^*}\right)$ can sometimes be measured on a ship or model.[6] Then the stability can be determined without knowing the steering engine or hydrodynamic constants, or using the equations of motion.

ANALYTICAL TREATMENT OF DISTURBANCES

As remarked earlier, the only disturbances treated in this paper are those due to steady forces and moments (constant inhomogeneous terms in equations (4)), and those due to impulsive forces and moments (initial conditions in the solutions). Moreover, only the relation of equation (3) (exponential lag) between δ and δ^* is considered in connexion with impulsive disturbances, since it probably provides a better description of the steering engine behaviour than equation (2) (constant lag) and is easier to handle analytically by the method of characteristic exponents. (When steady motion has been attained, there is of course no difference between equations (2) and (3), since δ is then constant.) It should be noted that the response of ships to external disturbances does not depend very much on the actual form of this disturbance. In view of this fact, these calculations represent a preliminary step in the treatment of a ship's response in rough water and air.

Steady Disturbances. The solution of the equations in the presence of steady disturbances under the assumption that the automatically steered ship is directionally stable, is obtained by setting ψ, θ, δ, and δ^* equal to the constant values ψ_0, θ_0, δ_0, and δ_0^*, respectively. A constant transverse external force to starboard, $F = \frac{1}{2}\rho AV^2C_N$, where ρ is the density of water and A the longitudinal plane area of the ship, adds a term C_N to the right side of the first of equations (4). If this force is applied at a distance λl forward of the centre of gravity of the ship, a term λC_N is also added to the right side of the second of equations (4). Equations (4) thus become in the steady state:

$$\left.\begin{aligned} -C_l\psi_0 &= -C\lambda\delta_0 + C_N \\ -C_m\psi_0 &= C\mu\delta_0 + \lambda C_N \end{aligned}\right\} \quad (14.1)$$

In similar fashion, equations (1) and either (2) or (3) become, respectively:

$$\delta_0^* = -\gamma\theta_0 \quad (14.2)$$

$$\delta_0 = \delta_0^* \quad (14.3)$$

Equations (14) are easily solved to give:

$$\left.\begin{aligned} &\theta_0 = -\frac{C_N(C_m - \lambda C_l)}{\gamma(C_mC\lambda + C_lC\mu)},\ \psi_0 = -\frac{C_N(C\mu + \lambda C\lambda)}{(C_mC\lambda + C_lC\mu)} \\ &\delta_0 = \delta_0^* = -\gamma\theta_0 \end{aligned}\right\} \quad (15)$$

The simplest case is that in which the ship is initially ($s = 0$) on straight course with rudder amidships, and the steady disturbance is applied suddenly without any accompanying impulsive disturbance. Then the initial values of ψ, θ, θ', and δ are all zero; the initial values of ψ' and θ'' are given by the first and second of equations (4), respectively, with the C_N terms included; and the initial values of ψ'' and θ''' are determined by differentiation of the expressions for ψ' and θ''. Since δ and δ^* can be expressed in terms of θ, this gives eight equations for the eight unknown constants: θ_1, θ_2, θ_3, θ_4, ψ_1, ψ_2, ψ_3, and ψ_4.

6A study of this is being carried out at present at the Experimental Towing Tank, Stevens Institute of Technology. See also [12].

Impulse Disturbances. The treatment of impulsive disturbances, either by themselves or in combination with steady disturbances, involves the determination of the coefficients ψ_i, etc., that appear in equations (5) where $i = 1, 2, 3, 4$. ψ_0, etc., are always given by equations (15), whether or not there is an impulsive disturbance present. When impulsive disturbance which is applied suddenly at $s = 0$, the initial values of bance which is appled suddenly at $s = 0$, the initial values of ψ and θ' are no longer zero, as previously assumed. This is because an ititial transverse force impulse gives the ship an initial transverse velocity, which is equivalent to an initial yaw angle; similarly, an initial moment impulse gives the ship an initial angular velocity. The explicit expressions for the eight equations are now given in terms of initial values that are denoted by the subscript a:

$$\left.\begin{aligned}
\theta_a &= \theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 \\
\theta'_a &= q_1\theta_1 + q_2\theta_2 + q_3\theta_3 + q_4\theta_4 \\
\theta''_a &= \frac{1}{n}(C\mu\delta_a + \lambda C_N - C_k\theta'_a + C_m\psi_a) = q_1^2\theta_1 + \\
&\quad q_2^2\theta_2 + q_3^2\theta_3 + q_4^2\theta_4 \\
\theta'''_a &= \frac{1}{n}(C\mu\delta'_a - C_k\theta''_a + C_m\psi'_a) = q_1^3\theta_1 + q_2^3\theta_2 + \\
&\quad q_3^3\theta_3 + q_4^3\theta_4 \\
\psi_1 + \psi_2 + \psi_a &= \psi_0 + \psi_3 + \psi_4 \\
\psi'_a &= \frac{1}{m_2}(C\lambda\delta_a - C_N + m\theta'_a - C_l\psi_a) = q_1\psi_1 + \\
&\quad q_2\psi_2 + q_3\psi_3 + q_4\psi_4 \\
\psi''_a &= \frac{1}{m_2}(C\lambda\delta'_a + m\theta''_a - C_l\psi'_a) = q_1^2\psi_1 + \\
&\quad q_2^2\psi_2 + q_3^2\psi_3 + q_4^2\psi_4 \\
\delta_a &= -(\bar{s}\delta'_a + \gamma\theta_a + \sigma\theta'_a)
\end{aligned}\right\} \qquad (16)$$

The coefficients obtained from a solution of equations (16) can be substituted into equations (5) to give expressions for ψ and θ as functions of s. From these the course angle ϕ (see Fig. 8) can be found from the relation $\phi = \theta - \psi$. The trajectory of the ship can then be obtained by integrating the course equations:

$$\frac{dx}{ds} = \cos\phi, \qquad \frac{dy}{ds} = \sin\phi \qquad (17)$$

where x and y are the rectangular co-ordinates of the ship's position, measured in ship lengths l. Equations (17) can be approximated in the linear region by assuming that $\phi \ll 1$, in which case $\cos\phi \cong 1$ and $\sin\phi \cong \phi$. Then $x \cong s$ and the second of equations (17) can be approximated by:

$$y = \int_0^x \phi_{(s)}\, ds \qquad (18)$$

The integration in equation (18) can of course be carried out analytically in the present linear approximation.

AUTOMATIC CHANGE IN COURSE

An important purpose of an automatic control system is to effect course changes in an optimum way when they are called for by the conning officer. This purpose should be considered in choosing the control parameters γ and σ, due allowance being made for the properties of the ship. A mathematical treatment of the course change problem can be given by solving the equations of motion for ship and control subject to the initial conditions $\psi = \theta' = \delta = 0$, $\theta = \theta_a$, where θ_a is the change in heading called for at $s = 0$ when the ship is on straight course with rudder amidships. Then γ and σ must be varied until a combination is found that makes θ approach zero in the shortest possible time.

There are two difficulties in the way of this ideal approach. First, the actual equations of motion for the ship, given by (11) and (12) of [1], are non-linear for large values of θ and can be handled only by numerical (step-by-step) integration. While a treatment of this type is possible and has actually been carried through (see [1], part 3, Section 7), it is not adapted to the determination of optimum values for the control parameters. This would require a large number of integrations to obtain $\theta_{(s)}$ and hence an enormous amount of numerical work. It is desirable, therefore, to simplify the equations of motion as much as possible. This may be done by assuming that the ship equations of motion have the linear form (4) and by maintaining the form of the steering control function given by equation (1). The further assumption is made that the steering engine delay $\bar{s}$ can be neglected; apart from the initial application of the rudder, this is justified by the rate of turn of the ship, which is usually small enough so that the rate at which the control calls for change of rudder is less than the rate of travel of the steering engine. The neglect of $\bar{s}$ reduces equation (8) for the characteristic exponents from a quartic to a cubic, so that only three exponentials enter into the solution. In all cases, one of the q_i's is real and negative, and the other two are generally complex conjugates of each other and have negative real parts. Thus the motion following initial application of the rudder is approximated as a sum of decreasing exponentials and a damped sinusoidal oscillation.

The second difficulty arises in the specification of what is meant by an optimum change in course. The appearance of complex q_i's implies that the ship may overshoot its course, although the overshoot can be unobservably small if the damping factor associated with the oscillatory motion is large enough. The dependence of θ on s following initial application of the rudder then has the form:

$$\begin{aligned}
\theta_{(s)} &= Ae^{q_1 s} + e^{as}(B\cdot\cos bs + C\cdot\sin bs) \\
&= Ae^{q_1 s} + (B^2 + C^2)^{\frac{1}{2}}\, e^{as}\cos\left(bs - \tan^{-1}\frac{C}{B}\right) \qquad (19)
\end{aligned}$$

where $q_{2,3} = a \pm ib$. The envelope of equation (19), $Ae^{q_1 s} + (B^2 + C^2)^{\frac{1}{2}}e^{as}$, is used here as a criterion of the approach of the ship to its new course. The course change is said to be optimum when this envelope is a minimum at some definite number of ship lengths after calling for the course change.

The calculation can proceed by assuming that the course change is large enough (one radian) so that the control initially calls for more than full rudder. There is a fixed interval during which the rudder moves out against its stop and stays, while the ship attains its steady angular velocity. Following this, the ship turns steadily under full rudder until the combination of heading deviation and its rate of change are such that the control calls fo rexactly full rudder. The constants A, B, and C in equation (19) are fitted at this point, and the envelope of θ computed from then on. Variation of γ and σ changes the point at which the control takes over as well as the shape of the θ envelope. The fixed interval s, during which steady turning with full rudder is attained, is omitted from the curves shown in Part 3; the curves are drawn as though the ship started immediately in the full rudder turn, so that θ decreases linearly with s near $s = 0$. The choice of optimum values for γ and σ is not affected by the omission since the initial motion is independent of the control parameters. The optimum control parameters found in this way do not depend appreciably on the course change that is called for, provided the change is large enough so that there is an interval during which the ship turns under full rudder. The problem for smaller course changes is completely equivalent to the impulsive disturbance problem, and requires a solution of equations (16) with the initial conditions given at the beginning of this section. The general behaviour of the ship in this instance can be inferred from the values of the characteristic exponents without additional computation. The characteristic exponents are, of course, the same as those which appear in the earlier consideration of directional stability. Thus, optimum small changes in course are obtained by making q as large (negative) as possible.

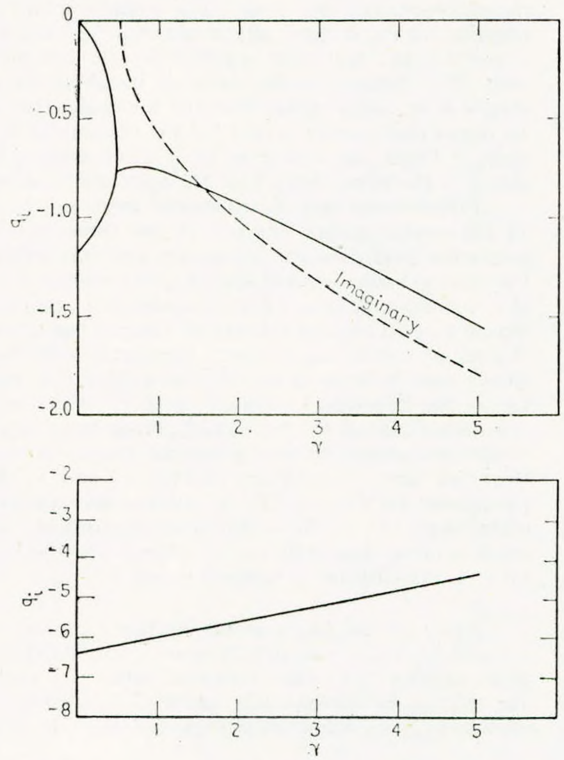


FIG. 9—*Ship "A"*

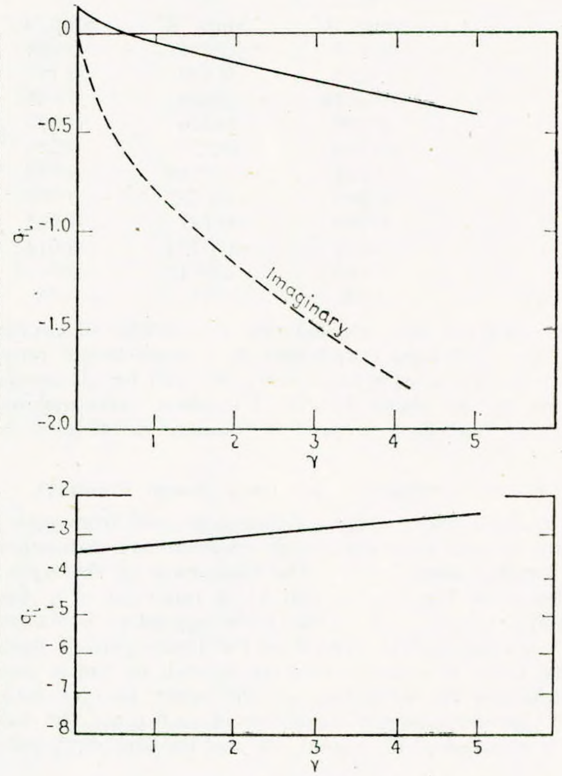


FIG. 10—*Ship "B"*

Characteristic exponents q_i for $\sigma = 0$, $\bar{s} = 0$
Three characteristic exponents when two are complex conjugates

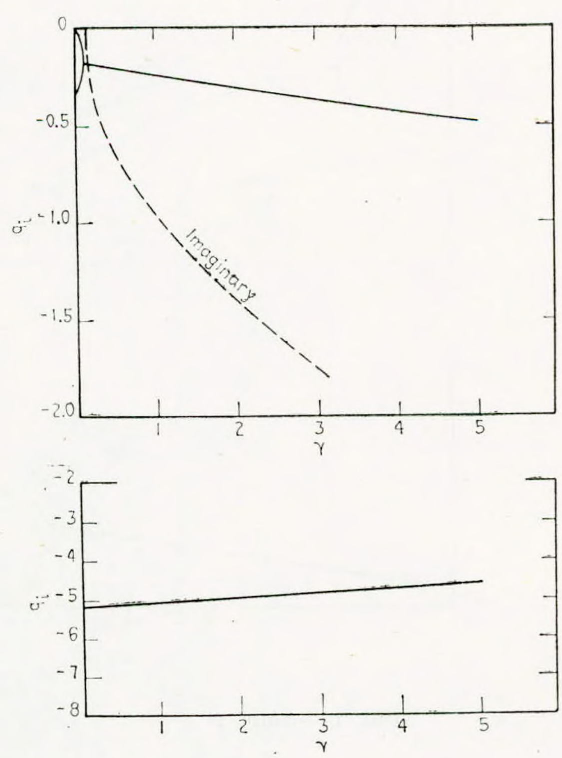


FIG. 11—*Ship "C"*

PART 3

NUMERICAL RESULTS FOR PARTICULAR SHIPS

The analytical methods described in Part 2 have been applied to the three ships: "A", "B", and "C". The numerical results are presented in four groups:

(1) Values of the characteristic exponents for $\bar{s} = 0$, limiting values of the control parameters and $\bar{s}$ for directional stability, and values of q for various cases.

(2) Nyquist plots $\bar{\delta}_k^*/\delta_k^*$ against k,

(3) The effect of a disturbance on ship "C",

(4) θ envelopes for course changes of one radian.

It should be noted in considering the subsequent calculations that γ is associated with rudder "power"; wherever γ appears in the equations, it is multiplied by $C\lambda$ or $C\mu$. The products $\gamma C\lambda$ and $\gamma C\mu$ may thus be altered by a change of either the proportionality γ, or of the rudder characteristics $C\lambda$ and $C\mu$. For the control of other bodies such as torpedoes, γ may assume values many times larger than for ships; the rudder power terms $\gamma C\lambda$ and $\gamma C\mu$ may not be greatly different.

VALUES OF SHIP PARAMETERS

The numerical values of the parameters are given in Table 1. These are all for straight-line motion; for example, C_m is the derivative of the moment coefficient with respect to yaw angle measured at zero yaw angle and zero path curvature. It should be remembered that the graphs of measured moment and force coefficient against yaw angle and path curvature are not straight lines except in the neighbourhood of straight course. Thus, while the parameters listed in Table 1 provide a reliable basis or studies of dynamic and directional stability,

TABLE 1

	Ship "A"	Ship "B"	Ship "C"
m_1	0·122	0·218	0·074
m_2	0·235	0·409	0·142
n	0·0116	0·024	0·0085
C_m	0·069	0·126	0·092
C_l	0·356	0·353	0·256
C_k	0·069	0·0588	0·032
C_f	0·063	0·0207	0·009
$m = m_1 - C_f$	0·059	0·197	0·065
$C\mu$	0·0287	0·0333	0·0143
$C\lambda$	0·0630	0·0715	0·0312
p_1 (unsteered)	−1·18	+0·16	−0·35

they can be used for only approximate calculations of course changes. The coefficients correspond to a speed-length ratio of $V/\sqrt{l} = 0{\cdot}8$. These same coefficients are valid for all speed-length ratios up to about $V/\sqrt{l} = 1{\cdot}1$ where wave-making becomes large. The effect of speed is discussed in reference 1.

CHARACTERISTIC EXPONENTS AND DIRECTIONAL STABILITY

Proportional Control Only. As may be seen from equations (8), the quartic equation for the characteristic exponents reduces to a cubic when $\bar{s} = 0$. The three roots of this equation are plotted in Figs. 9, 10, and 11 as functions of γ, for the three ships, when $\sigma = 0$. One of the exponents is always large and negative, and is plotted in the lower part of each figure. The other two exponents may be real, in which case they are indicated by solid lines in the upper part of each figure; or if they are complex conjugates of each other, the real part of each is indicated by a solid line and the imaginary part by a dotted line. When $\gamma = 0$, there is no control and one of the exponents becomes zero. The remaining two exponents are negative for the dynamically stable ship "A" and ship "C"; one is positive and the other negative for the dynamically unstable ship "B". Because of the dynamic instability of ship "B", the ship will be directionally unstable for sufficiently small γ; Fig. 10 shows that γ must exceed 0·7 for the ship to be directionally stable. From the definition of q given earlier, it is apparent that q is the upper branch of the solid curve on each chart.

Proportional and Rate Control with $\sigma = 1$. Figs. 12 and 13 are similar graphs for two of the three vessels under consideration (one directionally stable, and one unstable) showing the three q_i values plotted against γ but with $\sigma = 1$. This value of σ corresponds to $\sigma = l/V$ in equation (1), and means that in a typical case an angular velocity of ½ degree per second calls for 10 degrees of correcting rudder. Comparison of Figs. 9 and 12 shows that little is gained by the addition of rate control, as far as the directional stability of ship "A" is concerned; this was not calculated for "C" which, being less stable dynamically, would presumably show a somewhat greater improvement. On the other hand, comparison of Figs. 10 and 13 shows that the directional stability of "B" is substantially improved over the entire range of γ by the addition of rate control. This improvement is more than sufficient to change the motion from instability to stability for γ between 0 and 0·7.

Effect of the Lag $\bar{s}$ in the Rudder Control. On Figs. 14, 15, and 16, the connection between $\bar{s}$ and stability is shown by plots, against γ, of the maximum value of $\bar{s}$ that just permits the ship to be directionally stable; $\bar{s}$ is used as it appears in equation (3) to define an exponential lag. It is apparent that

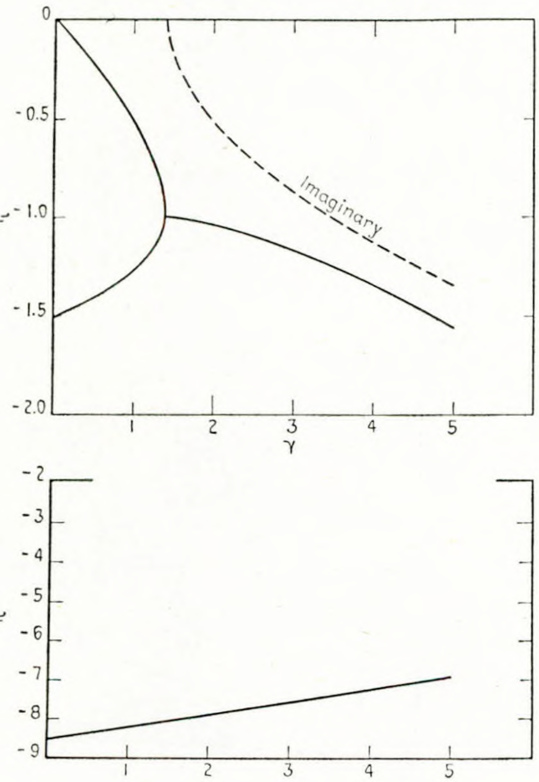


FIG. 12—*Ship "A"*

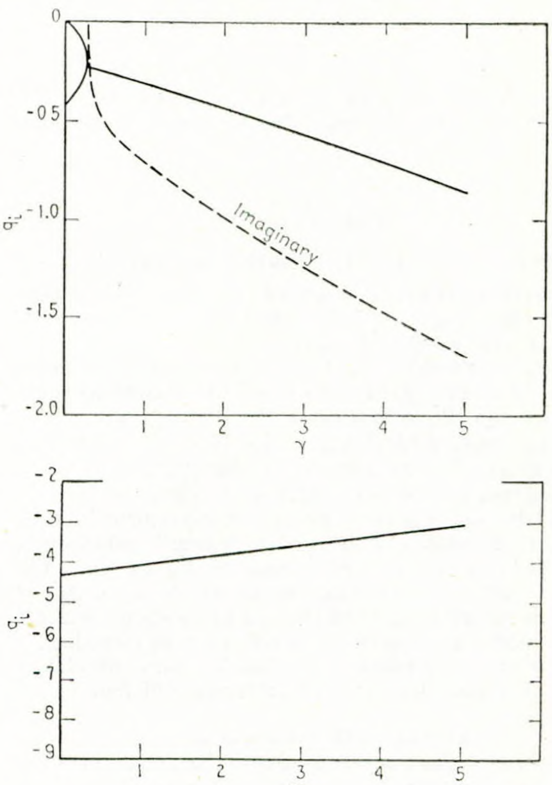


FIG. 13—*Ship "B"*

Characteristic exponents q_i for $\sigma = 1$, $\bar{s} = 0$

Three characteristic exponents when two are complex conjugates

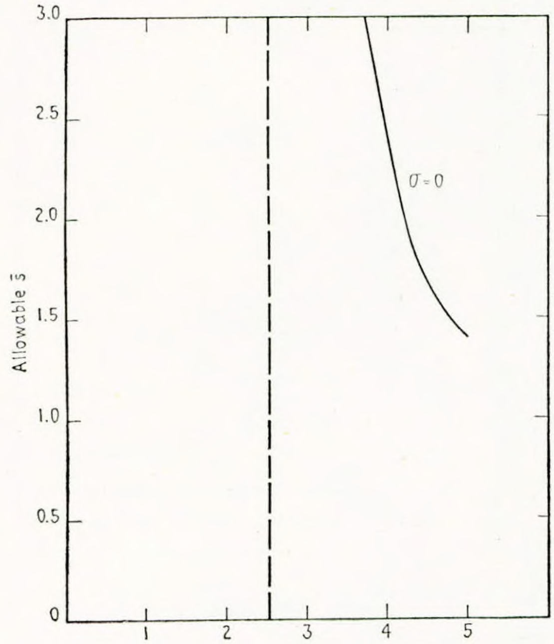


FIG. 14—*Ship "A"*

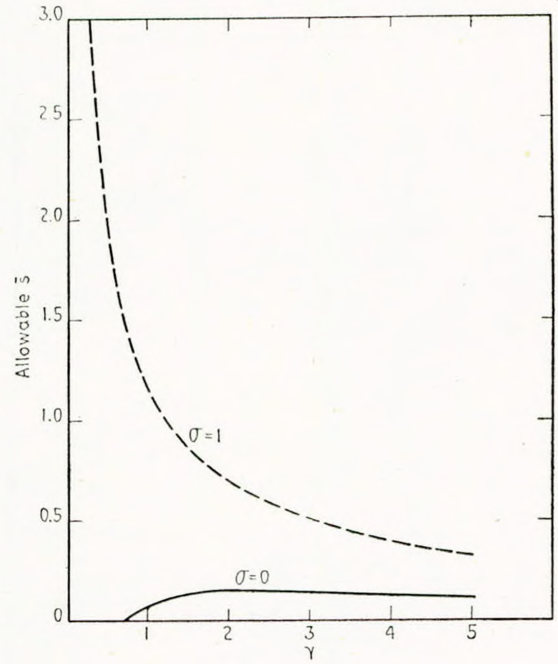


FIG. 15—*Ship "B"*

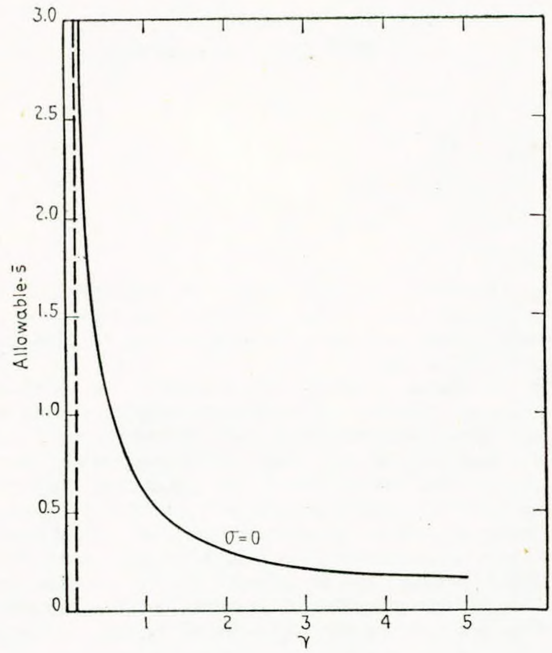


FIG. 16—*Ship "C"*

Allowable lag $\bar{s}$ for directional stability
$\delta + \bar{s}\delta' = \delta_{(s)}$ * *(exponential lag)*

ship "A" can tolerate much larger steering delays than "C", just as "C" can tolerate much larger delays than "B". For "A", $\bar{s}$ can be arbitrarily large if $\gamma < 2{\cdot}6$; similarly, $\bar{s}$ can be arbitrarily large for "C" if $\gamma < 0{\cdot}12$. As expected, however, "B" is directionally unstable for any actual (positive) $\bar{s}$ if $\gamma < 0{\cdot}7$ and $\sigma = 0$. When rate control is added to "B" ($\sigma = 1$), the curve of allowable $\bar{s}$ resembles that for "C".

Similar curves are shown in Figs. 17, 18, and 19 for the case of constant lag, as in equation (2); these curves are derived from the Nyquist plots that are presented in the next section. It is apparent that there is little difference between the results obtained from ships "B" and "C" from equations (2) (constant lag) and (3) (exponential lag) in the region of small $\bar{s}$; this would be expected from the discussion of Part 2. Ship "A", however, is so stable that large values of γ are required to make the allowable $\bar{s}$ small; in this case, the two curves need not approach each other closely.

Since equation (2) (constant lag) implies that whenever $\bar{s} > 0$ there is always an out-of-phase component of the rudder motion, it is found that for each value of $\bar{s}$ there is a maximum value of γ which will permit directional stability. This is quite different from the case with exponential lag, for which the out-of-phase component is not always present, and for which sufficiently small values of $\bar{s}$ will still permit stability with infinite values of γ.

From the discussion of equations (2) and (3), it is apparent that $\bar{s} = V\bar{t}/l$, where $\bar{t}$ is roughly equal to the rudder throw that is suddenly called for divided by the rate of rudder motion (generally about 3 degrees per second). Assuming that in normal steering, a few degrees of rudder at most is called for suddenly, $\bar{s}$ turns out to be about 0·1 for "A", a little larger for "B", and a little smaller for "C". Thus, $\bar{s}$ appears to be small enough so that the steering engine delay is of decisive importance only in connection with "B".

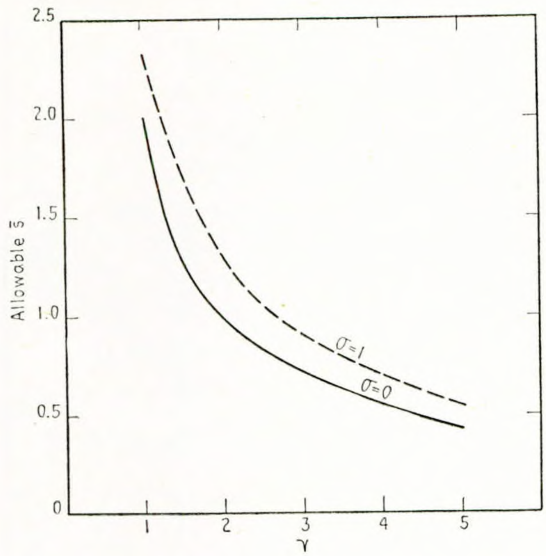


FIG. 17—*Ship "A"*

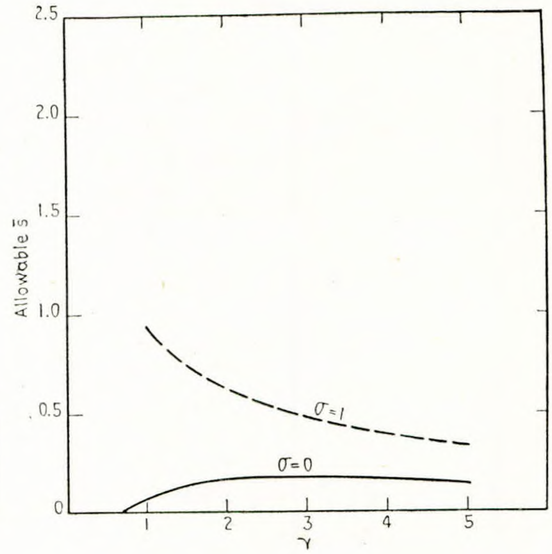


FIG. 18—*Ship "B"*

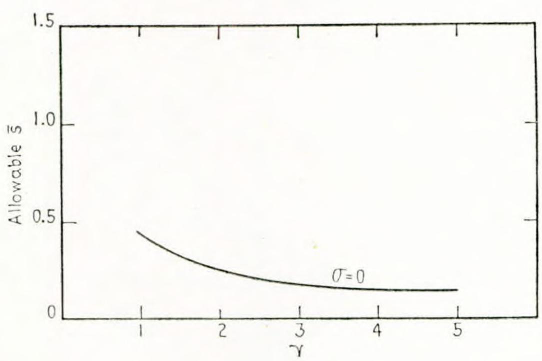


FIG. 19—*Ship "C"*

Allowable lag $\bar{s}$ for directional stability
$\delta + \bar{s}\delta' = \delta_{(s)}$ * *(constant lag)*

Effect of γ, σ, and $\bar{s}$ on the Directional Stability Index q. Figs. 20 through 23 are graphs of q against γ and against $\bar{s}$, where $\bar{s}$ appears as in equation (3) to define an exponential lag. For sufficiently large $\bar{s}$, a decrease of γ improves the stability slightly. For moderate $\bar{s}$, however, including values in the practical range, the directional stability improves with increasing γ. It should be remembered, however, that the value of $\bar{s}$ increases with γ (for reasons, see previous paragraph; essentially larger γ means larger rudder throws). Ship "A" is satisfactorily stable with $\sigma = 0$, "B" is just on the edge of instability, and "C" is in between. Fig. 23 shows the improvement that results in "B" when σ is increased from 0 to 1; her behaviour is then comparable with that of "A", and substantially better than that of "C". As remarked in Part 2, $1/|q|$ is approximately the number of ship lengths travelled during the time in which a disturbance falls off to $1/e$ of its initial value. Thus q should probably be less (more negative) than $-0{\cdot}5$ for satisfactory directional stability. This estimate is confirmed independently in the next section.

APPLICATION OF THE NYQUIST METHOD

As discussed in Part 2, the stability of an automatically steered ship can be inferred from plots of the magnitude and phase of the quantity $\delta_k{}^*/\delta_k$ against k, where $2\pi/k$ is the number of ship lengths travelled in one complete oscillation. The logarithm of the magnitude of the quantity and its phase are plotted in Figs. 24 through 28; these figures are based on equation (13), which assumes that the rudder engine delay is given by equation (2) (constant lag). The two critical points on these graphs are the k value at which the logarithm of the magnitude is zero, and the k value at which the phase is zero (if there is such a point).

Fig. 24 shows stability plots for ship "A" with $\sigma = 0$. Here γ appears only as a multiplying factor in equation (13), and hence affects the magnitude but not the phase of $\delta_k{}^*/\delta_k{}^*$. The solid lines are the logarithms of the magnitude for $\gamma = 1$, 2, and 5, and the dashed line is the phase (k is plotted on a logarithmic scale). It is apparent that this ship is directionally stable, since the phase is not zero until $k = \infty$, whereas the magnitude is unity (logarithm of the magnitude zero) for finite k. From equation (13), it follows that a constant steering engine delay $\bar{s}$, introduced as in equation (2), does not affect the magnitude but reduces the phase by $k\bar{s}$ radians. The dotted line in Fig. 24 was obtained by subtracting the phase $k\bar{s}$ from the dotted line, when $\bar{s} = 0{\cdot}1$. Since the k at which the phase is zero is still higher than those at which the magnitudes are zero, the ship is still stable with this delay and any of the γ values illustrated. The maximum allowable value of $\bar{s}$ for stability with $\gamma = 5$, say, can be found by noting that in this case the magnitude is unity for $k = 2{\cdot}2$, and the phase with $\bar{s} = 0$ is then 55 degrees $= 0{\cdot}96$ radians. Thus, $\bar{s}$ cannot exceed $0{\cdot}96/2{\cdot}2 = 0{\cdot}43$ if the ship is to be directionally stable with $\gamma = 5$ and $\sigma = 0$; this gives one point for plotting Fig. 17.

Fig. 25 shows similar plots for ship "B" with $\sigma = 0$. Since this ship is dynamically unstable when unsteered, the smallest k value at which the phase is zero must be *less* than the k value at

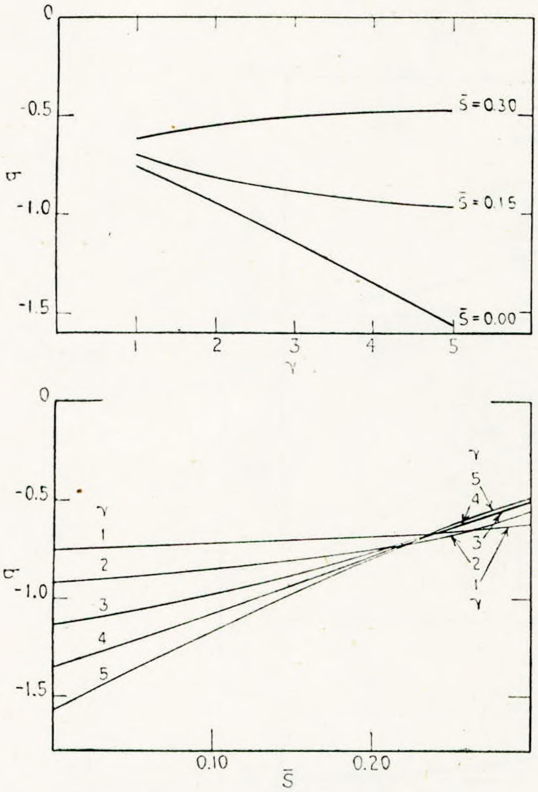


FIG. 20—*Ship "A". σ = 0*

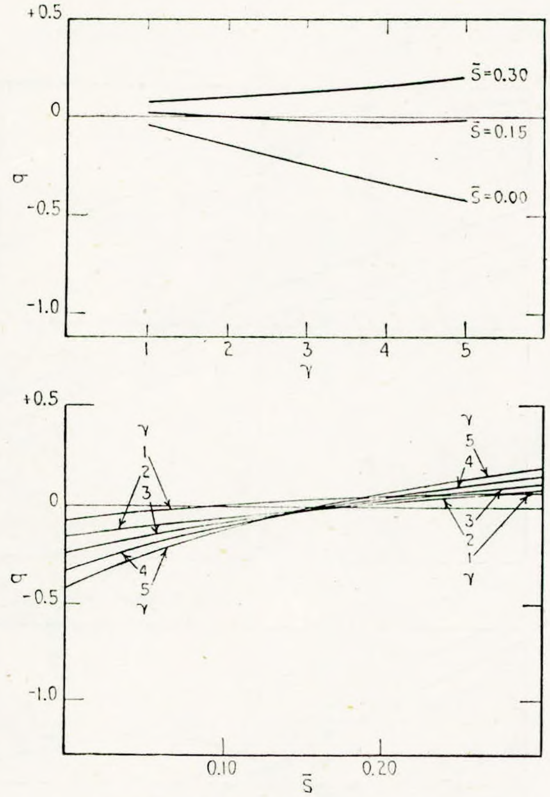


FIG. 21—*Ship "B". σ = 1*

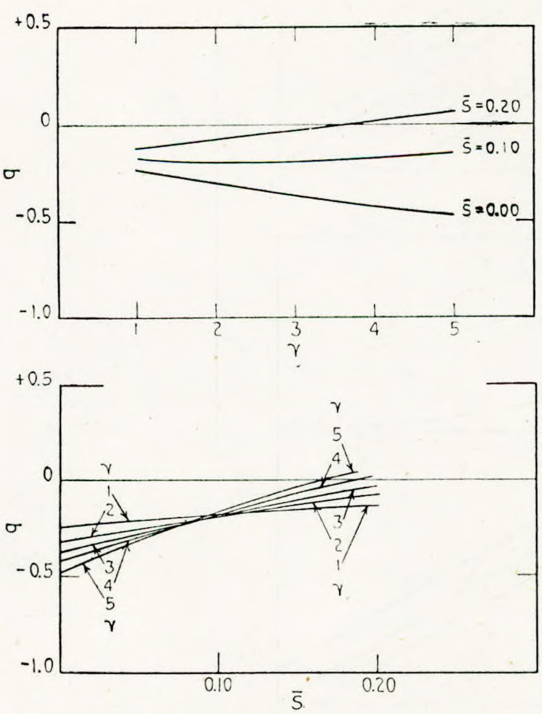


FIG. 22—*Ship "C". σ = 0*

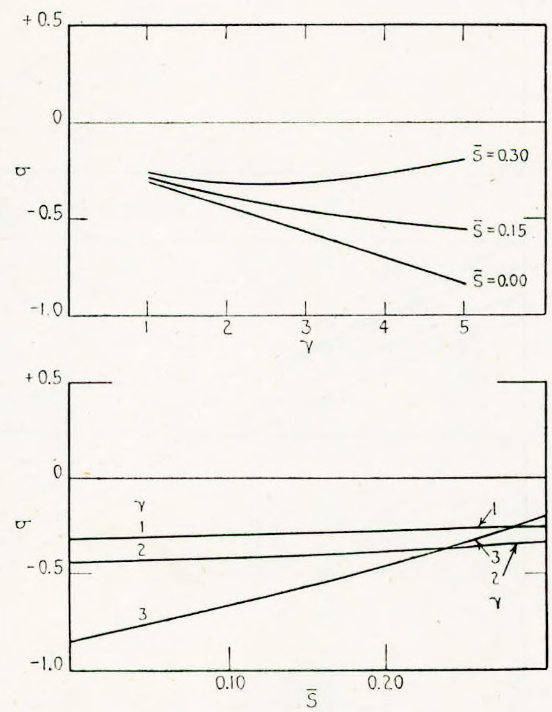


FIG. 23—*Ship "B". σ = 1*

Variation of directional stability index q with $\bar{s}$ and γ

$\delta + \bar{s}\delta' = \delta_{(s)}$ * *(exponential lag)*

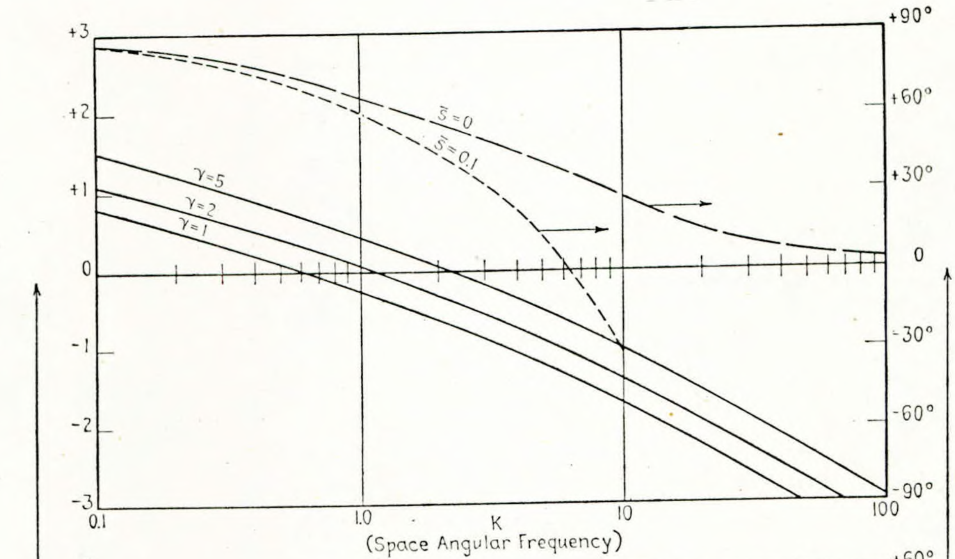


FIG. 24—*Ship "A"*

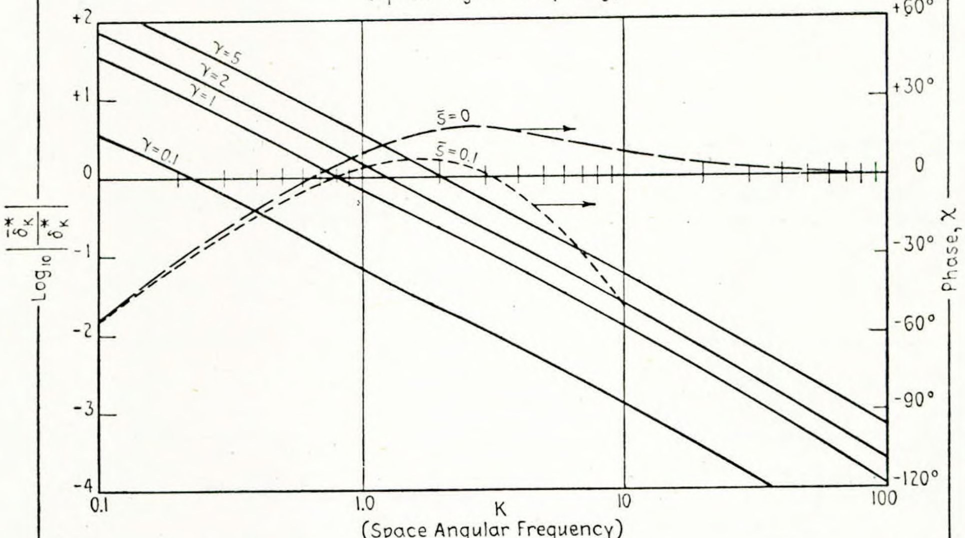


FIG. 25—*Ship "B"*

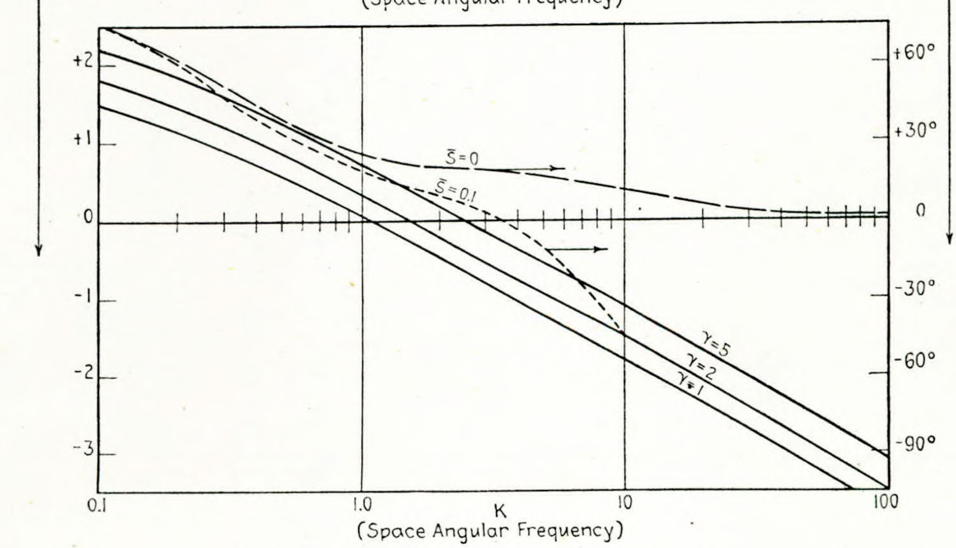
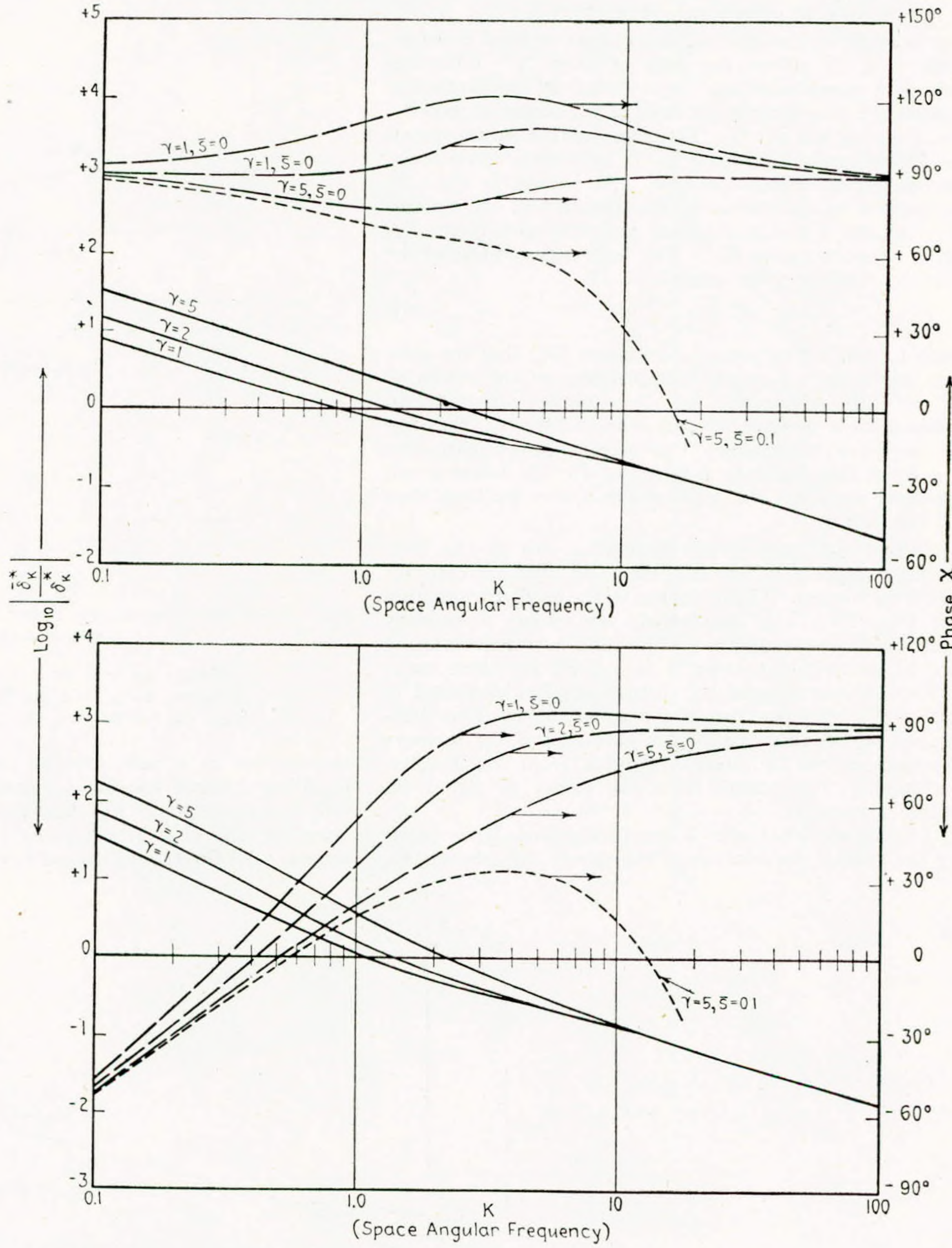


FIG. 26—*Ship "C"*

Stability plots by Nyquist method for $\sigma = 0$ (constant lag)

FIG. 27—*Ship "A"*

FIG. 28—*Ship "B"*



Stability plots by Nyquist method for $\sigma = 1$ (constant lag)

which the magnitude is unity, in order that the steered ship be directionally stable. With $\bar{s} = 0$, it is apparent that this is the case for $\gamma = 1$, 2, and 5, but that the ship is directionally unstable for $\gamma = 0{\cdot}1$. This is in agreement with the earlier results. Introduction of the lag $\bar{s} = 0{\cdot}1$ (dotted phase curve) puts the ship on the edge of instability with $\gamma = 1$, but is still stable with $\gamma = 2$ and 5.

The curves of Fig. 26 for "C" with $\sigma = 0$ are similar to those of Fig. 24 and require no further comment.

Effect of Introducing Rate Control in Nyquist Method ($\sigma = 1$). In Figs. 27 and 28 the effect of introducing rate control ($\sigma = 1$) is shown. "A" is still stable for all γ's shown with $\bar{s} = 0$ or $0{\cdot}1$, since in all cases the phase curves are zero for higher k than the magnitude curves. As expected, the crossing points for phase and magnitude are pushed farther apart by the rate control, thus giving a qualitative indication of increased directional stability. "B" is now stabilized for $\gamma = 1$, $\bar{s} = 0{\cdot}1$, by the introduction of rate control. It is interesting to note that the phase curves are no longer the same for all values of γ when rate control is added, since the phase has an extra term, $\tan^{-1}(k\sigma/\gamma)$, that involves γ when σ is other than zero.

A semi-quantitative indication of the attainment of satisfactory directional stability can be obtained from the Nyquist plots by taking over a rule-of-thumb from the field of electronic feed-back amplifier design, where the Nyquist method has been used for many years. A linear feed-back amplifier is generally said to be sufficiently stable if the logarithm of the magnitude of the ratio analogous to (13) is less than $-0{\cdot}5$ when the phase is zero, and if the phase lead is 30 degrees when the magnitude is unity [13]. Comparison of Figs. 20 through 23 with Figs. 24 through 28 indicates that a value of q less than about $-0{\cdot}5$ would be regarded as satisfactory in feed-back amplifier design. This confirms the estimate made in the last section.

EFFECT OF CONSTANT DISTURBANCE

As an example of the effect of a suddenly applied constant disturbance, Fig. 29 shows the path of ship "C" following application of a transverse force (represented by the coefficient C_N) at a point 0·2 ship lengths forward of the centre of gravity, when $\gamma = 2$, $\sigma = 0$ and $\bar{s} = 0$. The computation makes use of equations (15) through (18); since $\bar{s} = 0$, only three characteristic exponents appear instead of four. As shown in Fig. 29, the initial motion of the ship is in the direction of the applied force (to starboard if C_N is positive) and the subsequent displacement is proportional to C_N. The final course angle of the ship ϕ_0 can be obtained from equations (15):

$$\phi_0 = \theta_0 - \psi_0 = \frac{C_N[\gamma(C\mu + \lambda C\lambda) - (C_m - \lambda C_l)]}{\gamma(C_m C\lambda + C_l C\mu)} \qquad (20)$$

It is apparent from the structure of equation (20) that the ratio ϕ_0/C_N can be positive or negative, depending on the values of γ, λ and the ship parameters. In the situation illustrated in Fig. 29, this ratio is positive, so that with positive C_N the ship eventually moves to starboard. The path is shown with two different x scales that differ by a factor of 10; the lower graph shows the short term and the upper graph shows the long term motion of the ship.

As expected from the earlier discussion, one of the three characteristic exponents is real and the other two are complex conjugates of each other. This gives rise to the oscillatory motion shown in Fig. 29. The oscillations are rather completely damped out in about 10 lengths. This could have been inferred from Fig. 22, according to which $q = -0{\cdot}31$ for these parameters. Thus, in ten lengths the disturbance has decreased to roughly $e^{qs} = e^{-3\cdot1} \cong 5$ per cent of its initial value. This illustrates the general conclusion that the character of the recovery from disturbances can be inferred roughly from the q value, and in somewhat more detail from the values of all of the characteristic exponents.

Fig. 29 indicates that after a few oscillations, immediately following the sudden application of the steady disturbance, the ship settles on a new straight course. Fig. 30 indicates the resulting attitude for the assumed side force coefficient $C_N = 0{\cdot}020$; it corresponds to a lateral drift of 0·04 ship lengths in a travel of 100 ship lengths; i.e., a course change of only 0·02 degree. All forces and moments are balanced.

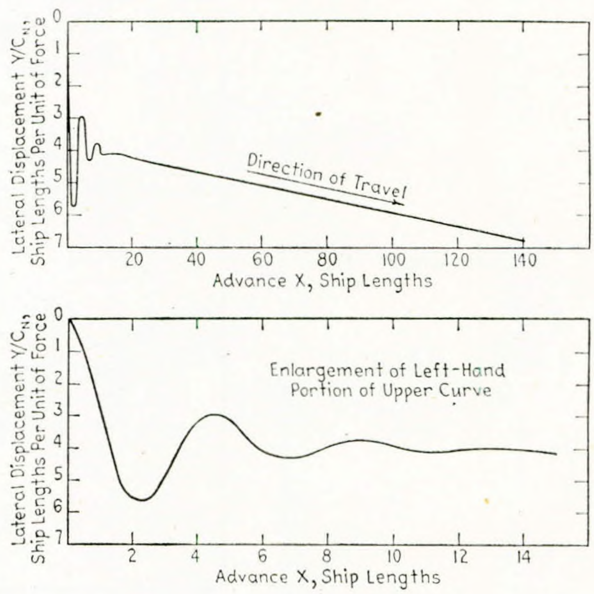
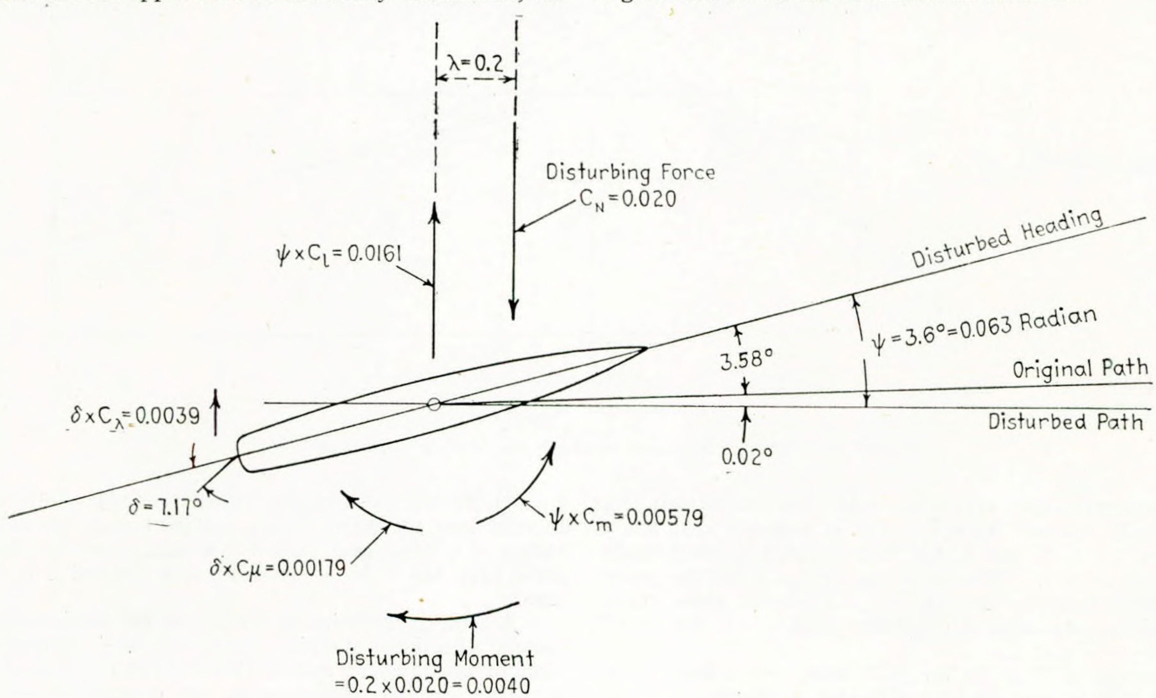


FIG. 29—*Course resulting from constant external force and moment—ship "C"*

Shown for $\gamma = 0$, $\delta = 0$, $\bar{s} = 0$, $\lambda = 0.2$
External force $= C_N(\rho/2)AV^2$ (to starboard)
External moment $= C_N\lambda(\rho/2)AV^2l$ (clockwise)



Coefficient of lateral force due to yaw $= \psi \times Cl = 0.063 \times 0.256 = 0.0161$
Coefficient of lateral force due to the rudder $= \delta \times C\lambda = 7.17 \times 0.0312/57.3 = 0.0039$
Coefficient of total (external) force $C_N = 0.0200$
Coefficient of yawing moment due to external force $= 0.02 \times 0.2 = 0.00400$
Coefficient of yawing moment due to yaw $= \psi \times C_m = 0.063 \times 0.092 = 0.00579$
Coefficient of total moment = coefficient of rudder moment $= \delta \times C\mu = 7.17 \times 0.0143/57.3 = 0.00179$

FIG. 30

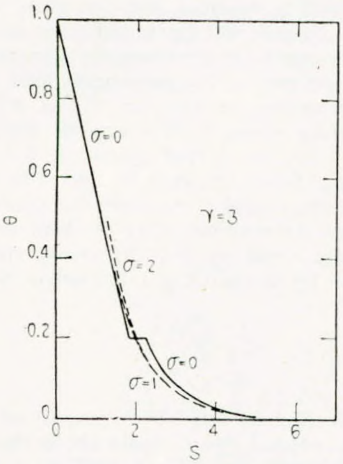


FIG. 31—*Ship "A"*

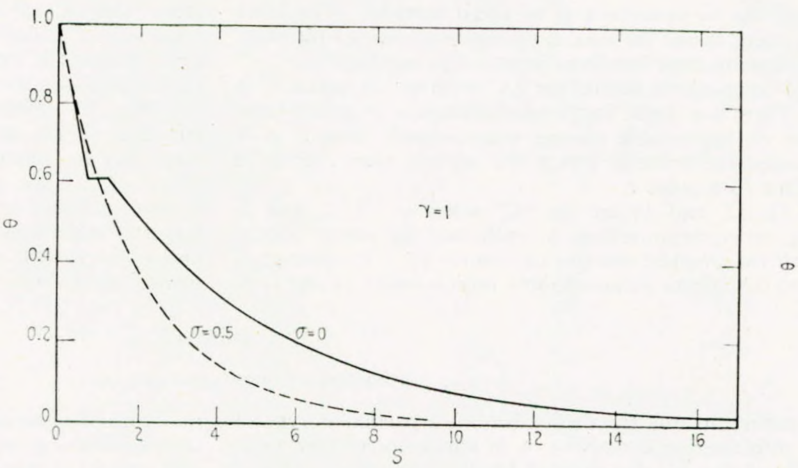


FIG. 32—*Ship "C"*

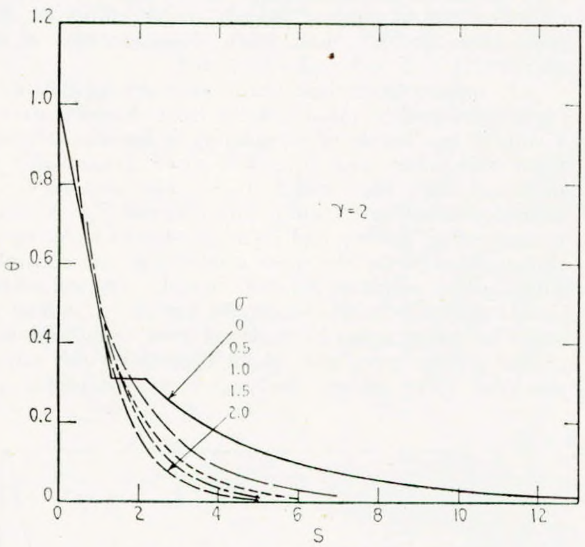


FIG. 33—*Ship "C"*

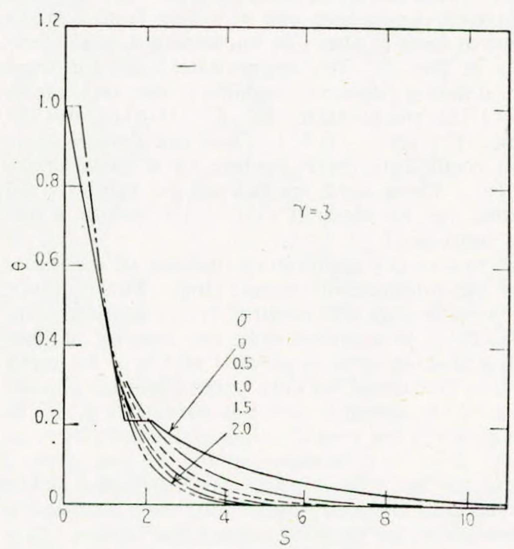


FIG. 34—*Ship "C"*

Envelopes of heading change during change of course for various values of σ and for $\bar{s} = 0$

It is interesting to observe that in the case of the particular ship and the assumed type of steady disturbance of magnitude corresponding to $C_N = 0{\cdot}020$, which can be visualized as due to side wind, the heading changes by 3·58 degrees to windward, while the course-made-good is practically identical with the original one. For a ship of different stability characteristics, and for a different kind of disturbance, the result, of course, would be different.

CHANGES IN COURSE

The results of the simplified calculation of automatic changes in course, outlined in Part 2, are plotted in Fig. 31 for ship "A" and in Figs. 32, 33, and 34 for "C". In each figure, θ is plotted in radians against the distance travelled s in ship lengths. The initial part of each curve is a straight line whose slope is the rate of turn under full rudder, measured in radians per ship length. The horizontal line segment that breaks across from the initial steady turn to the curved portion occurs at the value of θ for which the control ceases to call for full rudder. The remaining curved part is the envelope of θ: $Ae^{q^1 s} + (B^2 + C^2)^{\frac{1}{2}} e^{as}$. Since $\theta_{(s)}$ itself, as given by equation (19), fits continuously onto the initial straight line, it is not to be expected that the envelope will also. Thus the horizontal break and the

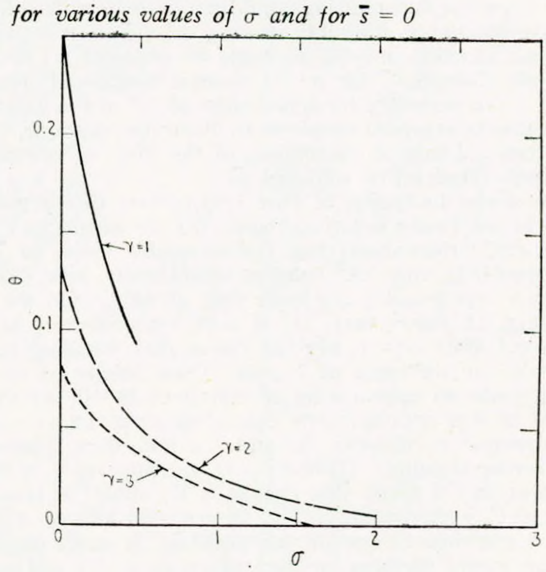


FIG. 35—*Ship "C"*

Variation of heading error envelope with γ and σ taken at $s = 5$ and for $\bar{s} = 0$

early part of the curve are not to be taken literally. The latter part of the curve shows the maximum heading error of the ship; the actual heading error oscillates within this envelope.

Fig. 31 contains the results for "A" with $\gamma = 3$, and $\sigma = 0$, 1, and 2. There is a slight improvement when σ increases from 0 to 1, but no appreciable change when σ goes from 1 to 2. As σ increases, the point at which the control takes over is of course shifted to smaller s.

Figs. 32, 33, and 34 are for "C" when $\gamma = 1$, 2, and 3, respectively, with computations in each case for several values of σ. With the smallest amount of control ($\gamma = 1$), change of σ from 0 to 0·5 effects a considerable improvement in the control. The change is still substantial, but not quite as marked, when $\gamma = 2$ and 3. In general, the initial introduction of a small amount of rate control (σ) improves the approach to new course considerably, and subsequent increments have less effect.

Fig. 35 summarizes the results for "C" at $s = 5$. The envelope of the heading error, 5 ship lengths after the ship starts on its steady turn, is plotted against σ for the three values of γ. The general conclusion to be drawn from Fig. 35 is that increases in both γ and σ improve the course change, but that little is gained by making either of them greater than about 2. By the same criterion, Fig. 31 shows that little is gained with ship "A" by introducing rate control at all.

PART 4

SUMMARY AND CONCLUSIONS

This paper presents the theory behind a particular type of automatic ship steering control—one in which the rudder angle is proportional to a combination of heading deviation and rate of change of heading, with a time delay. As discussed in Part 1, this is sufficiently representative of more general types of ship controls so that useful conclusions can be drawn from a study of it. The analytical work of Part 2 is implemented by numerical computations in Part 3. The computations are for three ships of widely differing dynamic stability: the very stable ship "A" ($p_1 = -1{\cdot}18$), the unstable "B" ($p_1 = +0{\cdot}16$), and the moderately stable "C" ($p_1 = -0{\cdot}35$). The calculations were carried out with coefficients corresponding to a speed-length ratio $V/\sqrt{l} = 0{\cdot}8$. These same coefficients are valid for all speed-length ratios up to about $V/\sqrt{l} = 1{\cdot}1$, when wave-making becomes substantial.

It is possible to set up a quantitative measure of the directional stability of the automatically steered ship. This quantity q is negative for a stable ship and positive for an unstable ship. For stable ships, $1/|q|$ is approximately the number of ship lengths in which a heading error is reduced to $1/e$ of its initial value. Thus q is a directional stability parameter that is completely analogous to the dynamic stability parameter p_1. The relation between q and p_1 for typical values of the control parameters ($\gamma = 1$, $\sigma = 0$, $\bar{s} = 0{\cdot}1$) is shown in Fig. 36 (constructed from three points for the three ships). As mentioned earlier (Part 2), it can be demonstrated analytically that increased q generally accompanies p_1, for the same control mechanism. It is apparent in Fig. 36 (lowest curve) that this simple relationship actually exists. The curve shown is for exponential lag—a similar curve for constant lag would not be materially different. Also, the maximum allowable steering delay $\bar{s}$ is seen to be an increasing function of $-p_1$, as might be expected. The figure shows the allowable $\bar{s}$ for $q = 0$ (neutral directional stability). Since $q < 0$ is necessary for actual ships ($q < -0{\cdot}5$ is indicated), Fig. 36 is to be regarded simply as an illustrative example, chosen on the basis of ease of calculation, of the effect of increasingly negative p_1 values on the allowable $\bar{s}$.

From the discussion of Part 3, it follows that q probably should be less (more negative) than $-0{\cdot}5$ for satisfactory steering. Fig. 20 then shows that for reasonable values of $\bar{s}$ (less than about 0·2), ship "A" behaves satisfactorily with a control for which σ is 0 and γ is greater than about 1. On the other hand, Fig. 21 shows that "B" is quite unsatisfactory as long as σ is 0. With $\sigma = 1$, Fig. 23 shows that this ship behaves well with γ in the range of 2 to 3. These results are not surprising, since an examination of equations (8) shows that an increase in σ is approximately equivalent to an increase in the hydrodynamic coefficients C_k and C_f, and these "damping" terms favour stability. (When $\bar{s} = 0$, a change in σ is exactly equivelent to C_μ times this change is C_k, and $C\lambda$ times this change in C_f.) According to Fig. 22, the behaviour of "C" with $\sigma = 0$ is marginal except for very small $\bar{s}$. It seems likely that this ship would perform satisfactorily with $\sigma = 1$ and a value of γ in the range 1 to 3. It thus appears that some rate control (σ term) is desirable for all but the most stable ships, and is essential for the steering of dynamically unstable ships.

Figs. 31 through 35 bear out these conclusions as far as course-changing is concerned. Here again the introduction of rate control is not necessary for "A", but effects a substantial improvement in "C". An even larger improvement would be expected in the less stable ship "B". Considerations of course-changing would suggest slightly larger values of the control parameters for "C" than would considerations of directional stability: $\gamma = 2$ to 3 and $\sigma = 1$ to 2.

It appears from these results that among existing ships, no limitations need be placed on the hydrodynamic parameters for a ship to be capable of satisfactory automatic steering in relatively still water and air. The more dynamically stable the unsteered ship, the smaller the σ value necessary for a fixed degree of directional stability (for constant $\bar{s}$). A single control incorporating both γ and σ terms should be adequate for all ships, although for the most stable ships the control could be simplified by omitting the rate control. The values of γ and σ should probably be left adjustable for the individual ship; they could be chosen either by trial and error or by analytical studies of the present type, and would depend on the value of $\bar{s}$ for the ship. The present analysis serves the useful purpose of

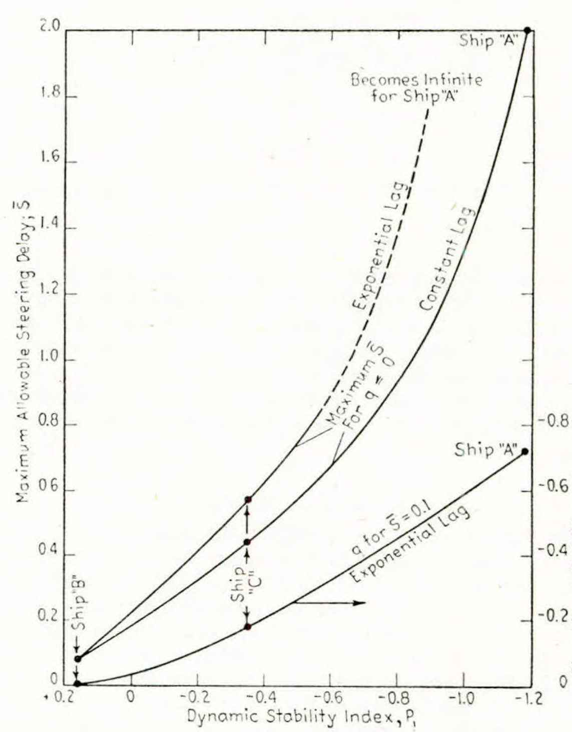


FIG. 36—*Maximum allowable $\bar{s}$ for directional stability ($q = 0$) as a function of dynamic stability and directional stability (q) when $\bar{s} = 0{\cdot}1$ for $\gamma = 1$ and $\sigma = 0$*

indicating the range of values required for existing types of ships. The effect of disturbances has been mentioned. The behaviour in rough water is now under study and further work on the subject is necessary.

The entire automatic control problem becomes more critical as the speed of the ship increases, since for a given time delay $\bar{t}$, the quantity $\bar{s}$ is proportional to the speed. Thus the analysis is relatively more important at the higher speeds. The converse is true as the length of the ship increases. Since $\bar{s} = V\bar{t}/l = (\bar{t}/\sqrt{l})\ (V/\sqrt{l})$, $\bar{s}$ decreases as the ship length increases for constant "speed-length ratio" $V/\sqrt{l}$. Thus the most critical situations can be expected to arise in connection with small ships run at high speed; an extreme example would be a high-speed submarine or torpedo. In such cases, the control problem will probably prove to be too delicate for rule-of-thumb adjustment, and will require analytical studies of the type illustrated in this paper.

ACKNOWLEDGMENT

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APPENDIX 1

EXPLANATORY LIST OF SYMBOLS

- t .... time, regarded as the independent variable of the motion; dot denotes time derivative
- l .... length of ship
- V .... speed of ship
- $s = \frac{V}{l}t$ .... number of ship lengths travelled in time t; prime denotes derivative with respect to s
- θ .... heading angle, in radians
- $\Omega = \theta'$ .... space angular velocity or instantaneous path curvature
- ψ .... yaw angle, in radians
- $\phi = \theta - \psi$ .... course angle, in radians
- δ .... actual rudder angle, in radians
- δ^* .... rudder angle called for by control, in radians; or in the Nyquist method, the rudder engine input when the control is disconnected from the rudder engine
- $\bar{\delta}^*$ .... in the Nyquist method, the steering control output when the control is disconnected from the rudder engine
- ω .... angular frequency of oscillation in Nyquist method
- $k = \frac{\omega l}{V}$ .... space angular frequency in Nyquist method
- γ, σ .... steering control parameters in equation (1)
- $\bar{t}$, $\bar{s}$ .... delay parameters in equation (2) or (3)
- m_1 .... longitudinal mass coefficient
- m_2 .... transverse mass coefficient
- n .... moment of inertia coefficient
- C_m .... derivative of moment coefficient with respect to yaw angle
- C_l .... derivative of force coefficient with respect to yaw angle
- C_k .... derivative of moment coefficient with respect to Ω
- C_f .... derivative of force coefficient with respect to Ω
- $C\mu$ .... derivative of moment coefficient with respect to rudder angle
- $C\lambda$ .... derivative of force coefficient with respect to rudder angle
- $m = m_1 - C_f$
- C_N .... coefficient of applied transverse force
- λ .... number of ship lengths that point of application C_N lies forward of centre of gravity
- p_1 .... dynamical stability parameter for unsteered ship
- q .... directional stability parameter for steered ship

APPENDIX 2

COURSE-KEEPING IN A STEADY CROSS WIND

Equation (26) in Part 3 gives the steady course angle ϕ_0 under the influence of a steady transverse force coefficient C_N applied at a point λ ship lengths forward of the centre of gravity of the ship. It is apparent that this can be made zero by choosing γ such that

$$\gamma = \gamma_0 = \frac{C_m - \lambda C_l}{C\mu + \lambda C\lambda}$$

γ_0 is independent of C_N, which determines the strength of the disturbance, but involves some of the ship parameters and λ. If it is assumed that the disturbance is a steady cross wind, with which a definite centre of pressure is associated, then the control constant γ_0 automatically maintains the ship on its proper path, even though the ship carries rudder and there is a heading deviation as well.

For $\lambda = 0{\cdot}2$, the values of γ_0 given by the above equation are:

Ship "A"	$\gamma_0 = 0{\cdot}9$
Ship "B"	$\gamma_0 = 2{\cdot}5$
Ship "C"	$\gamma_0 = 3{\cdot}8$

These lie in the practical range, especially since considerations of this type place no restrictions on σ, so that any departure of γ from the optimum for automatic steering can be compensated by a suitable choice of σ. The principal difficulty in using this as a criterion for the choice of γ is the uncertain value of λ and its possible variability under different wind conditions. More experimental data on λ should be obtained before γ_0 is adopted for the automatic steering control.

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[12] Millikan, Jr., William F., "Progress in Dynamic Stability and Control Research", *Journal* of the Institute of the Aeronautical Sciences, Volume 14, No. 9, page 493, 1947.

[13] Bode, H. W., "Network Analysis and Feedback Amplifier Design", chapter XVIII, 1945; this rule must be modified suitably for an unstable ship.

INSTITUTE ACTIVITIES

MEMBERSHIP ELECTIONS

Elected 19th July 1949

MEMBERS

Hosny Hafez Attia, M.Sc., Ph.D.
Peter Charig, Com'r(E), R.N.
Thomas Clark
Frederick Thomas Coates
William Graham Conkie
Albert Green
Charles Stanley Huxley
Ernest Edward Pryce Jones, Lieut.(E), R.N.
Gordon Maxwell Kersey
George Eric Penny
David Walker
George Scougal
Norbert Anthony Murray

ASSOCIATE MEMBER

William Leonard Coventry

ASSOCIATES

William John Baker
Ernest Reginald Blower
Patrick Burges
Kenneth Gordon Dixcee
John Edge
Peter Rowland Fairbairn
Thomas William Griffin
James Arthur Murray Hill
Henry John Jailler
Robert Knapton
Franciszek Antoni Krefft
James Palmer
J. K. Rolland, B.Sc.
David Scott
Wilfred Thomas Roger Scott
Barend James Viljoen
Reginald Percy Viner

GRADUATE

Peter James Paxton

TRANSFER FROM ASSOCIATE TO MEMBER

Leslie George Bowley
William Bruce
Thomas Raymond Cain
Clement George Crook
Arpad Desiderius Farkas
George Edward Johnson
William Douglas McKay
Peter Reginald Prior

TRANSFER FROM GRADUATE TO ASSOCIATE

Leonard Teasdale

TRANSFER FROM STUDENT TO GRADUATE

Ejaz Ahmad

AUTUMN GOLF MEETING, 1949

Chigwell was again chosen as the venue for the Autumn Golf Meeting which took place on Friday, 21st October. The weather remained fine during the whole of the day, and twenty-two players participated. The morning Medal Competition was won by Mr. G. M. McGavin with a score of 75; he received a leather brief case. Mr. J. A. Goddard and Mr. J. White tied for second place with a score of 78; Mr. White won second prize on the best performance in the second half of the round, and Mr. Goddard therefore gained the third prize. They received a leather zipp case containing hair brushes and half a dozen golf balls respectively.

In the afternoon Bogey Greensome Competition, Messrs. J. A. Rhynas and J. White gained first place with a score of 3 down; and Messrs. G. M. McGavin and J. C. Edmiston second place with 4 down. As both Mr. McGavin and Mr. White had won prizes in the morning, they had to forego the afternoon prizes, and therefore Mr. Rhynas and Mr. Edmiston were awarded the two first prizes, which were chromium travelling alarm clocks. This brought Mr. E. F. J. Baugh and Mr. H. M. Gorringe, the third couple, into second place with a score of 5 down, and they received leather wallets.

During the tea interval Mr. Robertson presented the prizes, and the company present passed a very hearty vote of thanks to the Committee of Chigwell Golf Club and to the Secretary, Mr. Hartness, for the arrangements which had been so ably made for the meeting.