4.

TORSIONAL VIBRATIONS WITH SPECIAL REFERENCE TO DIESEL ENGINES.

Introduction.

In a previous paper of this series (Vol. IV, p. 53), the subject of torsional vibrations in geared turbine installations was discussed. As this subject is of especial importance in the design of I.C. engines it is proposed to discuss it hereunder from this view-point only.

The phenomenon of torsional vibration consists of an angular vibration of the shaft and its attached masses about a node or nodes causing a repeated twisting and untwisting of the shaft and may be caused by quite small forces applied at regular intervals.

Any system consisting of a number of rotating masses joined bya flexible shaft is liable to be subject to torsional oscillations should a periodic irregularity in the driving force or resistance be present.

Such is the case with all reciprocating engine drives, especially when the prime mover is of the Diesel type, due to the high explosion and compression pressures ; thus in a 6-cylinder 4-cycle engine the torque will vary three times per revolution from approximately 40 to 130 per cent. of its mean value. If the engine shaft and attached shafting were absolutely rigid in torsion the only effect of this variation would be to cause a slight periodic speed variation in the engine and shafting. One of the purposes of a fly wheel is to reduce this speed variation to a minimum value. Actually the shafting is elastic and will be thrown into a state of vibration which at certain speeds may become of considerable amplitude.

The effects of such oscillations are quite distinct from vibrations due to lack of engine balance as the latter will occur at all speeds increasing to a maximum at full speed whereas the torsional oscillations will cause vibration over certain definite ranges of speed and having a maximum value at the centres of these ranges.

The serious consequences of neglecting the existence of torsional vibration in the shafting of reciprocating engines has been shown in the past by many shaft failures.

The first investigation into the possibility of such fractures being due to such an oscillation appears to have been made by Bauer in 1900 while in 1902 papers were published by Grimbel and Frahm on this subject ; the latter not only proving their existence theoretically but taking actual records of several cases of torsional oscillations by a method similar to that employed at present.

In later years with the advent of large high-powered Diesel propelling machinery and fast-running Diesel generators the problem has become very serious due principally to the fluctuating turning moment and heavy masses met with in Diesel practice.

The numerous cases of failures of shafts due to this cause have resulted in the mathematical investigation of the problem having been considerably extended, while practical methods have been evolved to obviate as far as possible the laborious calculations which would otherwise be necessary. It should now be possible in the design stage to predict with some accuracy the probable speeds at which prejudicial oscillations may occur. It should also be possible to estimate with some degree of accuracy the probable mechanical effects on the system of any major vibration. Such effects may include not only shear stresses in the main shafting, but also violent oscillations through the camshaft drive, etc.

These preliminary calculations are of course subject to revision as the design proceeds, but form an effective method of checking the effect of any alteration in design.

In the Service, this problem is not so widely realised, due to the rarity of cases being met. The direct drive and single reduction geared turbines employed are not as a rule liable to any noticeable oscillations of this character, while the Diesel generators fitted in all except the later vessels are of sensibly uniform design in which the first major critical speed is well below the normal running speed and is not noticeable to the ordinary observer unless the engine is run below its correct revolutions.

In the case of submarines there have always been certain speeds at which vibration was more pronounced, but as a rule the size and length of the shaft has been sufficiently large compared to the power of the engine and the weight of the moving parts, to damp down the vibrations to a safe figure. In certain earlier types of submarine shaft failures occurred, while in others it has been necessary to avoid running over certain ranges of speed at which heavy vibrations occur, in order to avoid the undue wear and tear that would otherwise occur.

Definition of a Torsional Oscillation.

Consider, for example, the torsional oscillation of a simple uniform shaft. If the ends of the shaft are twisted in opposite directions to each other and released, the shaft will attempt to recover its position of zero stress, one end moving clockwise, the other anti-clockwise and the centre remaining still. In this way, when the position of zero stress is reached, the ends will have a velocity and the momentum will cause them to over shoot the mark and twist the shaft again opposite to the initial twist, with a slightly decreased amplitude.

The shaft will carry on untwisting and twisting with a gradually decreasing amplitude until the vibration dies away altogether. Such a vibration occurs at a certain definite frequency called a "natural frequency." The centre of the shaft which remains at rest during the vibration is called the "node" of the vibration. Similarly, if the ends were twisted in the same direction and the centre in an opposite direction the shaft would vibrate with two nodes, at another natural frequency of the shaft higher than that for the one node vibration. The nodes for this vibration would be at one quarter and three quarters the shaft length. A three noded vibration may be caused by twisting the shaft at each third of its length and so on for any number of nodes, the natural frequency being higher as the number of nodes increases though in practice from the point of view of the stresses to which they give rise vibrations with more than two nodes may generally be neglected. The number of nodes in the vibration is generally defined as the " mode " of vibration.

Cause of Torsional Oscillation.

If now a simple harmonic impulse, *i.e.*, a twisting force whose value varies as a sine curve as time elapses, acts at the end of the shaft with a frequency that synchronizes with a natural frequency of the shaft, a vibration of the corresponding number of nodes (corresponding " Mode ") will result—this is called a " synchronous vibration." If the frequency of the impulse is a little below or above synchronism the resulting vibration takes on the frequency of the impulse with an amplitude smaller than that which occurs at the synchronous frequency. This is called a " forced vibration."

If, in addition, the shaft is revolving uniformly, the torsional vibration will not be affected but will be superimposed on the normal revolution. It is then possible to define the vibration by its Mode (number of nodes) and its " order " where the " order " is the number of complete cycles of the vibration that occur in *one* revolution of the shaft.

Consider now a typical indicator card from one cylinder of a Diesel engine as shown in Fig. I. This gives rise to a twisting

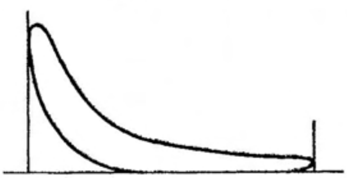


FIGURE I.

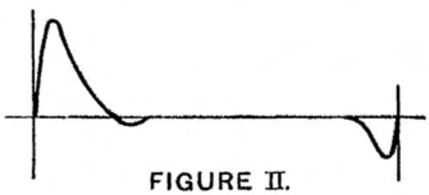


FIGURE II.

moment on the crankshaft as given by the T.M. diagram, Fig. II. this curve repeating itself for each cycle of the cylinder.

This curve has no direct mathematical formula but by the application of Fourier's theorem it can be replaced by a number of sine and cosine curves having 1, 2, 3, 4, etc. complete waves in a cycle (= 2 revolutions for a 4-cycle engine). Each of these curves is called an " harmonic component " of the original curve, the sum of all these curves being equal to the original curve, the determination of the value of these curves being known as harmonic analysis.

Fig. III illustrates the addition of the first five harmonics of a T.M. diagram. If then this turning moment was acting on the uniform shaft each of these harmonic components would stimulate a vibration of the shaft, the amplitude due to any given component varying with its closeness to synchronism with a natural frequency.

In the more complex case of a Diesel drive installation we have the engine and propeller shafting as an elastic system carrying heavy rotating masses such as the propeller, flywheel and cranks together with the moving parts of the engine. This system, taken as a whole, will act similarly to the plain uniform shaft and will have particular natural frequencies of torsional vibration, the values of which will depend on the magnitude of the masses and shafting and their disposition.

Each cylinder will be exerting a turning moment similar to that in Fig. II, the harmonic components of which will stimulate vibrations, the resulting vibration being the combination of the vibrations due to each cylinder.

Calculation of Frequency.—The problem of designing an installation which shall be free from objectionable vibration of this type devolves therefore into two principal stages :—

I. The calculation of the natural frequencies of the system as a whole (generally only those corresponding to the single and two noded modes need be considered).

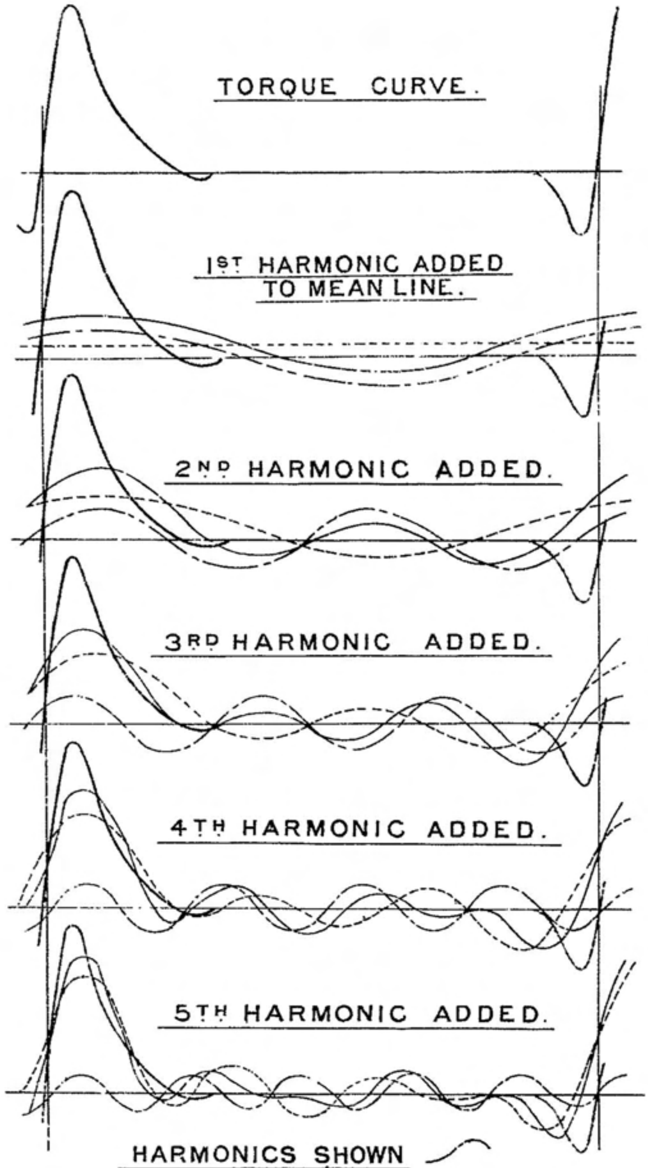
II. Estimation from the above, whether there exists a disturbing factor that is likely to cause torsional oscillation within the running range of the engine and whether after taking into account the damping influences at work the amplitude of such oscillation is likely to grow to a sufficient extent to lead to excessive stresses or irregular running.

The methods employed for calculating the natural frequencies will not be discussed here, but it may be said that they are based on the ordinary methods of applied mechanics and are somewhat lengthy and tedious. The principal point of interest for the purpose of this paper lies in the order of accuracy of the prediction so obtained. Given a length of simple shafting of known diameter with known masses at definite points the results would be absolutely accurate, but the average shaft system is more complex and entails a number of assumptions of which the accuracy is not known.

Thus the chief points of doubt met with are :—

(*a*) Incorrect data : the moments of inertia of complex masses such as armatures are very difficult to compute with any degree of accuracy nor is experimental verification easy to arrange.

(*b*) The torsional stiffness of a crankshaft as a whole is not readily calculable and will depend on the form of the webs, etc.



The Addition of the first five Harmonics in a typical twisting moment curve for the pressure forces in a 4-cycle Diesel-engine cylinder.

FIGURE III.

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(*c*) In the case of Diesel generators the axial plane in which the armature mass may be considered to be attached to the shafting system is not always determinable and will influence the calculated result considerably.

(*d*) A screw propeller in motion carries with it a certain amount of entrained water and the increase in the moment of inertia which must be made to allow for this is not calculable.

Nevertheless, it is possible in the case of an ordinary design to estimate the natural frequencies with reasonable accuracy and to predict with confidence whether alterations are required to assure freedom from objectionable vibrations or stresses.

Having obtained the natural frequencies of, say, the primary and secondary modes (single noded and two noded) of vibration, it is next necessary to discover whether any critical speeds may be expected within the running range and to what extent these are likely to be dangerous.

Determination of Critical Speeds.

The natural frequencies of vibration of the system being thus obtained, it is necessary to find at what shaft speeds the harmonic components of the torque will cause the shaft to vibrate with these frequencies. As was described previously the periods of the harmonic components were 1, 2, 3, 4 and so on per 2-shaft revolutions for a 4-cycle engine. If then one half the shaft speed divides into the natural frequency to give a whole number, the harmonic torque component corresponding to this number will recur at the same frequency as the natural, and will tend to set up a vibration of growing amplitude. This, of course, applies equally to each cylinder, each of which will, in turn, attempt to set up a vibration. If now we consider a case in which the order of the harmonic is the same or a multiple of the number of firing impulses in a revolution (*i.e.*, 3 for a 6-cylinder four-stroke engine), then the harmonic component of each cylinder will act upon the vibration at the same relative position of the vibration and the effect will be that each cylinder aids the vibration which will grow rapidly ; each cylinder will, in fact, pump into the system a certain amount of energy, the amount being proportional to the force acting and the distance through which it acts, namely, the magnitude of the harmonic torque component causing the vibration and the amplitude of the vibration at the particular cylinder. Such an oscillation is known as a Major critical speed : while, when the order is not a multiple of the number of cylinders, we get a Minor critical. In the latter case the cylinder forces do not act in unison, but due to the time interval between the start of their turning moment curves, the forces of some of the cylinders act in opposition to the rest.

The effect of each cylinder is proportional to its input of energy, which, as the harmonic torque component will be the same for each cylinder, will vary as the amplitude of vibration at the cylinder centre line. This amplitude is in turn proportional to the distance from the node, and will therefore vary from a maximum at the free end of the engine to a minimum at the end near the node. (See Figure 4) Knowing the crank angles and the order of firing, we can then sum up the effects of the various cylinders to give the

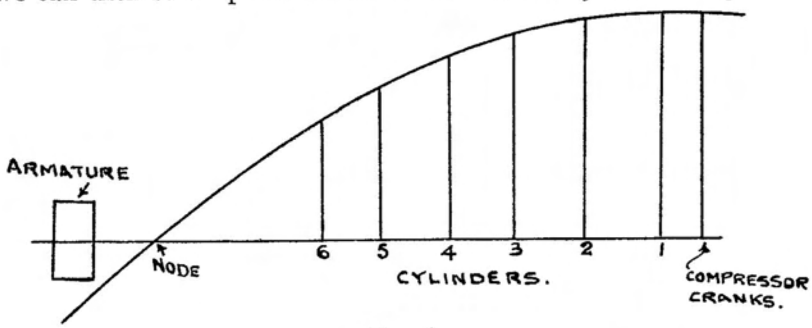

FIG. 4.

resultant effect and see to what extent the input of energy from the cylinders is cumulative. From this it is possible to estimate whether the vibration is liable to be excessive. As a rule, minor critical speeds are not dangerous, and can generally be neglected, but cases occur, especially at full speed or with a low order, where they are dangerous, and this possibility must not be overlooked in the design stage.

The effect of each critical extends over a range either side of the peak, and generally necessitates the avoidance of a speed extending several revolutions either side of the actual critical speed of a dangerous torsional vibration.

Effect of Damping.—Having illustrated how the vibration occurs, it is now necessary to investigate the effect of damping.

The effect of damping on a natural oscillation is to lower the frequency slightly, and to damp down gradually the oscillation which would otherwise continue indefinitely, once it has been started. The effect of damping on the *frequency* of oscillation is small, and can be neglected, being inside the limits of error of the calculations. The other effects of damping are, however, very important. Due to the viscous friction in the bearings, &c., and the internal molecular friction of the metal in the shaft (known as elastic hysteresis), any vibration once started would rapidly die away unless further energy is supplied to maintain the vibration. Similarly a propeller vibrating torsionally will carry with it a considerable quantity of water, and the resultant eddies will act as a strong damping factor.

The building up and maintenance of any torsional vibration depends, therefore, on energy being put into the vibrating system, and, so long as the cylinders are putting in more than is being absorbed by damping, the vibration will increase in amplitude. This growth will continue until finally the energy supplied exactly balances that absorbed by damping, whereafter the amplitude of the vibration will remain unchanged as long as the engine runs at the particular speed considered. The time taken to build up to this peak is generally sufficient, however, to allow the engine to be run quickly through a dangerous critical without any damage.

Estimate of Amplitude of vibration and Stresses caused thereby.

The estimation of the value of the amplitude just referred to, and thence that of the resultant fluctuating torque in the shaft involves the determination of this balance of energy supply and absorption. First the value of the harmonic component of the turning moment corresponding to the vibration is obtained by harmonic analysis of the turning moment diagram at the speed in question. The relative amplitude of vibration at each cylinder is calculated by assuming a known vibration at one free end, say 1 radian; this will diminish from the chosen value at the free end to zero at the node, so that the amplitude at each cylinder will have a definite relative value.

From this we can obtain in terms of the amplitude of vibration at the free end, the amount of energy put into the system by the cylinders; this is proportional to the magnitude of the force acting, and to the distance over which it acts, *i.e.*, to the value of the harmonic torque component and the amplitude of vibration. The energy put into the system having thus been estimated, we can obtain another estimate of that dissipated in viscous and hysteresis damping of which the latter is usually the most important. The hysteresis of steel has been determined experimentally, and and estimate of the effect of this damping can then be made by combining the experimental data with previous results. The energy absorbed by damping is thus obtained in terms of the amplitude of vibration: equating these two energies, the value of the actual amplitude is obtained.

When calculating the natural frequency of vibration a curve is obtained on the shaft line as base, the ordinates representing

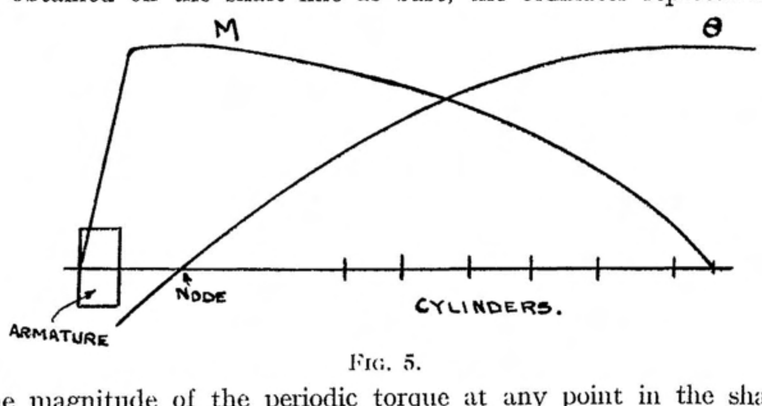

FIG. 5.

the magnitude of the periodic torque at any point in the shaft necessary to maintain a vibration having the assumed amplitude of, say, 1 radian at one free end. (See Fig. 5.) Knowing now the actual amplitude, we can therefore obtain the value of the periodic torque at all points in the system, and thus discover the maximum stress that is likely to occur.

The stress so obtained is an alternating shear stress for which the fatigue limit is about 25 per cent. of the ultimate tensile stress. As the presence of square shoulders, V notches and other discontinuities such as oil holes, &c., further reduces the endurance limit by about 50 per cent., the maximum allowable torsional vibration stress should be kept as low as possible, a reasonable limit being about 3 tons/in.^{2} for normal 30 tons/in.^{2} steel.

In propelling installations the amplitude at the propeller of the 1-node vibration is usually large in relation to the amplitude at the engine end, and the damping effect of the propeller is generally large enough to keep the amplitude small. For the 2-node vibration, however, this is not the case, and the shafting is likely to be subjected to severe stresses if run at or near the critical speeds for this mode of vibration.

In the case of Diesel generators with a relatively flexible length of shaft between the flywheel and armature, the major critical speeds of both the 1-node and 2-node vibrations may occur near the designed speed, rendering it difficult to select a running speed which would not entail dangerous stresses in the shafting and/or armature attachment. Particular attention should be given to this possibility in the design stage. By increasing the diameter of the journals at the flywheel and of the crankshaft, and attaching the armature close to the flywheel, it is possible in such cases to raise the natural frequency sufficiently to put the major critical speeds well above the designed speed.

Calculations on these lines on a submarine propelling installation indicated that as originally designed a major 4th order 2-noded vibration (8-cylinder engine) could be expected at 434 revs./min., and it was estimated that at full speed of 400 revs./min. there would be a periodic stress of between 4·5 and 5 tons per square inch superimposed on the normal stress due to torque. The nett effect was a stress fluctuating rapidly between + 5·5 and − 4·5 tons per square inch, which would necessarily cause fatigue at couplings, &c.

Alterations were, therefore, made to the shaft line in order to increase the shaft stiffness, especially in the vicinity of the nodes where the maximum stress occurs. This was done by increasing the diameters of the main motor armature shaft and of that between this point and the engine from 12 in. to 14 in. By this means the 4th order vibration was raised to 477 revs./min., and the stress range at 400 revs./min. (now due to 4½th and 5th order vibrations) reduced to from + 3·4 to − 2·8 tons per square inch which can be considered safe.

The stresses arising in this case are, however, comparatively low. In some cases stresses as high as ± 13 tons/in.^{2} have occurred at the critical speeds of certain generating sets, and have ended with failure of the shaft in spite of careful avoidance of the critical speed.

Although excessive stresses may not arise, the amplitudes of vibration at the position of the camshaft driving gear and at the cylinders may be such as to render it inadvisable to run at the critical speed in question in order to obviate undue noise and vibration generally.

In order to check these calculations an instrument known as a "Geiger" Torsiograph is extensively used. This consists of two *concentric* wheels, the external one, of small mass driven by a belt from a pulley on the engine shaft, and the other of large mass driven by the first wheel by means of a flexible spiral spring. Any small and rapid fluctuations (such as torsional vibration) in the speed of the driving pulley will be transmitted to the light wheel but will not affect the heavy one owing to its inertia. As a result the light pulley will vibrate relatively to the heavy internal wheel in proportion to the torsional vibration of the shaft which drives it. This relative movement is recorded through suitable mechanism, on a strip of moving paper. From these "Geiger" records the correctness of the calculations can be checked and also the dangerous range over which any vibration extends, obtained.

Effect of Firing Order and Engine Tune.

It is to be noted that at a minor critical the resultant amplitude of vibration may be dependent on the firing order of the cylinders.

In the majority of 6-cylinder S/M propelling installations, it is found that the 6th order 2-node critical occurs within or just above the running range of speed and this being a major order the effect of the cylinders is cumulative whatever the firing order.

Cases have been met, however, with fast-running 6-cylinder engines, in which the 4½th order harmonic, occurring very near the maximum speed, has caused serious vibration and this vibration has been considerably reduced by a suitable change of firing order: this can be briefly explained, as follows.

In a 6-cylinder engine with a firing order 1, 4, 2, 6, 3, 5, numbering the cylinders from the forward end, the effect of the 4½th order harmonic is such that cylinders 1, 2 and 3 are putting energy into the system in direct opposition to cylinders 4, 5 and 6. (See Fig. 6.)

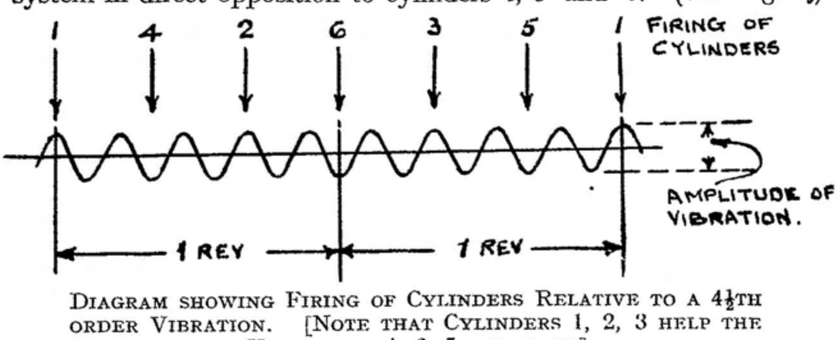

DIAGRAM SHOWING FIRING OF CYLINDERS RELATIVE TO A 4½TH ORDER VIBRATION. [NOTE THAT CYLINDERS 1, 2, 3 HELP THE VIBRATION; 4, 6, 5 OPPOSE IT.]

FIG. 6.

As the node is generally close abaft the after cylinder, the amplitude of vibration will be least at No. 6 cylinder and increase to a maximum at No. 1. The harmonic components of each cylinder should be equal and the nett input of energy is, therefore, proportional to the difference (sum of amplitudes of 1, 2 and 3—sum of amplitudes of 4, 5 and 6).

If now we alter the firing order to 1, 3, 5, 6, 4, 2, we put cylinders 1, 4 and 5 in opposition to 2, 3 and 6 and the difference in the sum of the amplitudes is considerably decreased with a corresponding decrease in the amplitude of the vibration.

It is seen, however, that this depends on the harmonic components being equal. This is dependent on the cylinders contributing energy equally and having similar turning moment diagrams, *i.e.*, the engine must be in tune. Should the engine be out of tune the position is not so clear, but any preponderance of cylinders No. 1, 4 and 5 over cylinders No. 2, 3 and 6 will result in an increased vibration, so that it would appear very desirable to keep an engine which is known to have a $4\frac{1}{2}$th order critical or similarly any minor critical in good tune to avoid increase in torsional stresses.

Elimination of Torsional Oscillations.

Any engine, of which the driving force or resistance has a periodic irregularity, will have certain speeds at which torsional vibrations occur, and it is the aim of the designer either to raise these outside the running range or, if this is not possible to keep the stresses below a certain limit so that the engine may run at the critical speed without damage or undue vibration.

The usual method of raising the natural frequency is by increasing the shaft stiffness, especially near the nodes and by decreasing the rotating and reciprocating masses. If this is not sufficient the frequency should be raised or lowered to the most favourable position and the shaft stiffened at the points of maximum stress to reduce the stresses to a safe figure. If necessary a suitable firing order must be used to keep the minor criticals from becoming of importance.

Generally, it should be possible to avoid any troublesome criticals in the designs stage, but the case of an engine that has been built without due regard to this factor is more difficult an elimination of the criticals involves either very extensive alterations to the crankshaft and shafting or a reduction in the revolutions with corresponding reduction in output, which is usually unacceptable.

In some cases it is possible to make use of a damping device whereby the engine can run through an otherwise dangerous critical —the revolutions over a certain range either side of the critical being run through quickly. The best known of this type is the Lanchester damper (much used in motor cars), which consists of a flywheel connected to the free end of the shaft through friction surfaces. When the shaft vibrates the friction surfaces slip, thereby absorbing much of the energy of vibration and preventing the vibration from becoming too large.

For large Diesels this has so far only proved satisfactory for enabling the engine to run through and not to run at the critical for any period ; frequent adjustment is also necessary.

Moreover, the heat generated in such a device may become excessive if it is desired to absorb the energy of a major vibration.

The general question of damping has yet to be investigated, and in consequence it is better to design a large engine with a view to avoiding the major criticals rather than rely on any damping device. This is a comparatively simple matter when dealing with generating sets which are to run at constant speed, but it is often found necessary, in the case of high-speed propelling machinery, to allow one or more of the criticals to remain within the running range and to avoid these speeds on service. A damper is then useful to reduce stresses while running through the bad ranges of speed.